بسم الله الرحمن الرحيم

King Saud University
College of Science
Physics & Astronomy Dept.

PHYS 103 (GENERAL PHYSICS)

CHAPTER 9: LINEAR MOMENTUM-III

LECTURE NO. 15

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Lecture Outline

- *Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6*
- *9.4 Two-Dimensional Collisions*
- *PROBLE M-SOLV ING HI NTS*
- *Example 9.10 Collision at an Intersection*
- *9.5 The Center of Mass*
- *9.5 The Center of Mass (Integral form)*
- *Quiz 9.10*
- *Example 9.13 The CM of Three Particles*
- *Example 9.14 The Center of Mass of a Rod*
- *Example 9.15 The CM of a Right Triangle*
- *Lecture Summary*
- *Interactive Flash*
- *End of Presentation*

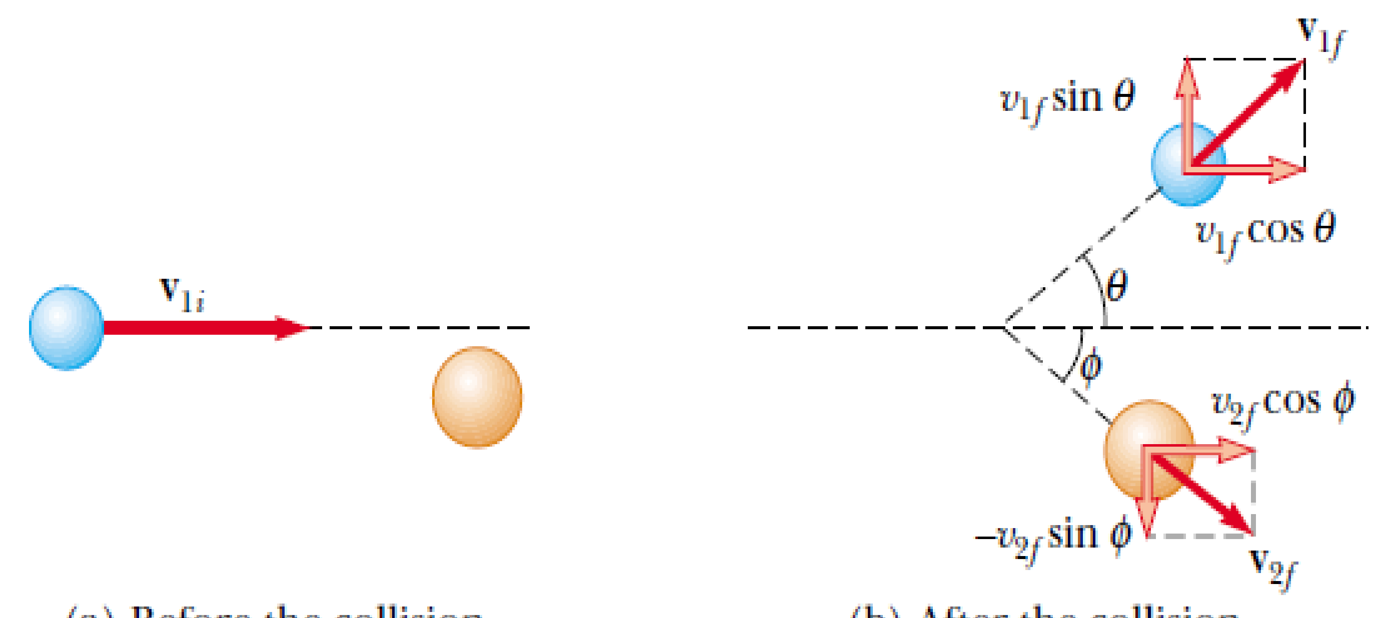
9.4 Two-Dimensional Collisions

- For two dimensional collisions, we obtain two component equations for conservation of momentum:

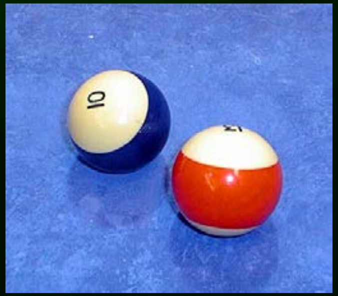
$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

consider a 2-D problem in which particle 1 of mass *m_1 collides* with particle 2 of mass m_2 *, where particle 2 is initially at rest, as in Figure*



(a) Before the collision (b) After the collision



9.4 Two-Dimensional Collisions (continued)

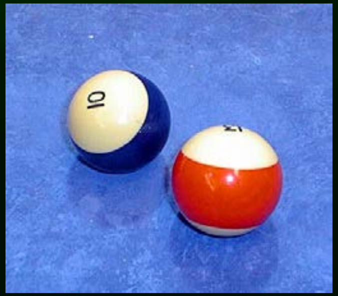
- Applying the law of conservation of momentum in component form and noting that the initial y component of the momentum of the two-particle system is zero, we obtain:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi \quad (9.24)$$

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi \quad (9.25)$$

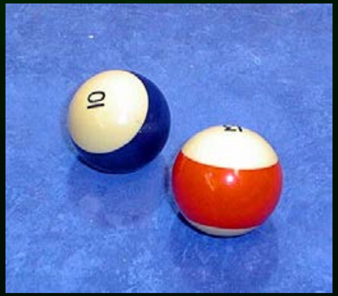
- where the minus sign in Equation 9.25 comes from the fact that after the collision, particle 2 has a y component of velocity that is downward.
- If the collision is elastic, we can also use Equation 9.16 (conservation of kinetic energy) with $v_{2i} = 0$ to give:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (9.26)$$



PROBLE M-SOLV ING HI NTS

- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the x and y components of the momentum of each object before and after the collision.
- Write expressions for the total momentum of the system in the x and y directions before and after the collision and equate the two..
- If the collision is inelastic, kinetic energy of the system is not conserved, and additional information is probably required.
- If the collision is perfectly inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is elastic, kinetic energy is conserved, and you can equate the total kinetic energy before and after the collision.

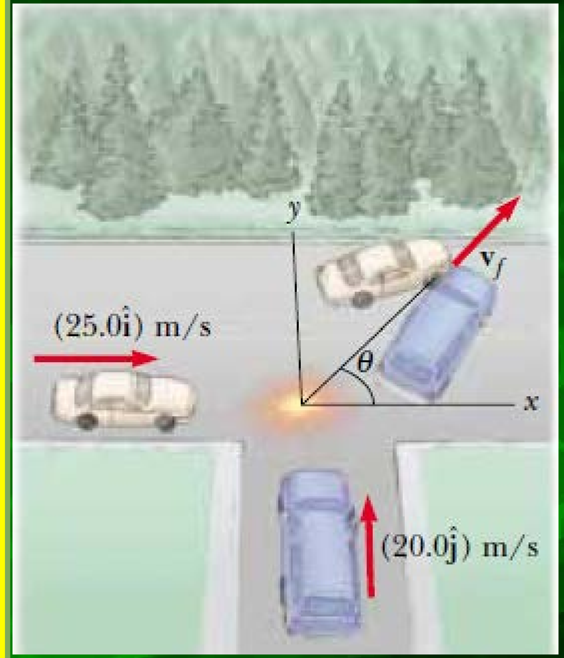


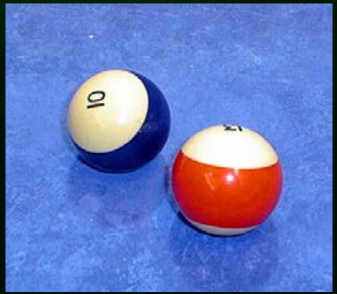
Example 9.10 Collision at an Intersection

- A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.14. *Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).*
- ***Solution***:
- We shall apply the conservation of momentum in each direction.

$$x:\ m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \qquad (1)$$

$$y:\ m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy} \qquad (2)$$





Example 9.10 (continued)

- Solving to find final velocity and direction:

$$(1)\rightarrow:\ (1500)(25)+(2500)(0)=(1500+2500)v_{fx} \qquad (3)$$

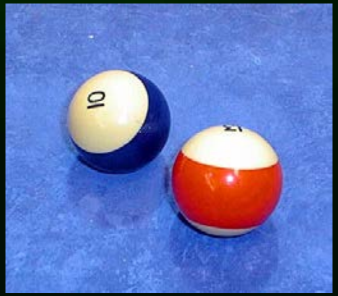
$$\therefore v_{fx}=\frac{37500}{4000}=9.37\,m/s$$

$$(2)\rightarrow:\ (1500)(0)+(2500)(20)=(1500+2500)v_{fy} \qquad (4)$$

$$\therefore v_{fy}=\frac{50000}{4000}=12.5\,m/s$$

$$(1)+(2)\rightarrow: v_f=\sqrt{v_{fx}^2+v_{fy}^2}=\sqrt{9.37^2+12.5^2}=15.6\,m/s$$

$$\theta=\tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)=\tan^{-1}\left(\frac{12.5}{9.37}\right)=53.1^o$$



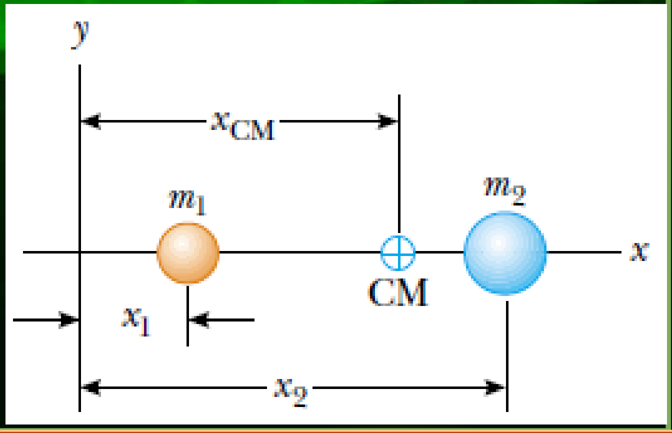
9.5 The Center of Mass

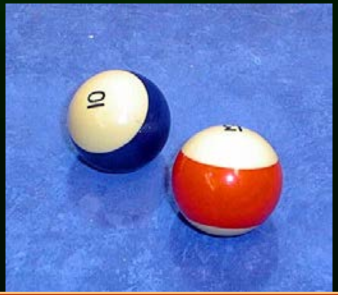
- The center of mass of the pair of particles described in Figure is located on the x axis and lies somewhere between the particles. Its x coordinate is given by:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad (9.27)$$

- We can extend this concept to a system of many particles with masses *mi*

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots + m_n x_n}{m_1 + m_2 + m_3 + \ldots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\sum_i m_i x_i}{M} \qquad (9.28)$$





9.5 The Center of Mass (continued)

- We can do the same for y and z:

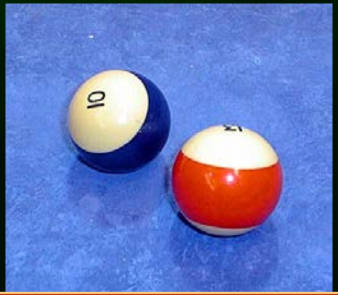
$$y_{CM} = \frac{\sum_i m_i y_i}{M} \qquad z_{CM} = \frac{\sum_i m_i z_i}{M} \qquad (9.29)$$

$$\rightarrow r_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{\sum_i m_i x_i \hat{i} + \sum_i m_i y_i \hat{j} + \sum_i m_i z_i \hat{k}}{M}$$

$$\therefore r_{cm} = \frac{\sum_i m_i r_i}{M} \qquad (9.30)$$

with: $r_i = x_i \hat{i} + y_i \hat{j} + z_i \hat{k}$



9.5 The Center of Mass (Integral form)

- When mass is distributed uniformly over the space:

$$m_i \rightarrow \Delta m_i$$

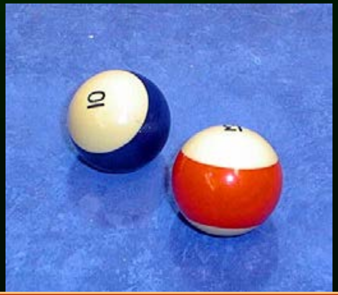
$$x_{CM} = \lim_{\Delta m_i \to 0} \frac{\sum_i x_i \Delta m_i}{M} = \frac{1}{M}\int x dm \qquad (9.31)$$

Also:

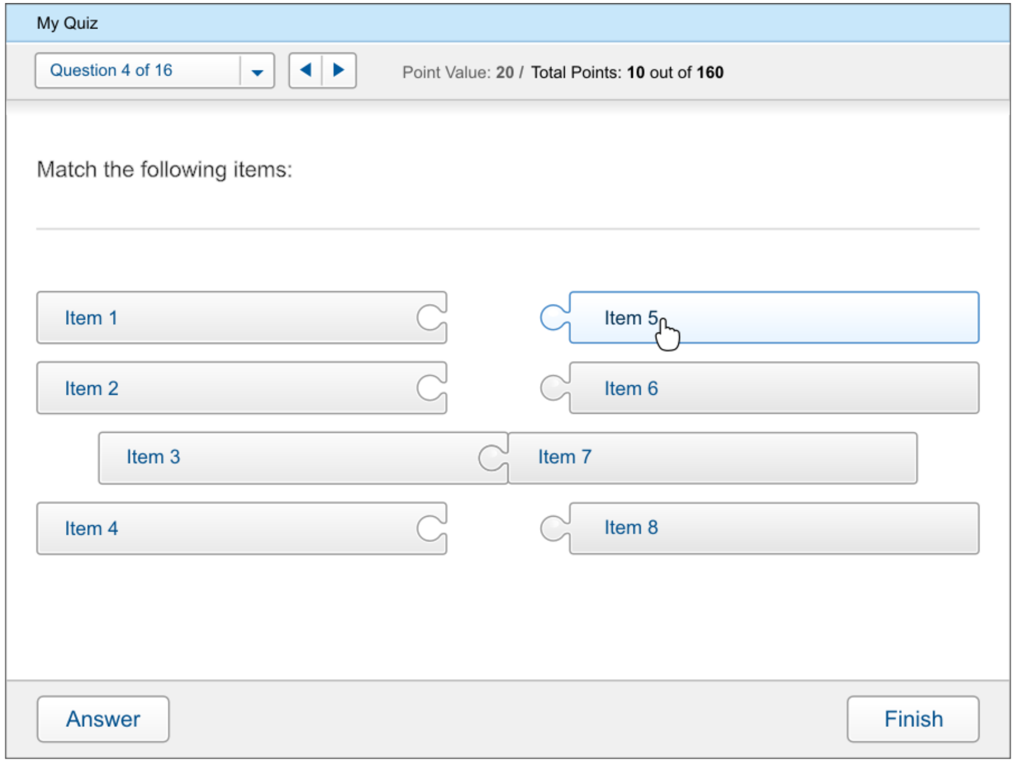
$$y_{CM} = \frac{1}{M}\int y dm \quad \text{and} \quad z_{CM} = \frac{1}{M}\int z dm \qquad (9.32)$$

in general:

$$r_{CM} = \frac{1}{M}\int r dm \qquad (9.33)$$



Quiz 9.10



Click the **Quiz** button on iSpring Pro toolbar to edit your quiz

Example 9.13 The CM of Three Particles

- A system consists of three particles located as shown in Figure . *Find the center of mass of the system.*

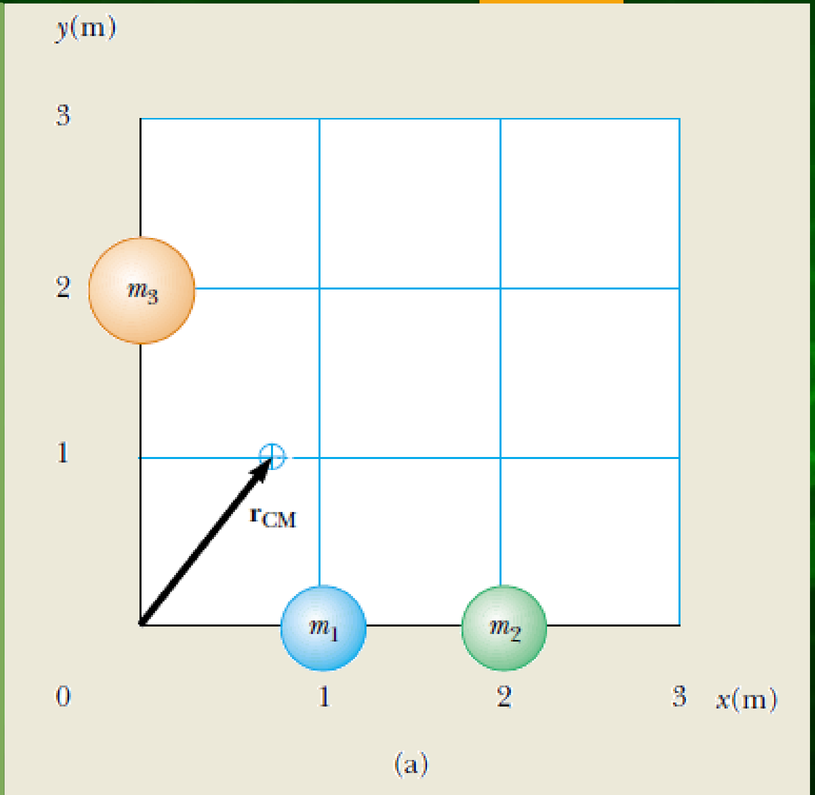
$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_l x_3}{m_1 + m_2 + m_3}$$

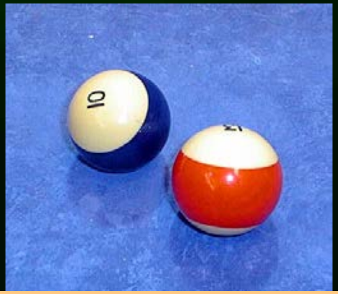
$$= \frac{(1)(1) + (1)(2) + (2)(0)}{1+1+2} = 0.75\,\text{m}$$

$$y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_l y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(1)(0) + (1)(0) + (2)(2)}{1+1+2} = 1\,\text{m}$$

$$\therefore\ \text{r}_{\text{CM}} = x_{CM}\,\hat{i} + y_{CM}\,\hat{j} = (0.75\hat{i} + 1.0\hat{j})\,m$$





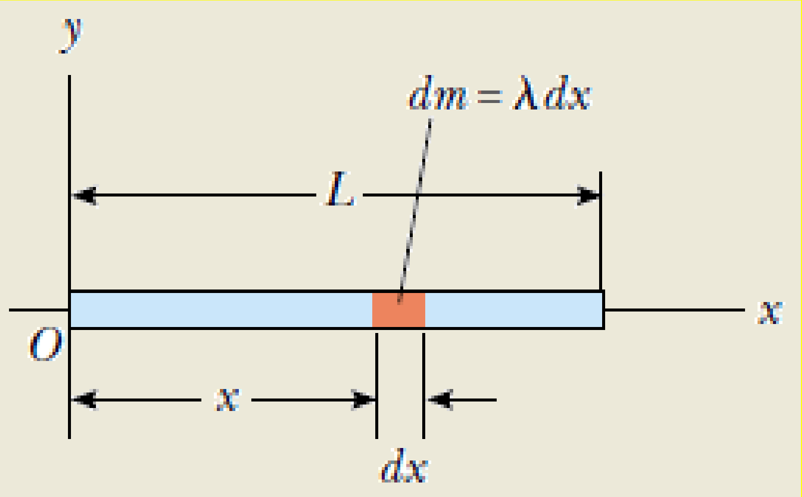
Example 9.14 The Center of Mass of a Rod

- Show that the center of mass of a rod of mass M and length L lies midway between its ends, assuming the rod has a uniform mass per unit length

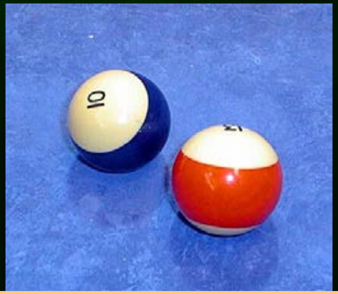
$$\because x_{CM} = \frac{1}{M}\int x\,dm$$

- We shall find an expression for the quantity dm: use dm = λdx where λ is the longitudinal density.

$$\therefore x_{CM} = \frac{1}{M}\int_0^L x\,\lambda dx = \frac{\lambda}{M}\left.\frac{x^2}{2}\right|_0^L = \frac{\lambda L^2}{2M}$$

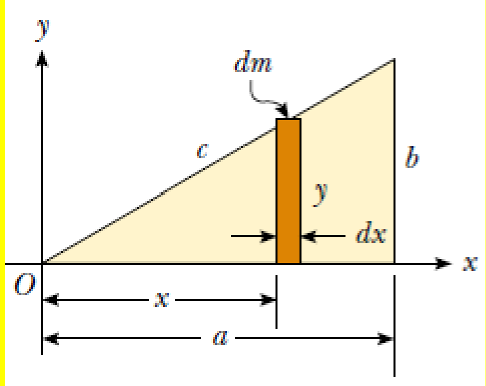


$$\because \lambda = \frac{M}{L} \Rightarrow \qquad x_{CM} = \frac{1}{2}L$$



Example 9.15 The CM of a Right Triangle

- Find CM of the uniform triangle shown in figure.
- ***Solution:*** we use surface density ρ to express *dm* as: *ρydx*



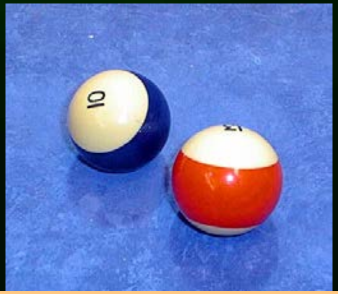
$$\because x_{CM} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^a x(\rho y\,dx) \quad (1)$$

Expressing y in x: $\because \frac{b}{a} = \frac{y}{x} \Rightarrow y = \frac{b}{a}x \quad (2)$

$$(2)\text{ in }(1): x_{CM} = \frac{1}{M}\int_0^a x\left(\rho\frac{b}{a}x\,dx\right) = \rho\frac{1}{M}\frac{b}{a}\int_0^a x^2 dx$$

$$\therefore x_{CM} = \rho\frac{1}{M}\frac{b}{a}\left|\frac{x^3}{3}\right|_0^a = \rho\frac{1}{M}\frac{b}{a}\frac{a^3}{3} = \left(\frac{1}{2}ab\rho\right)\frac{2}{3}\frac{a}{M}$$

$$\Rightarrow x_{CM} = \frac{2}{3}a$$



Lecture Summary

- For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1v_{1ix} + m_2v_{2ix} = m_1v_{1fx} + m_2v_{2fx}$$

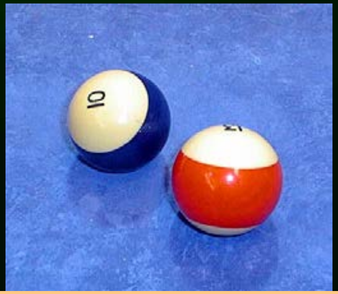
$$m_1v_{1iy} + m_2v_{2iy} = m_1v_{1fy} + m_2v_{2fy}$$

- The center of mass of the pair of particles:

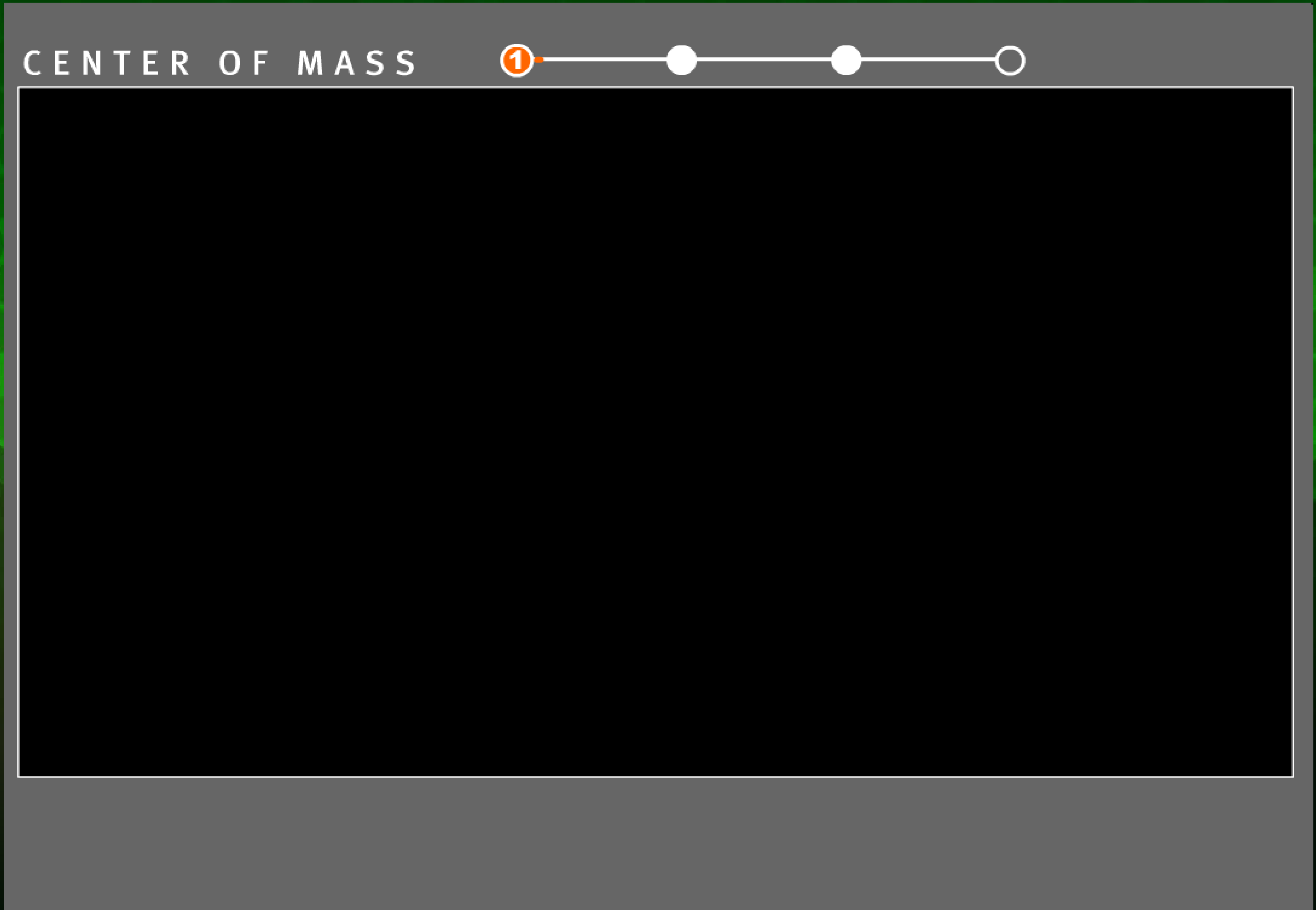
$$x_{CM} = \frac{\sum_i m_i x_i}{M} \qquad y_{CM} = \frac{\sum_i m_i y_i}{M} \qquad z_{CM} = \frac{\sum_i m_i z_i}{M}$$

- When mass is distributed uniformly over the space:

$$x_{CM} = \frac{1}{M}\int x\,dm \qquad y_{CM} = \frac{1}{M}\int y\,dm \qquad z_{CM} = \frac{1}{M}\int z\,dm$$



Interactive Flash

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