# PHYS 111
# 1ST semester 1439-1440

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**Lecture 10**

# Chapter 26
# Capacitance and Dielectrics

# 26.3 Combinations of Capacitors

- Parallel Combination

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

- Series Combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(series combination)}$$

## Example 26.4 Equivalent Capacitance

Interactive

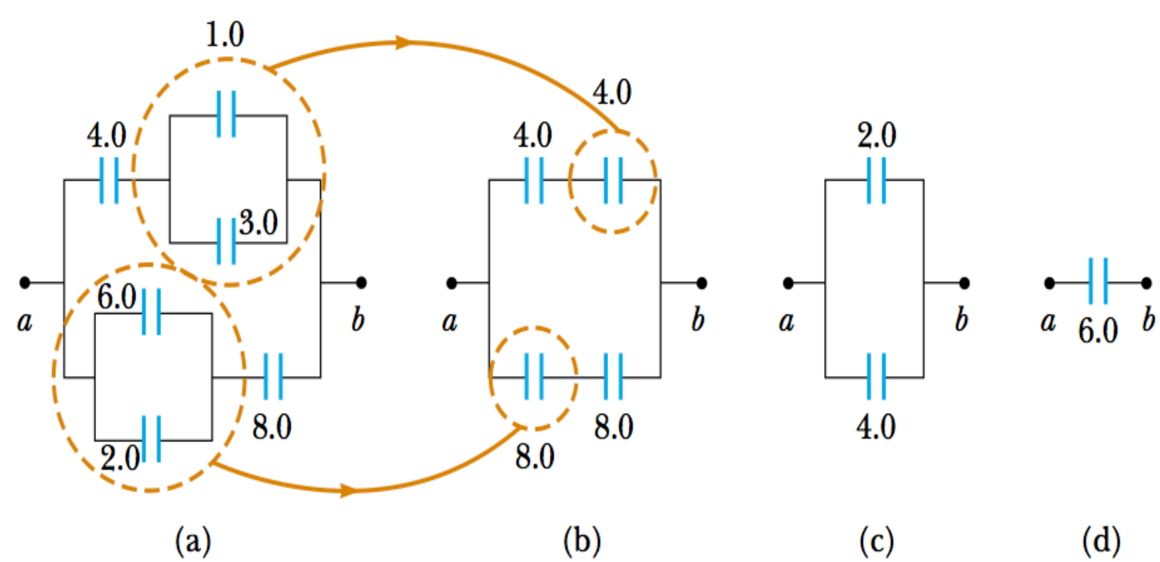
Find the equivalent capacitance between $a$ and $b$ for the combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.

***Solution*** Using Equations 26.8 and 26.10, we reduce the combination step by step as indicated in the figure. The 1.0-$\mu$F and 3.0-$\mu$F capacitors are in parallel and combine according to the expression $C_{eq} = C_1 + C_2 = 4.0\ \mu$F. The 2.0-$\mu$F and 6.0-$\mu$F capacitors also are in parallel and have an equivalent capacitance of 8.0 $\mu$F. Thus, the upper branch in Figure 26.11b consists of two 4.0-$\mu$F capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = 2.0\ \mu\text{F}$$

The lower branch in Figure 26.11b consists of two 8.0-$\mu$F capacitors in series, which combine to yield an equivalent capacitance of 4.0 $\mu$F. Finally, the 2.0-$\mu$F and 4.0-$\mu$F capacitors in Figure 26.11c are in parallel and thus have an equivalent capacitance of 6.0 $\mu$F.



# 26.4 Energy Stored in a Charged Capacitor

- Suppose that *q* is the charge on the capacitor at some instant during the charging process.
- At the same instant, the potential difference across the capacitor is

$$\Delta V = q/C$$

- The work necessary to transfer an increment of charge *dq* from the plate carrying charge -*q* to the plate carrying charge *q*

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

- The total work required to charge the capacitor from *q*=0 to some final charge *q*=Q is

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

- The potential energy stored in a charged capacitor is:

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}Q\,\Delta V = \tfrac{1}{2}C(\Delta V)^2$$

# 26.4 Energy Stored in a Charged Capacitor

- For a **parallel-plate capacitor**, the potential difference is related to the electric field through the relationship $\Delta V = Ed$, and its capacitance is $C = \varepsilon_0 A/d$

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

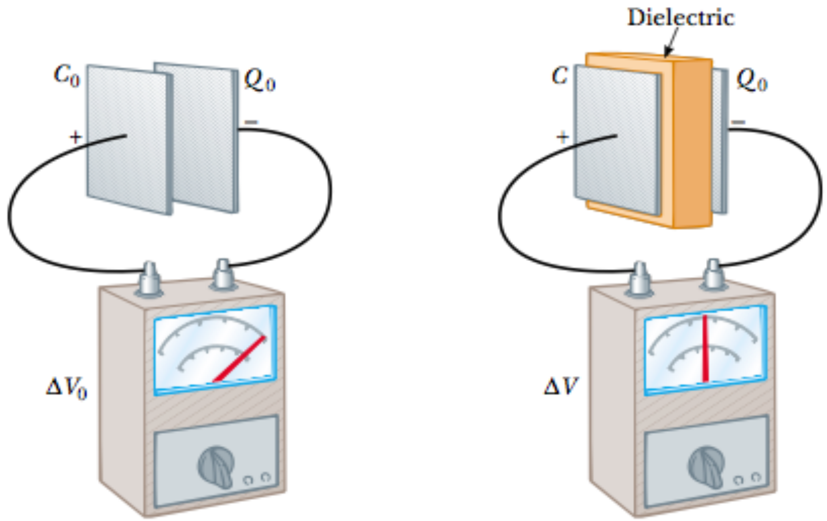
- The *energy per unit volume* $u_E = U/Ad$, known as the *energy density*, is

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

- The energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

# 26.5 Capacitors with Dielectrics

- A **dielectric** is a nonconducting material, such as rubber, glass, or waxed paper.
- When a dielectric is inserted between the plates of a capacitor, the capacitance **increases** by a dimensionless factor κ, which is called the **dielectric constant** of the material.
- Consider a parallel-plate capacitor that **without** a dielectric has a charge $Q_0$ and a capacitance $C_0$. The potential difference across the capacitor is $\Delta V_0 = Q_0/C_0$.
- The voltages with and without the dielectric are related by the factor κ as $\Delta V = \Delta V_0/\kappa$
- Because $\Delta V < \Delta V_0$, we see that $\kappa > 1$.
- The charge $Q_0$ on the capacitor does not change.



(a) (b)

# 26.5 Capacitors with Dielectrics

- The capacitance is

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

- That is, the capacitance ***increases*** by the factor κ.
- For a parallel-plate capacitor, where $C_0 = \varepsilon_0 A/d$, the capacitance when the capacitor is filled with a dielectric is given by

$$C = \kappa \frac{\epsilon_0 A}{d}$$

## Example 26.6 A Paper-Filled Capacitor

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

**(A)** Find its capacitance.

***Solution*** Because $\kappa = 3.7$ for paper (see Table 26.1), we have

$$C = \kappa \frac{\epsilon_0 A}{d}$$

$$= 3.7 \left( \frac{(8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2})(6.0 \times 10^{-4}\,\mathrm{m^2})}{1.0 \times 10^{-3}\,\mathrm{m}} \right)$$

$$= 20 \times 10^{-12}\,\mathrm{F} = \boxed{20\,\mathrm{pF}}$$

## Example 26.7 Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge $Q_0$, as shown in Figure 26.20a. The battery is then removed, and a slab of material that has a dielectric constant $\kappa$ is inserted between the plates, as shown in Figure 26.20b. Find the energy stored in the capacitor before and after the dielectric is inserted.

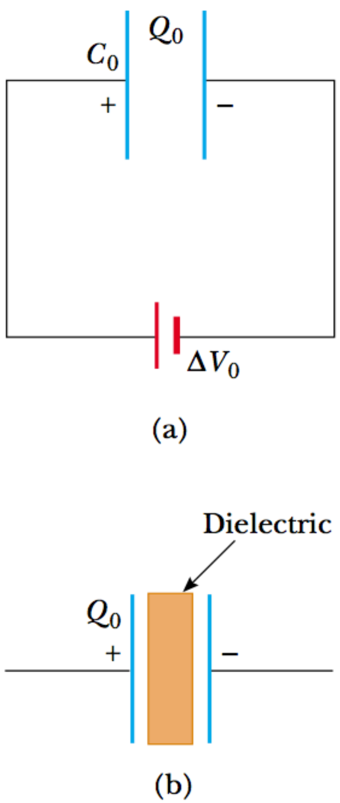



**Figure 26.20** (Example 26.7) (a) A battery charges up a parallel-plate capacitor. (b) The battery is removed and a slab of dielectric material is inserted between the plates.

***Solution*** From Equation 26.11, we see that the energy stored in the absence of the dielectric is

$$U_0 = \frac{Q_0^2}{2C_0}$$

After the battery is removed and the dielectric inserted, the *charge on the capacitor remains the same.* Hence, the energy stored in the presence of the dielectric is

$$U = \frac{Q_0^2}{2C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, so $U$ becomes

$$U = \frac{Q_0^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Because $\kappa > 1$, the final energy is less than the initial energy. We can account for the "missing" energy by noting that the dielectric, when inserted, is pulled into the device (see Section 26.7). An external agent must do negative work to keep the dielectric from accelerating. This work is simply the difference $U - U_0$. (Alternatively, the positive work done by the system on the external agent is $U_0 - U$.)