PHYS 111
1ST semester 1439-1440

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Lecture 14

Chapter 27
Current and Resistance

SUMMARY

The **electric current** I in a conductor is defined as

$$I \equiv \frac{dQ}{dt} \tag{27.2}$$

where dQ is the charge that passes through a cross section of the conductor in a time interval dt. The SI unit of current is the **ampere** (A), where 1 A = 1 C/s.

The average current in a conductor is related to the motion of the charge carriers through the relationship

$$I_{\text{av}} = nqv_d A \tag{27.4}$$

where n is the density of charge carriers, q is the charge on each carrier, v_d is the drift speed, and A is the cross-sectional area of the conductor.

The magnitude of the **current density** J in a conductor is the current per unit area:

$$J \equiv \frac{I}{A} = nqv_d \tag{27.5}$$

The current density in an ohmic conductor is proportional to the electric field according to the expression

$$\mathbf{J} = \sigma \mathbf{E} \tag{27.7}$$

The proportionality constant σ is called the **conductivity** of the material of which the conductor is made. The inverse of σ is known as **resistivity** ρ (that is, $\rho = 1/\sigma$). Equation 27.7 is known as **Ohm's law,** and a material is said to obey this law if the ratio of its current density **J** to its applied electric field **E** is a constant that is independent of the applied field.

The **resistance** R of a conductor is defined as

$$R \equiv \frac{\Delta V}{I} \tag{27.8}$$

where ΔV is the potential difference across it, and I is the current it carries.

The SI unit of resistance is volts per ampere, which is defined to be 1 **ohm** (Ω); that is, $1\ \Omega = 1\ \text{V/A}$. If the resistance is independent of the applied potential difference, the conductor obeys Ohm's law.

For a uniform block of material of cross sectional area A and length ℓ, the resistance over the length ℓ is

$$R = \rho \frac{\ell}{A} \tag{27.11}$$

where ρ is the resistivity of the material.

In a classical model of electrical conduction in metals, the electrons are treated as molecules of a gas. In the absence of an electric field, the average velocity of the electrons is zero. When an electric field is applied, the electrons move (on the average) with a **drift velocity** $\mathbf{v}_d$ that is opposite the electric field and given by the expression

$$\mathbf{v}_d = \frac{q\mathbf{E}}{m_e} \tau \tag{27.14}$$

where τ is the average time interval between electron–atom collisions, m_e is the mass of the electron, and q is its charge. According to this model, the resistivity of the metal is

$$\rho = \frac{m_e}{nq^2\tau} \tag{27.17}$$

where n is the number of free electrons per unit volume.

The resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)] \tag{27.19}$$

where α is the **temperature coefficient of resistivity** and ρ_0 is the resistivity at some reference temperature T_0.

If a potential difference ΔV is maintained across a circuit element, the **power,** or rate at which energy is supplied to the element, is

$$\mathcal{P} = I\,\Delta V \tag{27.22}$$

Because the potential difference across a resistor is given by $\Delta V = IR$, we can express the power delivered to a resistor in the form

$$\mathcal{P} = I^2R = \frac{(\Delta V)^2}{R} \tag{27.23}$$

The energy delivered to a resistor by electrical transmission appears in the form of internal energy in the resistor.

Chapter 28
Direct Current Circuits

28.2 Resistors in Series

- In a series connection, if an amount of charge Q exits resistor R_1, charge Q must also enter the second resistor R_2.

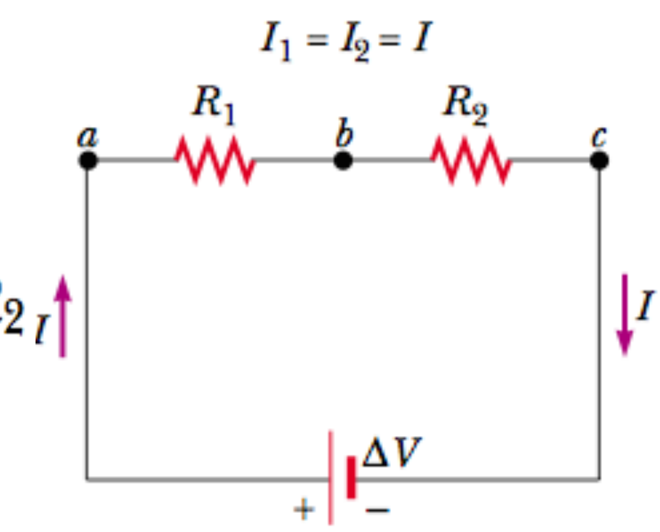
> for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.

- The potential difference applied across the series combination of resistors will divide between the resistors.

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

- The equivalent resistance

$$\Delta V = IR_{eq} = I(R_1 + R_2) \longrightarrow R_{eq} = R_1 + R_2$$



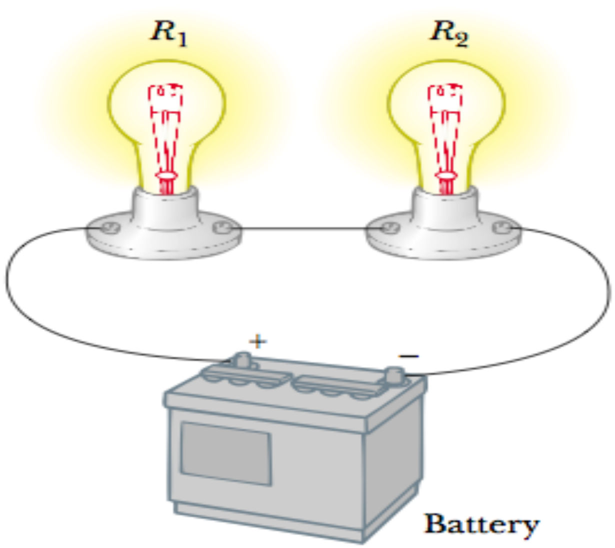
28.2 Resistors in Series

- The **equivalent resistance** of three or more resistors connected in **series** is

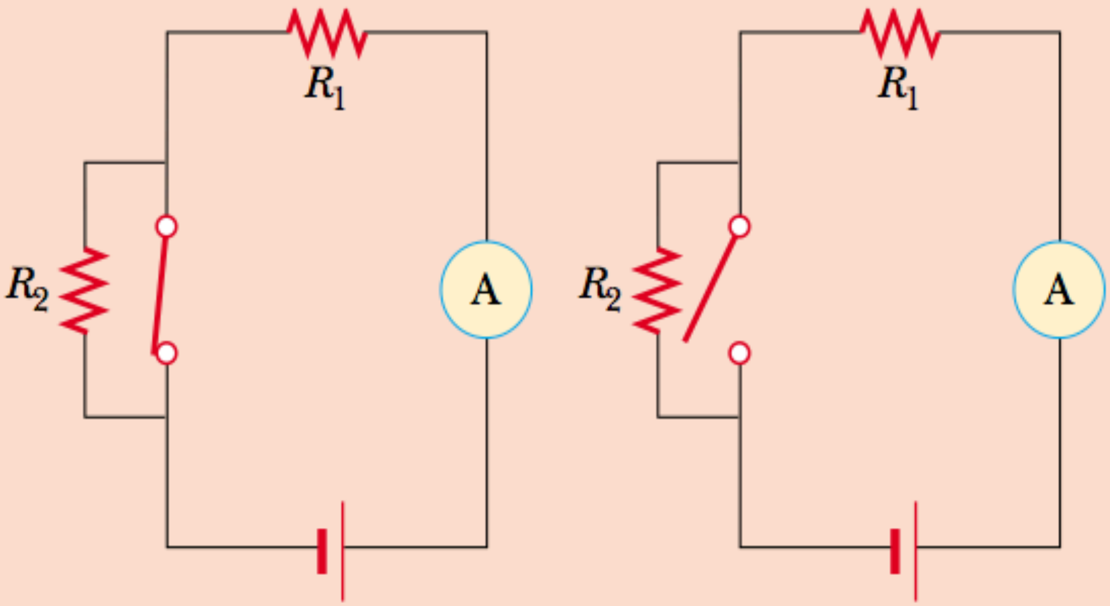
$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

- The equivalent resistance of a **series** connection of resistors is the numerical **sum** of the individual resistances and is always **greater** than any individual resistance.

- Note that if the filament of one lightbulb were to fail, the circuit would no longer be complete (resulting in an open-circuit condition) and the second bulb would also go out.



Quick Quiz 28.4 With the switch in the circuit of Figure 28.5 closed (left), there is no current in R_2, because the current has an alternate zero-resistance path through the switch. There is current in R_1 and this current is measured with the ammeter (a device for measuring current) at the right side of the circuit. If the switch is opened (Fig. 28.5, right), there is current in R_2. What happens to the reading on the ammeter when the switch is opened? (a) the reading goes up; (b) the reading goes down; (c) the reading does not change.



Switch closed   Switch open

28.2 Resistors in parallel

- When charges reach point *a*, called a *junction,* they split into two parts, with some going through R_1 and the rest going through R_2.
- A **junction** is any point in a circuit where a current can split.
- The current *I* that enters point *a* must equal the total current leaving that point:

$$I = I_1 + I_2$$

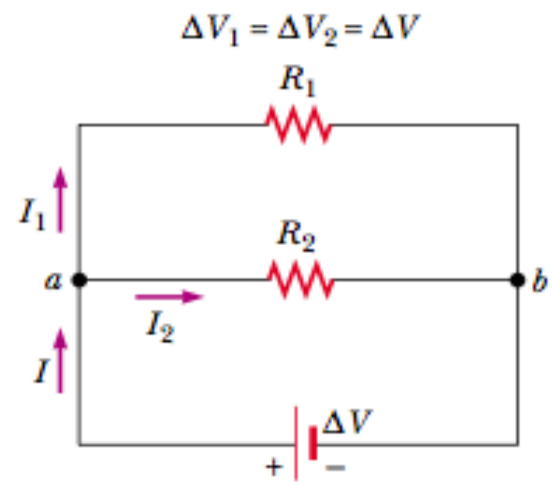
when resistors are connected in parallel, the potential differences across the resistors is the same.

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{eq}}$$

- The equivalent resistance of two resistors in parallel is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$



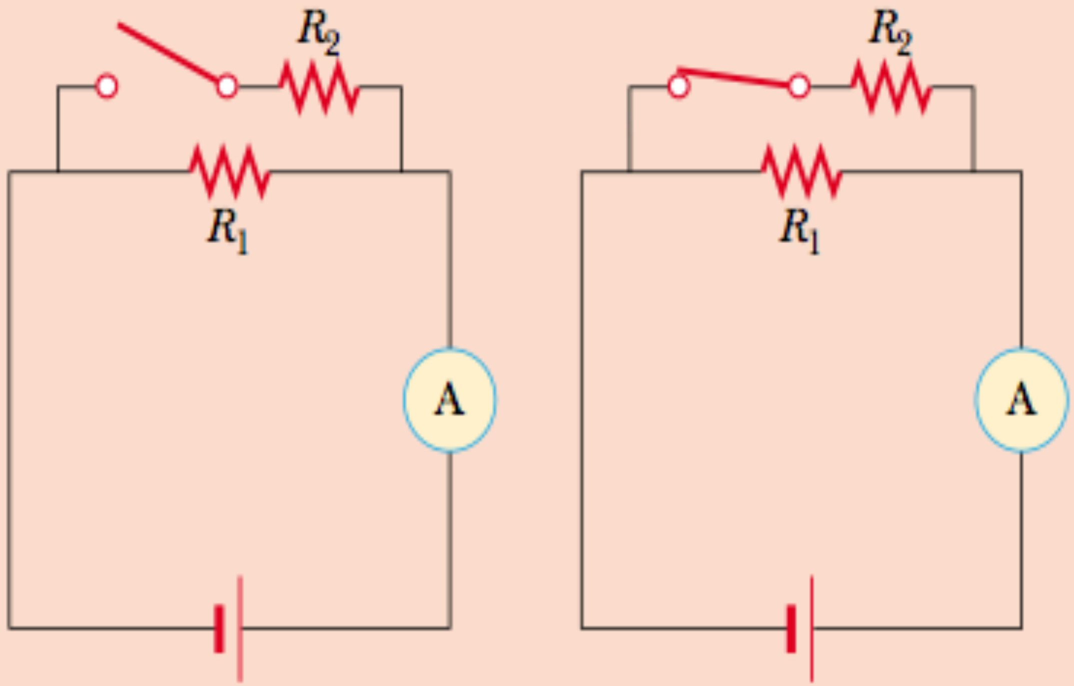
28.2 Resistors in parallel

- The **equivalent resistance** of three or more resistors connected in **parallel** is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

- The inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always **less** than the smallest resistance in the group.

Quick Quiz 28.7 With the switch in the circuit of Figure 28.7 open (left), there is no current in R_2. There is current in R_1 and this current is measured with the ammeter at the right side of the circuit. If the switch is closed (Fig. 28.7, right), there is current in R_2. What happens to the reading on the ammeter when the switch is closed? (a) the reading goes up; (b) the reading goes down; (c) the reading does not change.



Switch open Switch closed

Example 28.4 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 28.9a.

(A) Find the equivalent resistance between points *a* and *c*.

Solution The combination of resistors can be reduced in steps, as shown in Figure 28.9. The 8.0-Ω and 4.0-Ω resistors are in series; thus, the equivalent resistance between *a* and *b* is 12.0 Ω (see Eq. 28.5). The 6.0-Ω and 3.0-Ω resistors are in parallel, so from Equation 28.7 we find that the equivalent resistance from *b* to *c* is 2.0 Ω. Hence, the equivalent resistance from *a* to *c* is 14.0 Ω.

(B) What is the current in each resistor if a potential difference of 42 V is maintained between *a* and *c*?

Solution The currents in the 8.0-Ω and 4.0-Ω resistors are the same because they are in series. In addition, this is the same as the current that would exist in the 14.0-Ω equivalent resistor subject to the 42-V potential difference. Therefore, using Equation 27.8 ($R = \Delta V/I$) and the result from part (A), we obtain

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42\text{ V}}{14.0\ \Omega} = 3.0\text{ A}$$

This is the current in the 8.0-Ω and 4.0-Ω resistors. When this 3.0-A current enters the junction at *b*, however, it splits, with part passing through the 6.0-Ω resistor (I_1) and part through the 3.0-Ω resistor (I_2). Because the potential difference is ΔV_{bc} across each of these parallel resistors, we see that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A. We could have guessed this at the start by noting that the current in the 3.0-Ω resistor has to be twice that in the 6.0-Ω resistor, in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check of our results, note that $\Delta V_{bc} = (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 = 6.0$ V and $\Delta V_{ab} = (12.0\ \Omega)I = 36$ V; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.

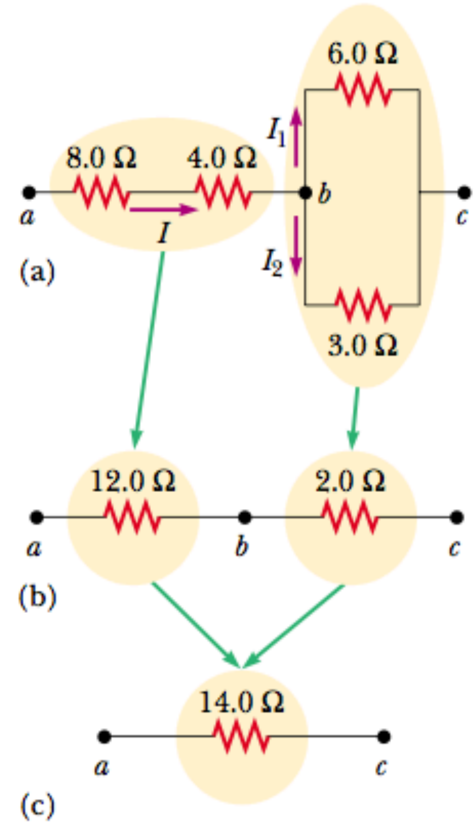


Figure 28.9 (Example 28.4) The original network of resistors is reduced to a single equivalent resistance.

Example 28.6 Three Resistors in Parallel

Interactive

Three resistors are connected in parallel as shown in Figure 28.11a. A potential difference of 18.0 V is maintained between points *a* and *b*.

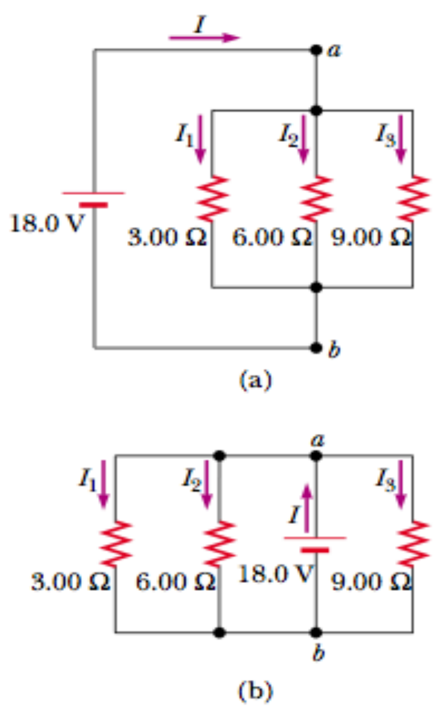
(A) Find the current in each resistor.

Solution The resistors are in parallel, and so the potential difference across each must be 18.0 V. Applying the relationship $\Delta V = IR$ to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0\text{ V}}{3.00\ \Omega} = 6.00\text{ A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0\text{ V}}{6.00\ \Omega} = 3.00\text{ A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0\text{ V}}{9.00\ \Omega} = 2.00\text{ A}$$



(B) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

Solution We apply the relationship $\mathcal{P} = I^2R$ to each resistor and obtain

3.00-Ω: $\mathcal{P}_1 = I_1{}^2R_1 = (6.00\text{ A})^2(3.00\ \Omega) = 108\text{ W}$

6.00-Ω: $\mathcal{P}_2 = I_2{}^2R_2 = (3.00\text{ A})^2(6.00\ \Omega) = 54.0\text{ W}$

9.00-Ω: $\mathcal{P}_3 = I_3{}^2R_3 = (2.00\text{ A})^2(9.00\ \Omega) = 36.0\text{ W}$

This shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 198 W.

(C) Calculate the equivalent resistance of the circuit.

Solution We can use Equation 28.8 to find R_{eq}:

$$\frac{1}{R_{eq}} = \frac{1}{3.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{9.00\ \Omega}$$

$$R_{eq} = \frac{18.0\ \Omega}{11.0} = 1.64\ \Omega$$

Figure 28.11 (Example 28.6) (a) Three resistors connected in parallel. The voltage across each resistor is 18.0 V. (b) Another circuit with three resistors and a battery. Is this equivalent to the circuit in part (a) of the figure?