PHYS 111
1ST semester 1439-1440

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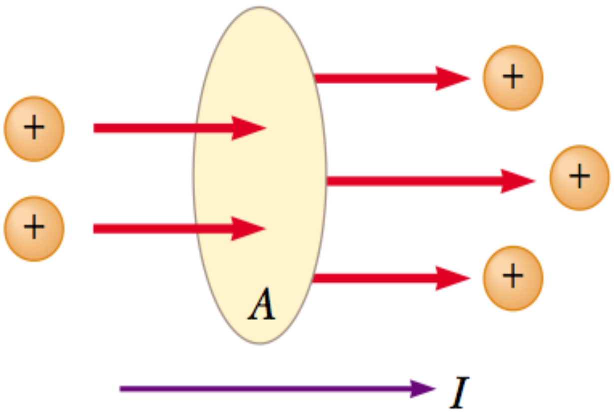
Lecture 12

Chapter 27
Current and Resistance

27.1 Electric Current

- The **current** is the rate at which charge flows through this surface.
- If ΔQ is the amount of charge that passes through this area in a time interval Δt, the **average current** I_{av} is equal to the charge that passes through A per unit time:



$$I_{av} = \frac{\Delta Q}{\Delta t}$$

- If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** I as the differential limit of average current:

$$I \equiv \frac{dQ}{dt}$$

- The SI unit of current is the **ampere** (A):

$$1\ \text{A} = \frac{1\ \text{C}}{1\ \text{s}}$$

27.1 Electric Current

- It is conventional to assign to the current the same direction as the flow of **positive** charge.
- The direction of the current is opposite the direction of flow of electrons.
- It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier.** For example, the mobile charge carriers in a metal are electrons.

Microscopic Model of Current

- Consider the current in a conductor of cross-sectional area *A*.
- The volume of a section of the conductor of length Δx.
- If *n* is the number of mobile charge carriers per unit volume (the charge carrier density), the number of carriers in the gray section is $nA\,\Delta x$.
- The total charge ΔQ is

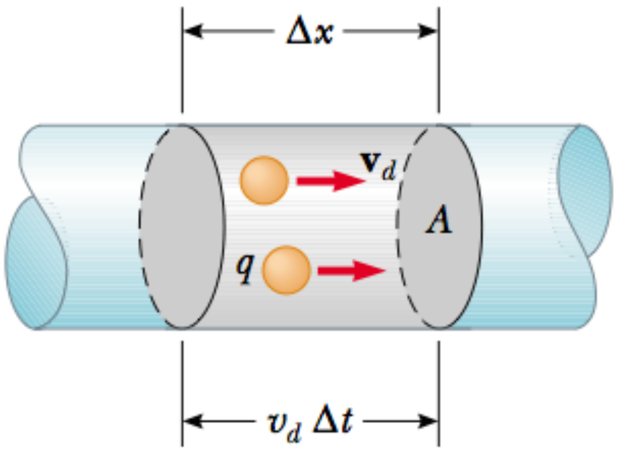
ΔQ = number of carriers in section x charge per carrier = $(nA\,\Delta x)q$

where *q* is the charge on each carrier.

$$\Delta Q = (nAv_d\,\Delta t)q$$

- The Current in a conductor in terms of microscopic quantities

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_dA$$



- The speed of the charge carriers v_d is an average speed called the **drift speed.**

Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \text{m}^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ \text{g/cm}^3$.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3}\left(\frac{1.00 \times 10^6\ \text{cm}^3}{1\ \text{m}^3}\right)$$

$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$

$$= \frac{10.0\ \text{C/s}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.31 \times 10^{-6}\ \text{m}^2)}$$

$$= 2.22 \times 10^{-4}\ \text{m/s}$$

27.2 Resistance

- Consider a conductor of cross-sectional area *A* carrying a current *I*. The **current density** *J* in the conductor is defined as the current per unit area.

$$J \equiv \frac{I}{A} = nqv_d$$

- where *J* has SI units of A/m^2.
- This expression is valid only if the current density is uniform and if the surface of cross-sectional area *A* is perpendicular to the direction of the current.
- The current density is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.
- In general, current density is a **vector** quantity:

$$\mathbf{J} = nq\mathbf{v}_d$$

- A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor.

27.2 Resistance

- In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

- Where σ is called the **conductivity** of the conductor.
- Materials that obey previous equation are said to follow **Ohm's law,**

for many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

27.2 Resistance

- If the field is assumed to be uniform, the potential difference is

$$\Delta V = E\ell$$

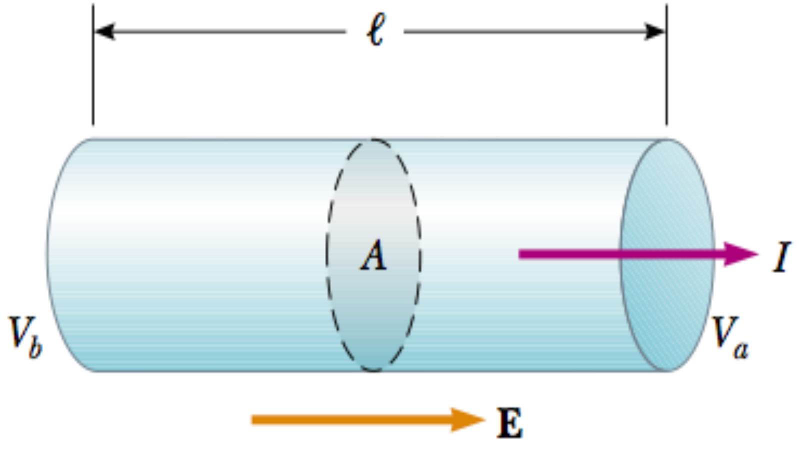
- The magnitude of the current density in the wire

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

- Because *J=I/A*, we can write the potential difference as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

- The quantity *R* is called the **resistance** of the conductor.
- We can define the resistance as the ratio of the potential to the current.



$$R \equiv \frac{\Delta V}{I}$$

27.2 Resistance

- The resistance has SI units of volts per ampere. One volt per ampere is defined to be one **ohm** (Ω):

$$1\,\Omega \equiv \frac{1\,\mathrm{V}}{1\,\mathrm{A}}$$

- The inverse of conductivity is **resistivity** ρ

$$\rho = \frac{1}{\sigma}$$

where ρ has the units ohm-meters $(\Omega \cdot \mathrm{m})$. Because $R = \ell/\sigma A$, we can express the resistance of a uniform block of material along the length ℓ as

$$R = \rho \frac{\ell}{A}$$

Example 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

Solution From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= 1.41 \times 10^{-5}\ \Omega$$

Similarly, for glass we find that

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega \cdot \text{m}) \left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2} \right)$$

$$= 1.5 \times 10^{13}\ \Omega$$

As you might guess from the large difference in resistivities, the resistances of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

Example 27.3 The Resistance of Nichrome Wire

Interactive

(A) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\cdot\text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = 4.6\ \Omega/\text{m}$$

(B) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = 2.2\ \text{A}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.