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King Saud University
College of Science
Physics & Astronomy Dept.

PHYS 103 (GENERAL PHYSICS)

CHAPTER 10: ROTATION OF A RIGID OBJECT ABOUT A FIXED AXIS

LECTURE NO. 16

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Lecture Outline

- *We shall discuss the following topics:*

- *10.1 Angular Position*
- *10.1 Angular Speed*
- *Quiz 10.1*
- *10.1 Angular Acceleration*
- *10.2 Rotational Motion with Cons. Ang. Acc.*
- *Example 10.1 Rotating Wheel*
- *10.3 Angular and Linear Quantities*
- *Quiz 10.5-10.6*
- *10.4 Rotational Kinetic Energy*
- *Quiz 10.7*
- *Example 10.3 The Oxygen Molecule*
- *Example 10.4 Four Rotating Objects*
- *Lecture Summary*
- *Interactive Flash*
- *End of Presentation*

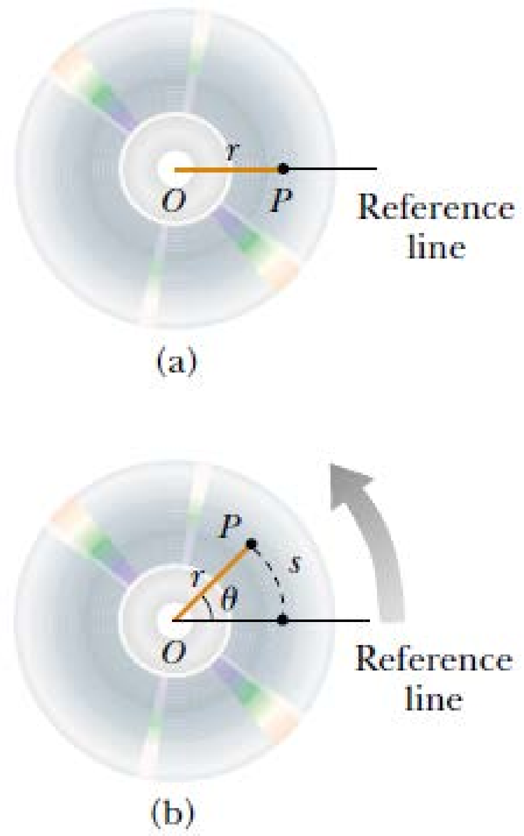
10.1 Angular Position

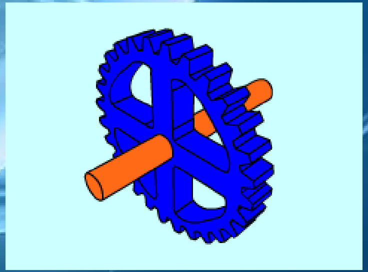
- Consider a particle at P is at a fixed distance r from the origin and rotates about it in a circle of radius r.
- The particle moves through an arc of length s, as in Figure . The arc length *s* is related to the angle θ through the relationship:

$$s = r\theta \qquad (10.1a)$$

$$\Rightarrow \theta = \frac{s}{r} \qquad (10.1b)$$

- Note the dimensions of θ in Equation 10.1b. Because θ is the ratio of an arc length and the radius of the circle, it is a pure number. However, we commonly give ! the artificial unit radian (rad).





10.1 Angular Speed

- As a particle travels from position 1 to position 2 in a time interval Δ, the reference line of length *r* sweeps out an angle $\Delta\theta = \theta_f - \theta_i$. This quantity Δθ is defined as the ***angular displacement*** of the rigid object:

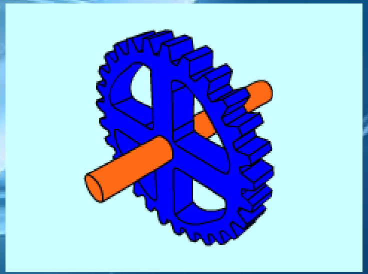
$$\Delta\theta = \theta_f - \theta_i$$

- We define the average angular speed as:

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} \qquad (10.2)$$

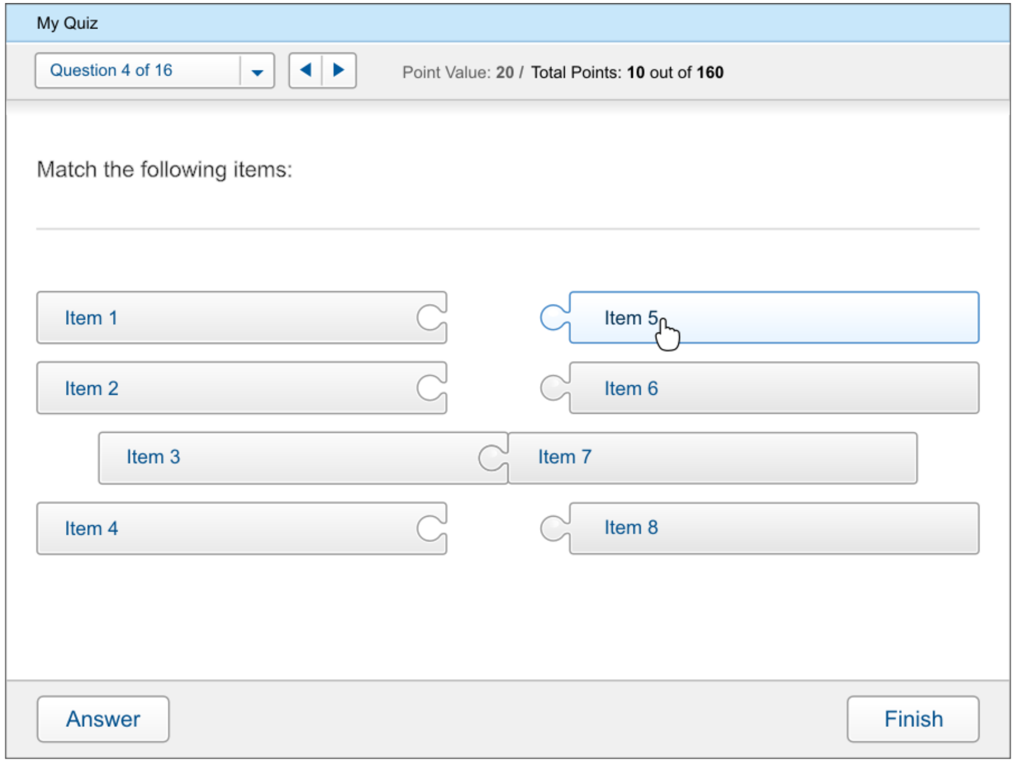
- the instantaneous angular speed is:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad (10.3)$$



Angular speed has units of radians per second (rad/s)

Quiz 10.1


My Quiz
Question 4 of 16
Point Value: 20 / Total Points: 10 out of 160
Match the following items:
Item 1
Item 5
Item 2
Item 6
Item 3
Item 7
Item 4
Item 8
Answer
Finish

10.1 Angular Acceleration

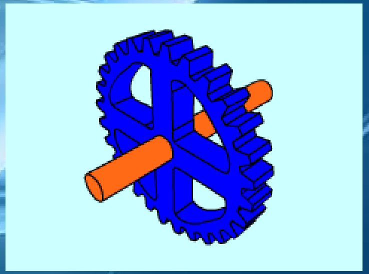
- The average angular acceleration of a rotating rigid object is defined as:

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \qquad (10.3)$$

- the instantaneous angular acceleration is defined as:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad (10.5)$$

- Angular acceleration has units of radians per second squared (rad/s^2)
- **When a rigid object is rotating about a *fixed axis*, every particle on the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration.**
- That is, the quantities θ, ω, and α characterize the rotational motion of the entire rigid object as well as individual particles in the object.



10.2 Rotational Motion with Cons. Ang. Acc.

- In our study of linear motion, we found that the simplest form of motion to analyze is motion under constant linear acceleration.
- Likewise, for rotational motion about a fixed axis, the simplest motion to analyze is motion under constant angular acceleration.

with $\theta_i = 0,\ \alpha = \text{constant}$

$$\omega_f = \omega_i + \alpha t \qquad (10.6)$$

$$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (10.7)$$

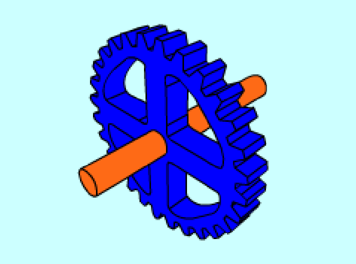
$$\omega_f^2 = \omega_i^2 + 2\alpha\theta_f \qquad (10.8)$$

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

- Notice that these expressions are of the same mathematical form as those for linear motion under constant linear acceleration with the change:

$$x \to \theta,\ v \to \omega,\ a \to \alpha$$



Example 10.1 Rotating Wheel

- A wheel rotates with a constant angular acceleration of 3.50 rad/s^2.

(A) If the angular speed of the wheel is 2.00 rad/s at $t_i = 0$, through what angular displacement does the wheel rotate in 2.00 s?

$$\because \quad \theta_i = 0$$

$$\therefore \Delta\theta = \theta_f = (2)(2) + \frac{1}{2}(3.5)(2)^2 = 11\,rad = 630^o$$

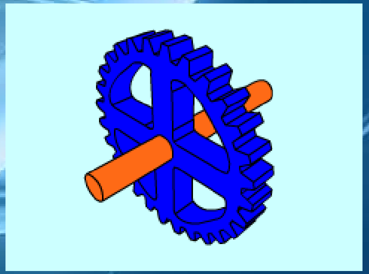
(B) Through how many revolutions has the wheel turned during this time interval?

$$\Delta\theta = 630^o \left(\frac{1 \ \text{rev}}{360^o}\right) = 1.75 \ \text{rev}$$

(C) What is the angular speed of the wheel at t = 2.00 s?

$$\because \omega_f = \omega_i + \alpha t$$

$$\therefore \omega_f = 2 + (3.5)(2) = 9\,rad/s$$



10.3 Angular and Linear Quantities

- We shall find relations between linear and angular quantities:

$$\because \omega_f = \omega_i + \alpha t$$

$$\because v = \frac{ds}{dt} = \frac{d}{dt} r\theta = r\frac{d\theta}{dt} = r\omega \qquad (10.10)$$

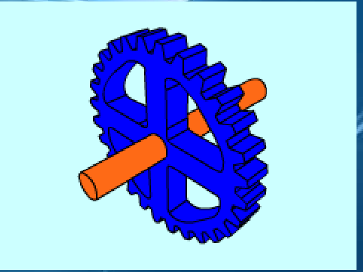
$$\because a_t = \frac{dv}{dt} = \frac{d}{dt} r\omega = r\frac{d\omega}{dt} = r\alpha \qquad (10.11)$$

$$\because a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2 \qquad (10.12)$$

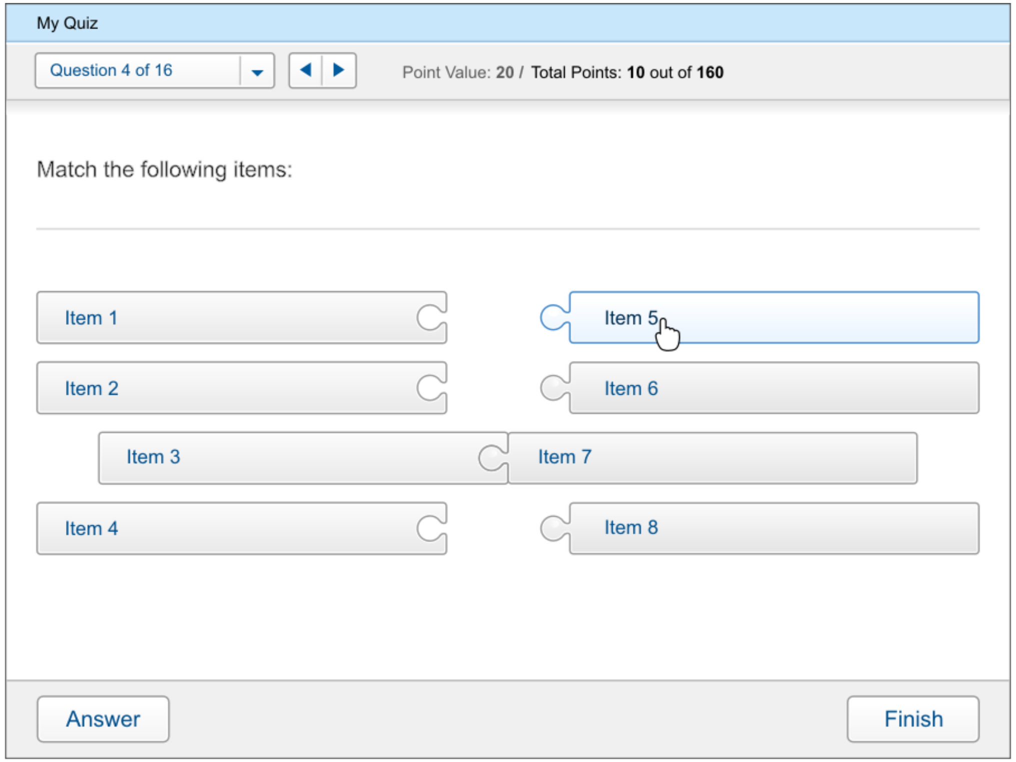
$$\therefore a = \sqrt{a_t^{\,2} + a_c^{\,2}} = \sqrt{r^2\alpha^2 + r^2\omega^4}$$

$$\therefore a = r\sqrt{\alpha^2 + \omega^4} \qquad (10.13)$$

- a_t : tangential acceleration, a_c: central acceleration, a: total acceleration



Quiz 10.5-10.6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10.4 Rotational Kinetic Energy

- Let us consider an object as a collection of particles and assume that it rotates about a fixed z axis with an angular speed $\boldsymbol{\omega}$ If the mass of the ith particle is $\boldsymbol{m_i}$ and its tangential speed is $\boldsymbol{v_i}$, its kinetic energy is:

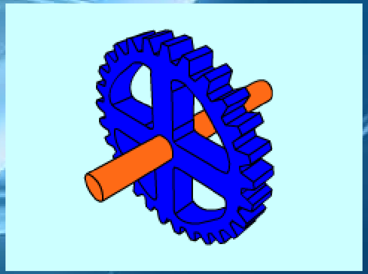
$$K_i = \frac{1}{2} m_i {v_i}^2$$

$$\because v_i = r_i \omega$$

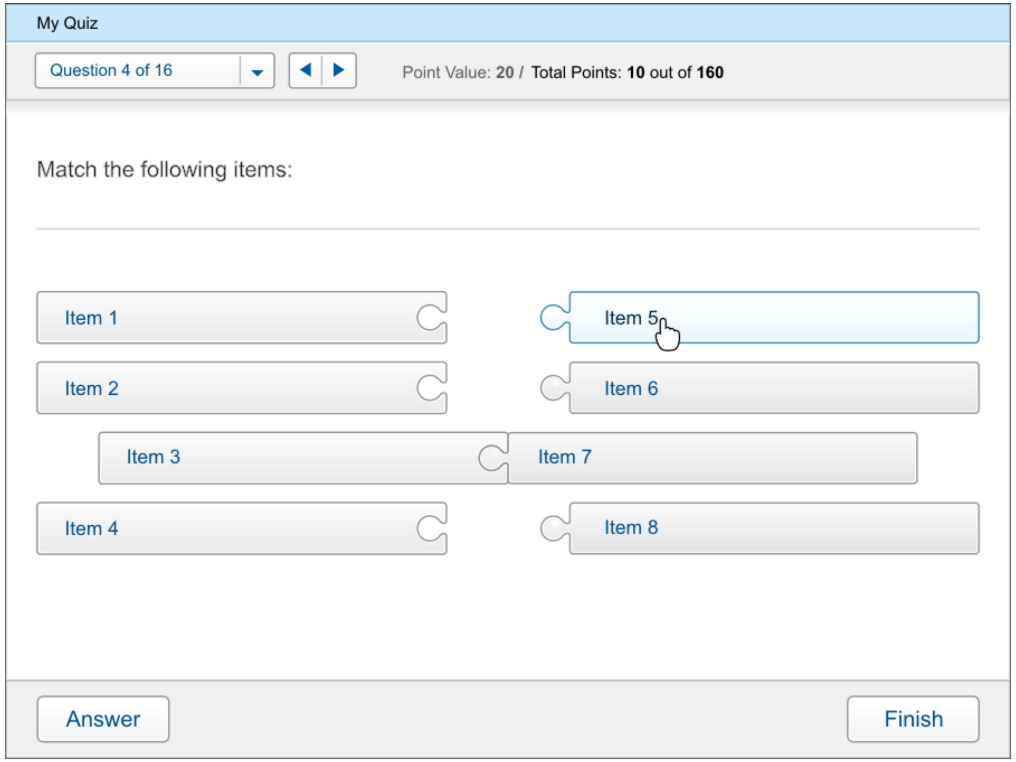
$$\therefore K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i {v_i}^2 = \frac{1}{2} \sum_i m_i {r_i}^2 \omega^2 = \frac{1}{2} \left(\sum_i m_i {r_i}^2 \right) \omega^2$$

define the moment of inertia I as: $I = \sum_i m_i {r_i}^2$ (10.15)

$$\therefore K_R = \frac{1}{2} I \omega^2 \qquad (10.16)$$



Quiz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Example 10.3 The Oxygen Molecule

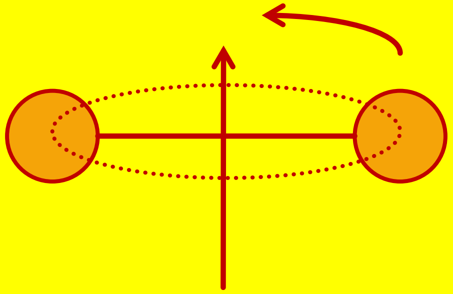
▶Consider an oxygen molecule (O_2) rotating in the xy plane about the z axis. The rotation axis passes through the center of the molecule, perpendicular to its length. The mass of each oxygen atom is 2.66x10-^{26} kg, and at room temperature the average separation between the two atoms is d = 1.21x10-^{10} m. (The atoms are modeled as particles.)

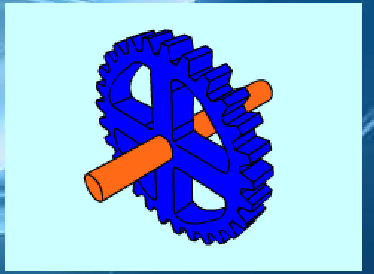
(A) *Calculate the moment of inertia of the molecule about the z axis.*

$$\because\ I=\sum_i m_i r_i^2 = m\left(\frac{d}{2}\right)^2 = m\left(\frac{d}{2}\right)^2 = \frac{1}{2}md^2 \qquad (1)$$

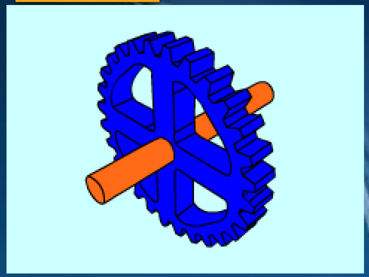
$$\therefore I = \frac{1}{2}(2.66\times10^{-26})(1.21\times10^{-10})^2 = 1.95\times10^{-46}\,kg.m^2 \qquad (2)$$

This is a very small number, consistent with the Minuscule masses and distances involved





Example 10.4 Four Rotating Objects

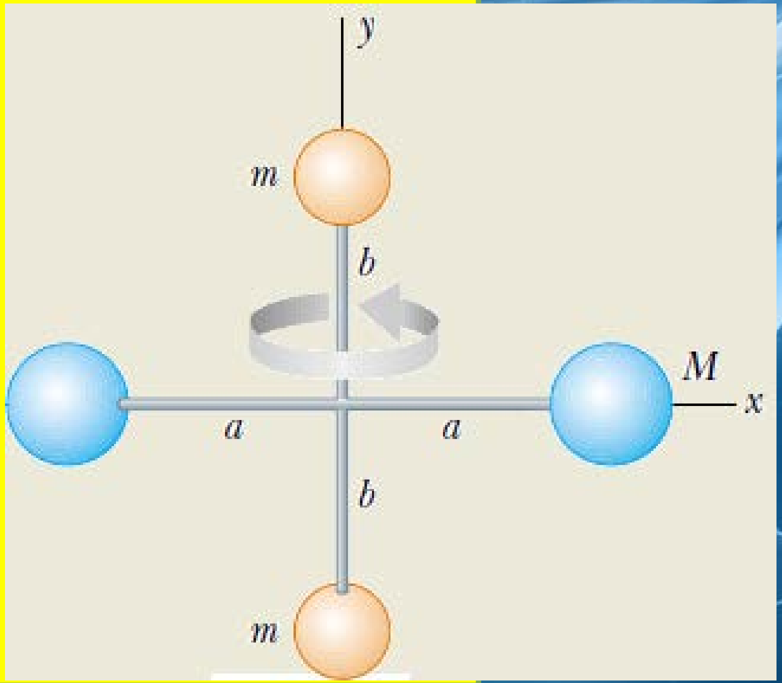


▶Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the xy plane. We shall assume that the radii of the spheres are small compared with the dimensions of the rods.

▶*(A) If the system rotates about the y axis with an angular speed ω, find the moment of inertia and the rotational kinetic energy about this axis.*

$$\because\ I_y = \sum_i m_i r_i^{\,2} = Ma^2 + Ma^2 + m(0) + m(0) = 2Ma^2 \quad (1)$$

$$\therefore K_R = \frac{1}{2} I_y \omega^2 = \frac{1}{2}(2Ma^2)\omega^2 = Ma^2\omega^2 \quad (2)$$



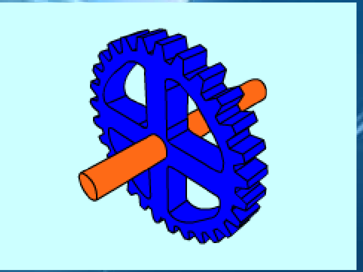
▶*(B) Same but in the xy plane about the z axis*

$$I_z = \sum_i m_i r_i^{\,2} = Ma^2 + Ma^2 + mb^2 + mb^2 = 2Ma^2 + 2\mathrm{mb}^2$$

$$\therefore K_R = \frac{1}{2} I_z \omega^2 = \frac{1}{2}(2Ma^2 + 2\mathrm{mb}^2)\omega^2 = (Ma^2 + 2\mathrm{mb}^2)\omega^2$$

Lecture Summary

- If a particle moves in a circular path of radius *r through an angle* θ *(measured in radians)*, the arc length it moves through is $s = r\theta$.
- The angular position of a rigid object is defined as the angle θ between a reference line attached to the object and a reference line fixed in space. The angular displacement of a particle moving in a circular path or a rigid object rotating about a fixed axis is $\Delta\theta = \theta_f - \theta_i$.
- The instantaneous angular speed of a particle moving in a circular path or of a rigid object rotating about a fixed axis is: $\omega = d\theta/dt$
- The instantaneous angular acceleration of a particle moving in a circular path or a rotating rigid object is: $\alpha = d\omega/dt$
- When a rigid object rotates about a fixed axis, every part of the object has the same angular speed and the same angular acceleration.



Lecture Summary (Continued)

- If an object rotates about a fixed axis under constant angular acceleration, one can apply equations of kinematics that are analogous to those for linear motion under constant linear acceleration:

with $\theta_\text{i} = 0,\ \alpha = \text{constant}$

$$\omega_\text{f} = \omega_\text{i} + \alpha t \qquad (10.6)$$

$$\theta_\text{f} = \omega_\text{i} t + \frac{1}{2}\alpha t^2 \qquad (10.7)$$

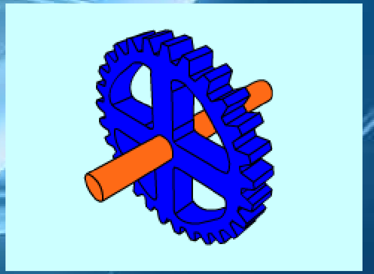
$$\omega_\text{f}^2 = \omega_\text{i}^2 + 2\alpha\theta_\text{f} \qquad (10.8)$$

- Relationships between linear and rotational quantities:

$$s = r\theta, \qquad v = r\omega, \qquad a_t = r\alpha$$

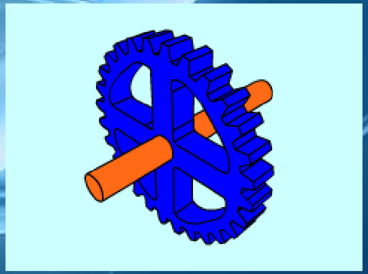
- The moment of inertia of a system of particles is defined as

$$I = \sum_i m_i r_i^2$$



Interactive Flashes

- In next few slides please watch the interactive flashes.
- It is recommended to go through all of them.



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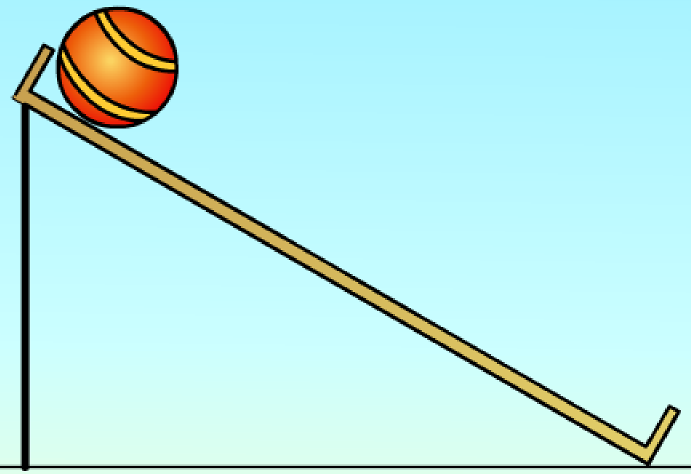

Problem Statement

Determine the moment of inertia of an object rolling down an incline.

The length of the incline is 10 m. The mass of the object is 1 kg and its radius is 1 m. Assume the object rolls down the incline without slipping.

Hint: All of the objects appear round when viewed from the side, however it could be a sphere or a cylinder and it could be solid or hollow.


THE END!