PHYS 111
1^{ST} semester 1439-1440

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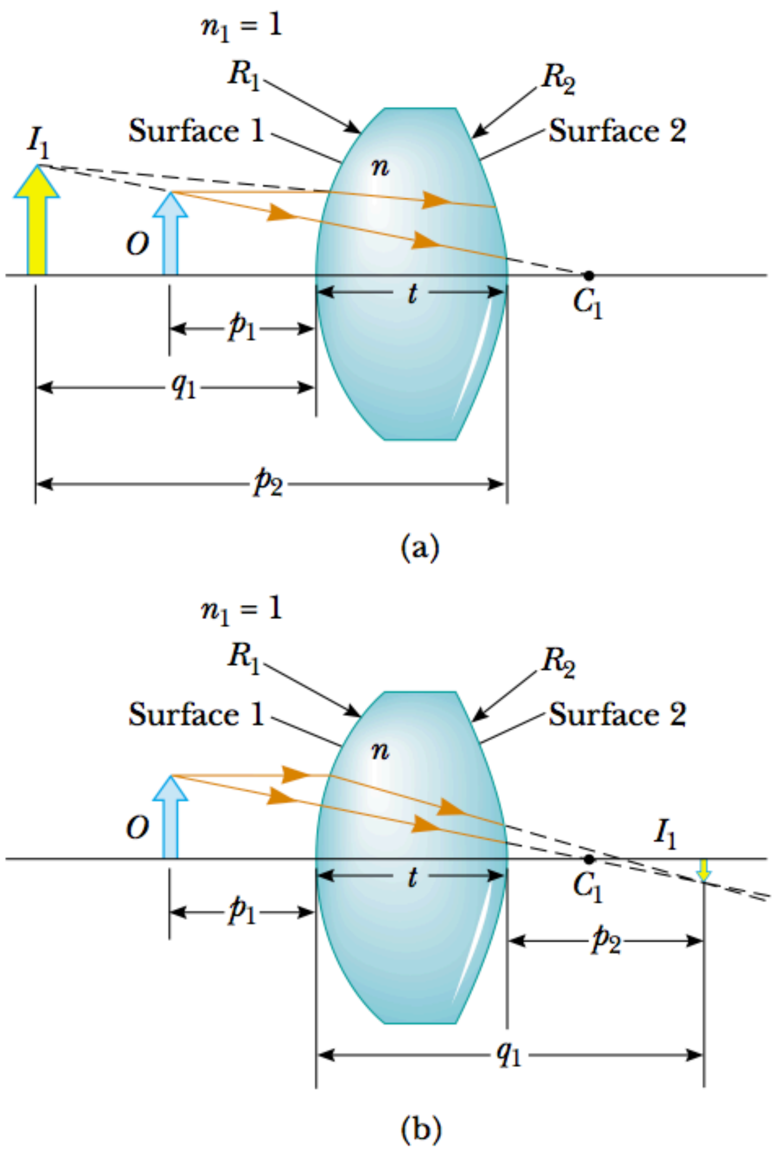
Lecture 19

Chapter 36
Image Formation

36.4 Thin Lenses

- Lenses are commonly used to form images by refraction in optical instruments, such as cameras, telescopes, and microscopes.
- Light passing through a lens experiences refraction at two surfaces.
- The image formed by one refracting surface serves as the object for the second surface.
- Consider a lens having an index of refraction *n* and two spherical surfaces with radii of curvature R_1 and R_2,
- An object is placed at point *O* at a distance p_1 in front of surface 1.



36.4 Thin Lenses

- For a thin lens, the expression relates the image distance *q* of the image formed by a thin lens to the object distance *p* and to the lens properties (index of refraction and radii of curvature).

$$\frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

- It is valid only for paraxial rays and only when the lens thickness is much less than R_1 and R_2.
- The **focal length** *f* of a thin lens is the image distance that corresponds to an infinite object distance,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

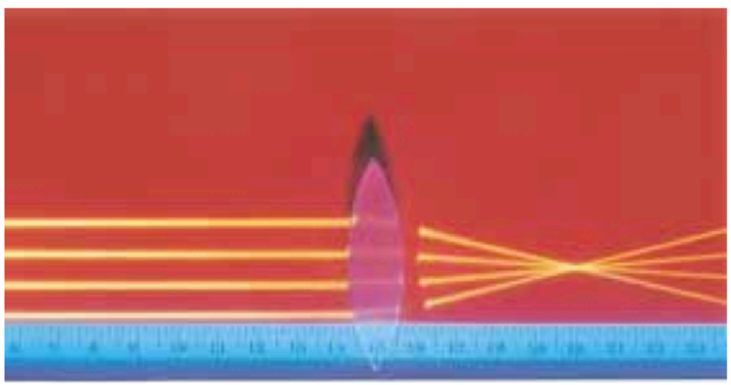
Lens makers' equation

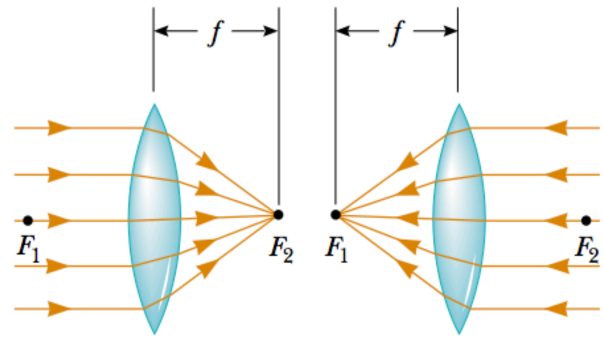
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Thin lenses equation

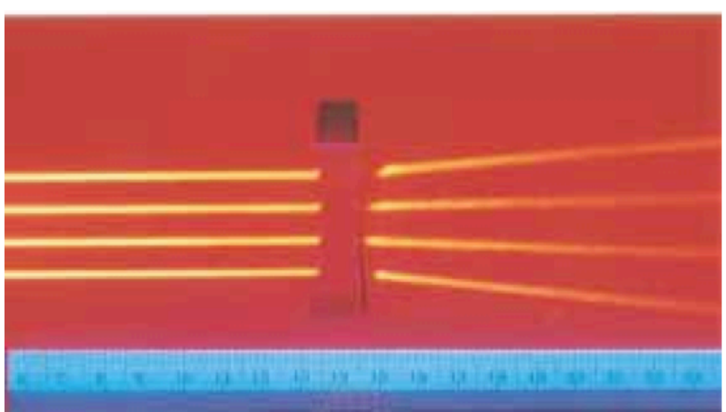
36.4 Thin Lenses

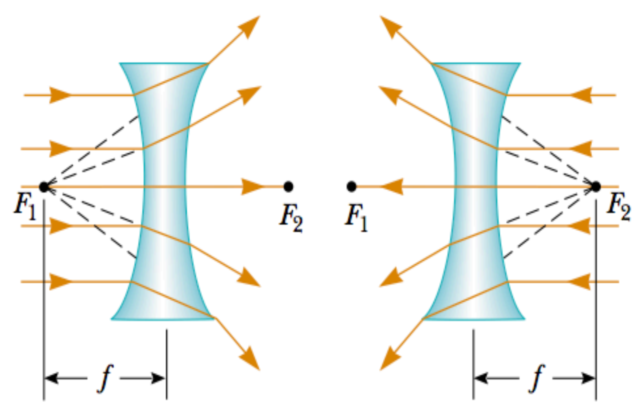
- Because light can travel in either direction through a lens, each lens has **two** focal points, one for light rays passing through in one direction and one for rays passing through in the other direction.
- For a **biconvex** lens (two convex surfaces, resulting in a converging lens)





- For a **biconcave** lens (two concave surfaces, resulting in a diverging lens).





36.4 Thin Lenses

- To obtain the signs of *p* and *q*,

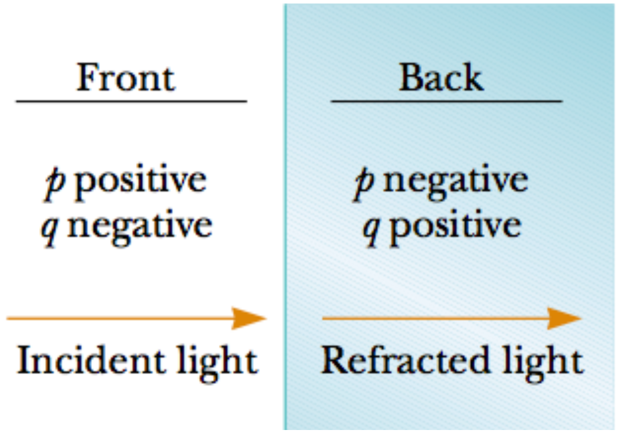


Table 36.3

Sign Conventions for Thin Lenses

Quantity	Positive When	Negative When
Object location (p)	Object is in front of lens (real object)	Object is in back of lens (virtual object)
Image location (q)	Image is in back of lens (real image)	Image is in front of lens (virtual image)
Image height (h')	Image is upright	Image is inverted
R_1 and R_2	Center of curvature is in back of lens	Center of curvature is in front of lens
Focal length (f)	Converging lens	Diverging lens

36.4 Thin Lenses

- **Magnification of Images**

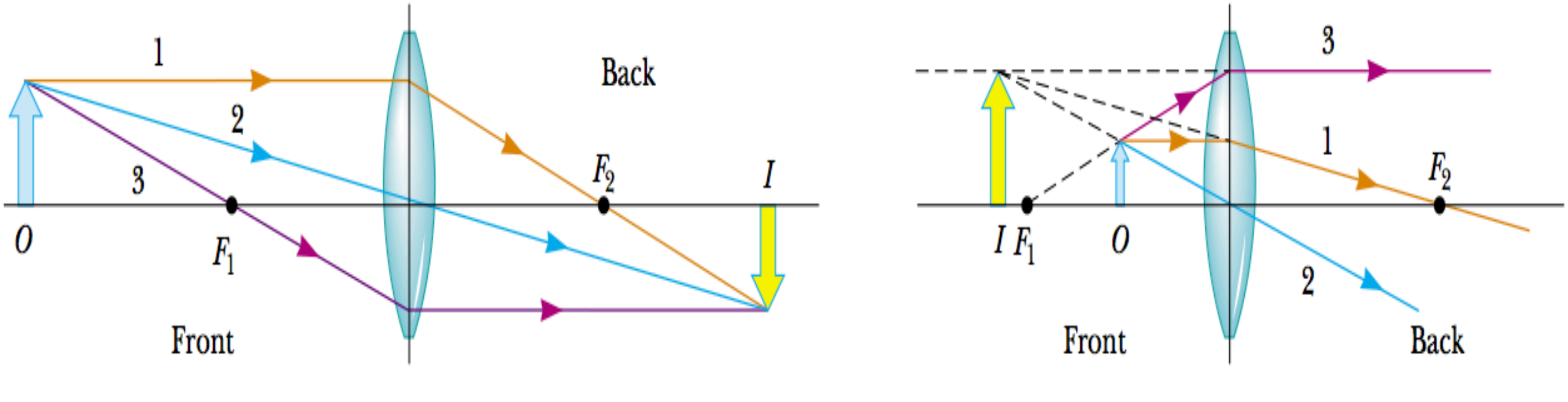
$$M = \frac{h'}{h} = -\frac{q}{p}$$

- When *M* is **positive**, the image is upright and on the same side of the lens as the object.
- When *M* is **negative**, the image is inverted and on the side of the lens opposite the object.

Ray Diagrams for Thin Lenses

- To locate the image of a ***converging*** lens the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray passes through the focal point on the back side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn through the focal point on the front side of the lens (or as if coming from the focal point if $p < f$) and emerges from the lens parallel to the principal axis.



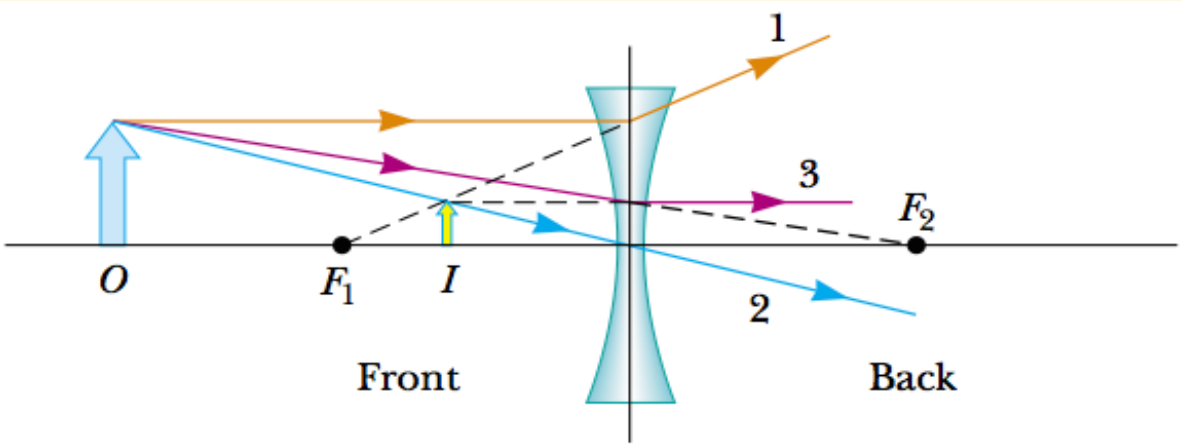
(a)

(b)

Ray Diagrams for Thin Lenses

- To locate the image of a ***diverging*** lens, the following three rays are drawn from the top of the object:

- Ray 1 is drawn parallel to the principal axis. After being refracted by the lens, this ray emerges directed away from the focal point on the front side of the lens.
- Ray 2 is drawn through the center of the lens and continues in a straight line.
- Ray 3 is drawn in the direction toward the focal point on the back side of the lens and emerges from the lens parallel to the principal axis.



(c)

Example 36.9 Images Formed by a Converging Lens

Interactive

A converging lens of focal length 10.0 cm forms images of objects placed

(A) 30.0 cm,

(B) 10.0 cm, and

(C) 5.00 cm from the lens.

In each case, construct a ray diagram, find the image distance and describe the image.

Solution

(A) First we construct a ray diagram as shown in Figure 36.30a. The diagram shows that we should expect a real, inverted, smaller image to be formed on the back side of the lens. The thin lens equation, Equation 36.16, can be used to find the image distance:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

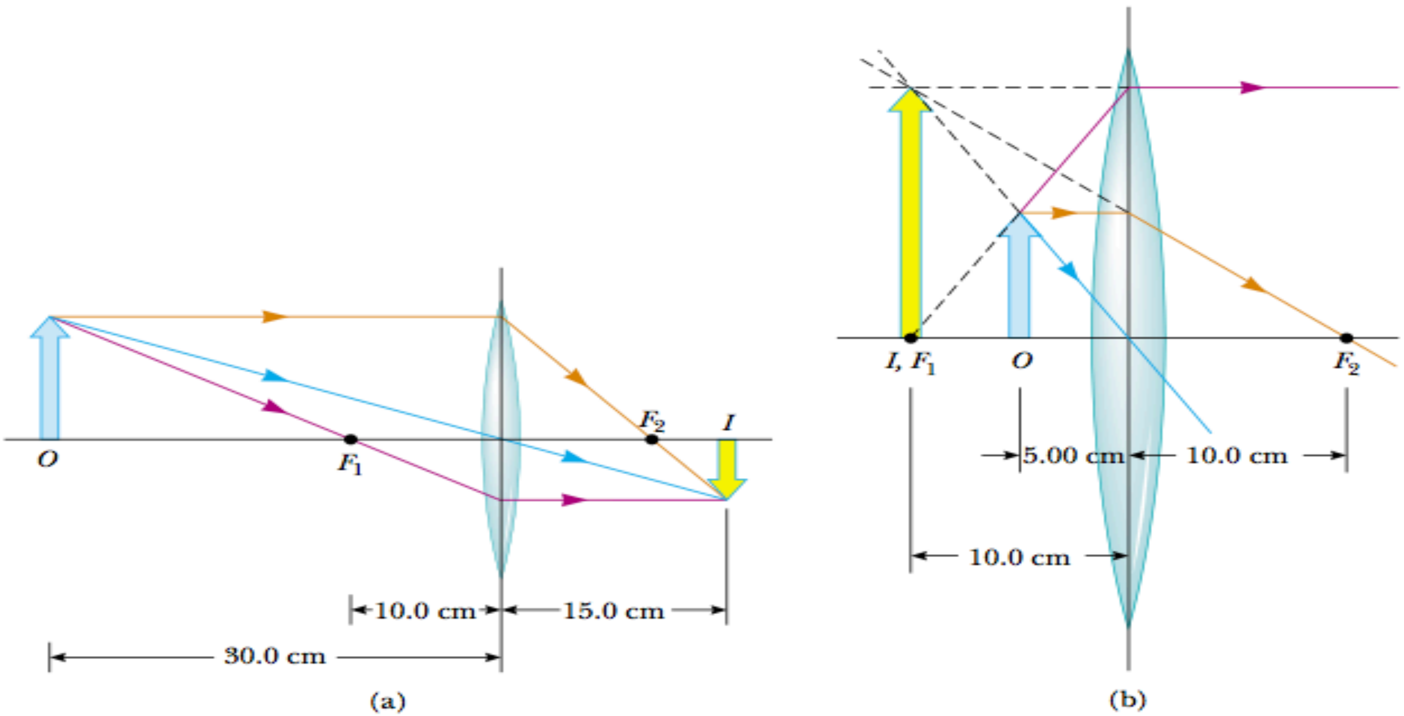


Figure 36.30 (Example 36.9) An image is formed by a converging lens. (a) The object is farther from the lens than the focal point. (b) The object is closer to the lens than the focal point.

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = +15.0 \text{ cm}$$

The positive sign for the image distance tells us that the image is indeed real and on the back side of the lens. The magnification of the image is

$$M = -\frac{q}{p} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = -0.500$$

Thus, the image is reduced in height by one half, and the negative sign for M tells us that the image is inverted.

(B) No calculation is necessary for this case because we know that, when the object is placed at the focal point, the image is formed at infinity. This is readily verified by substituting $p = 10.0$ cm into the thin lens equation.

(C) We now move inside the focal point. The ray diagram in Figure 36.30b shows that in this case the lens acts as a magnifying glass; that is, the image is magnified, upright, on the same side of the lens as the object, and virtual. Because the object distance is 5.00 cm, the thin lens equation gives

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{10.0 \text{ cm}}$$

$$q = -10.0 \text{ cm}$$

and the magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-10.0 \text{ cm}}{5.00 \text{ cm}}\right) = +2.00$$

The negative image distance tells us that the image is virtual and formed on the side of the lens from which the light is incident, the front side. The image is enlarged, and the positive sign for M tells us that the image is upright.

Example 36.10 The Case of a Diverging Lens Interactive

Repeat Example 36.9 for a *diverging* lens of focal length 10.0 cm.

Solution

(A) We begin by constructing a ray diagram as in Figure 36.31a taking the object distance to be 30.0 cm. The diagram shows that we should expect an image that is virtual, smaller than the object, and upright. Let us now apply the thin lens equation with $p = 30.0$ cm:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{1}{-10.0 \text{ cm}}$$

$$q = \boxed{-7.50 \text{ cm}}$$

The magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-7.50 \text{ cm}}{30.0 \text{ cm}}\right) = \boxed{+0.250}$$

This result confirms that the image is virtual, smaller than the object, and upright.

(B) When the object is at the focal point, the ray diagram appears as in Figure 36.31b. In the thin lens equation, using $p = 10.0$ cm, we have

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{q} = \frac{1}{-10.0 \text{ cm}}$$

$$q = \boxed{-5.00 \text{ cm}}$$

The magnification of the image is

$$M = -\frac{q}{p} = -\left(\frac{-5.00 \text{ cm}}{10.0 \text{ cm}}\right) = \boxed{+0.500}$$

Notice the difference between this situation and that for a converging lens. For a diverging lens, an object at the focal point does not produce an image infinitely far away.

(C) When the object is inside the focal point, at $p = 5.00$ cm, the ray diagram in Figure 36.31c shows that we expect a virtual image that is smaller than the object and upright. In

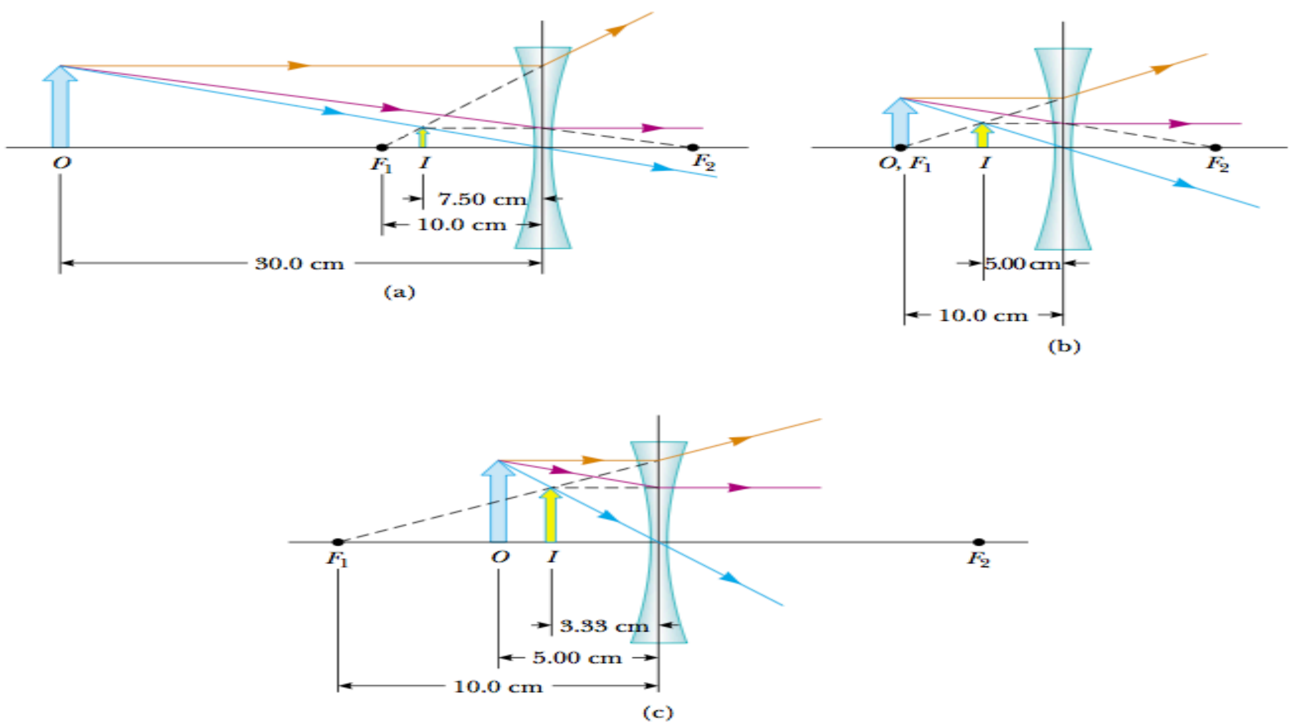


Figure 36.31 (Example 36.10) An image is formed by a diverging lens. (a) The object is farther from the lens than the focal point. (b) The object is at the focal point. (c) The object is closer to the lens than the focal point.

this case, the thin lens equation gives

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q} = \frac{1}{-10.0 \text{ cm}}$$

$$q = -3.33 \text{ cm}$$

and the magnification of the image is

$$M = -\left(\frac{-3.33 \text{ cm}}{5.00 \text{ cm}}\right) = +0.667$$

This confirms that the image is virtual, smaller than the object, and upright.

Investigate the image formed for various object positions and lens focal lengths at the Interactive Worked Example link at **http://www.pse6.com.**

Example 36.11 A Lens Under Water

A converging glass lens ($n = 1.52$) has a focal length of 40.0 cm in air. Find its focal length when it is immersed in water, which has an index of refraction of 1.33.

Solution We can use the lens makers' equation (Eq. 36.15) in both cases, noting that R_1 and R_2 remain the same in air and water:

$$\frac{1}{f_{\text{air}}} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_{\text{water}}} = (n'-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n' is the ratio of the index of refraction of glass to that of water: $n' = 1.52/1.33 = 1.14$. Dividing the first equation by the second gives

$$\frac{f_{\text{water}}}{f_{\text{air}}} = \frac{n-1}{n'-1} = \frac{1.52-1}{1.14-1} = 3.71$$

Because $f_{\text{air}} = 40.0$ cm, we find that

$$f_{\text{water}} = 3.71 f_{\text{air}} = 3.71(40.0 \text{ cm}) = 148 \text{ cm}$$

The focal length of any lens is increased by a factor $(n-1)/(n'-1)$ when the lens is immersed in a fluid, where n' is the ratio of the index of refraction n of the lens material to that of the fluid.

Combination of Thin Lenses

- If **two** thin lenses are used to form an image,
  - First, the image formed by the first lens is located as if the second lens were not present.
  - Then a ray diagram is drawn for the second lens, with the image formed by the first lens now serving as the object for the second lens. The second image formed is the final image of the system.
  - If the image formed by the first lens lies on the back side of the second lens, then that image is treated as a **virtual object** for the second lens (that is, in the thin lens equation, *p* is negative).

- **Focal length for a combination of two thin lenses in contact**

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Example 36.12 Where Is the Final Image?

Interactive

Two thin converging lenses of focal lengths $f_1 = 10.0$ cm and $f_2 = 20.0$ cm are separated by 20.0 cm, as illustrated in Figure 36.32a. An object is placed 30.0 cm to the left of lens 1. Find the position and the magnification of the final image.

Solution Conceptualize by imagining light rays passing through the first lens and forming a real image (because $p > f$) in the absence of the second lens. Figure 36.32b shows these light rays forming the inverted image I_1. Once the light rays converge to the image point, they do not stop. They continue through the image point and interact with the second lens. The rays leaving the image point behave in the same way as the rays leaving an object. Thus, the image of the first lens serves as the object of the second lens. We categorize this problem as one in which we apply the thin lens equation, but in stepwise fashion to the two lenses.

To analyze the problem, we first draw a ray diagram (Figure 36.32b) showing where the image from the first lens falls and how it acts as the object for the second lens. The location of the image formed by lens 1 is found from the thin lens equation:

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$$

$$\frac{1}{30.0 \text{ cm}} + \frac{1}{q_1} = \frac{1}{10.0 \text{ cm}}$$

$$q_1 = +15.0 \text{ cm}$$

The magnification of this image is

$$M_1 = -\frac{q_1}{p_1} = -\frac{15.0 \text{ cm}}{30.0 \text{ cm}} = -0.500$$

The image formed by this lens acts as the object for the second lens. Thus, the object distance for the second lens is 20.0 cm − 15.0 cm = 5.00 cm. We again apply the thin lens equation to find the location of the final image:

$$\frac{1}{5.00 \text{ cm}} + \frac{1}{q_2} = \frac{1}{20.0 \text{ cm}}$$

$$q_2 = -6.67 \text{ cm}$$

The magnification of the second image is

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-6.67 \text{ cm})}{5.00 \text{ cm}} = +1.33$$

Thus, the overall magnification of the system is

$$M = M_1 M_2 = (-0.500)(1.33) = -0.667$$

To finalize the problem, note that the negative sign on the overall magnification indicates that the final image is inverted with respect to the initial object. The fact that the absolute value of the magnification is less than one tells us that the final image is smaller than the object. The fact that q_2 is negative tells us that the final image is on the front, or left, side of lens 2. All of these conclusions are consistent with the ray diagram in Figure 36.32b.

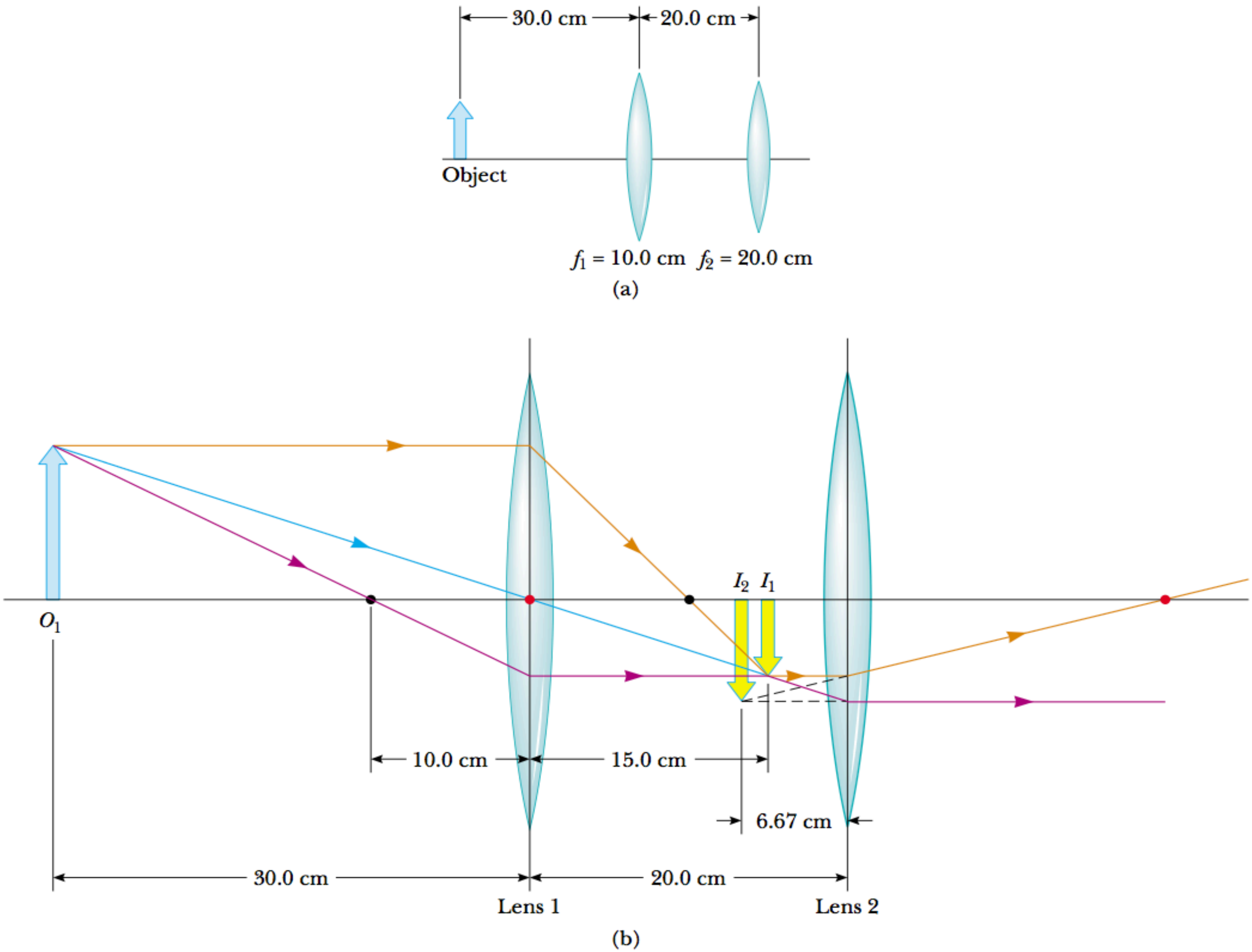


Figure 36.32 (Example 36.12) (a) A combination of two converging lenses. (b) The ray diagram showing the location of the final image due to the combination of lenses. The black dots are the focal points of lens 1 while the red dots are the focal points of lens 2.