بسم الله الرحمن الرحيم

King Saud University
College of Science
Physics & Astronomy Dept.

PHYS 103 (GENERAL PHYSICS)

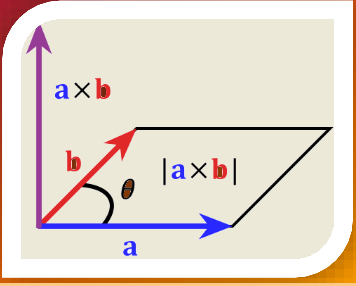
CHAPTER 3: VECTORS

LECTURE NO. 4

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

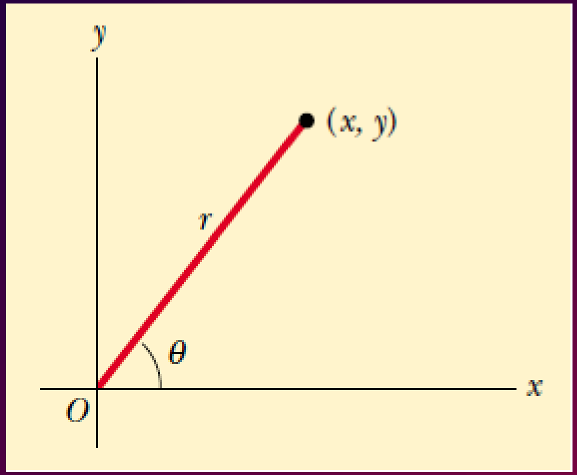
Lecture Outline

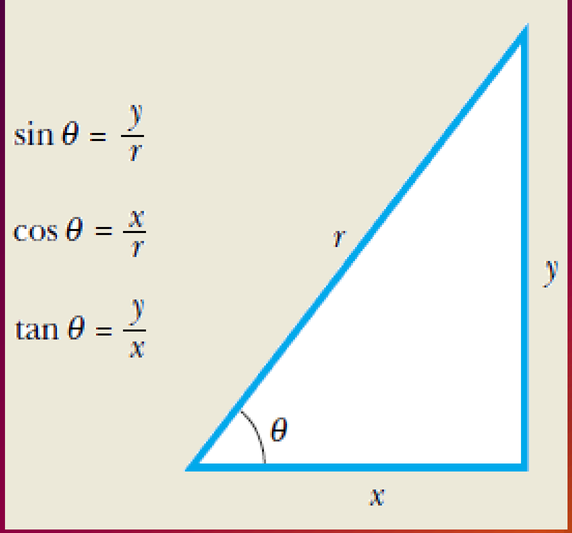
- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6
- *3.1 Coordinate Systems (Cartezian & Polar)*
- *3.2 Vector and Scalar Quantities*
- *3.3 Some Properties of Vectors (Addition, subtraction, …)*
- *3.4 Components of a Vector and Unit Vectors*
- *Examples*
- *Lecture Summary*
- *Activities (Interactive Flashes)*
- *Quizzes*

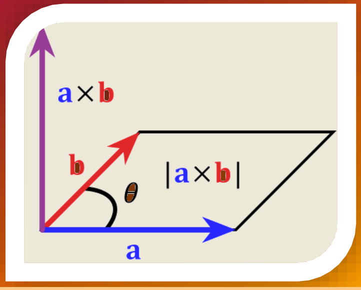


3.1 Coordinate Systems

- Many aspects of physics involve a description of a location in space. This usually is implemented using: *coordinate systems*
- *Cartesian coordinate system* is one simple system in which horizontal and vertical axes intersect at a point defined as the origin.
- Sometimes it is more convenient to represent a point in a plane by its plane polar coordinates (r, θ), In this *polar coordinate system*, r is the distance from the origin to the point having Cartesian coordinates (x, y), and θ is the angle between a line drawn from the origin to the point and a fixed axis







3.1 Conversion Between Coordinate Systems

- we can obtain the Cartesian coordinates from Polar coordinates by using the equations:

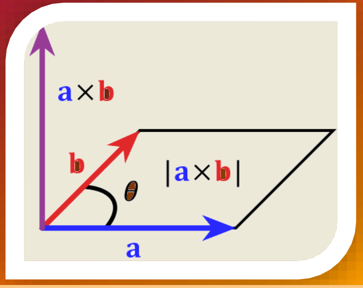
$$x = r\cos\theta \qquad (3.1)$$

$$y = r\sin\theta \qquad (3.2)$$

$$\tan\theta = \frac{y}{x} \qquad (3.3)$$

$$r = \sqrt{x^2 + y^2} \qquad (3.4)$$

- These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when positive θ is an angle measured counterclockwise from the positive x axis.
- If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.



Example 3.1 Polar Coordinates

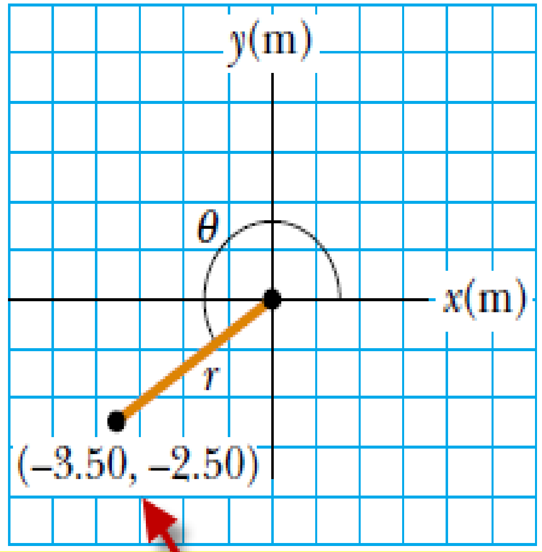
- The Cartesian coordinates of a point in the xy plane are (x, y) = (-3.50, -2.50) m, as shown in the figure. Find the polar, (r, θ), coordinates of this point.
- Solution:

$$\because r = \sqrt{x^2 + y^2} = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.30\, m$$

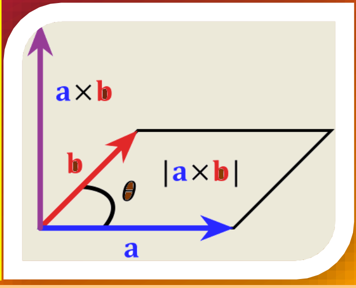
$$\because \tan\theta = \frac{y}{x} = \frac{-2.5}{-3.5} = 0.714$$

$$\therefore \theta = 216^o$$

- Note that you must use the signs of x and y to find that the point lies in the *third quadrant* of the coordinate system. That is, θ = 216° and not 35.5°.



x is (-) and y is (-) Hence, the point must be in the ***third quadrant***



3.2 Vector and Scalar Quantities

- *A scalar quantity* is completely specified by a single value with an appropriate unit and *has no direction*.
- *A vector quantity* is completely specified by a number and appropriate units *plus a direction*.
- Below examples of scalar and vector quantities

scalar quantities	vector quantities
Temperature	*Velocity*
Density	*Acceleration*
Distance	*Force*
Mass	*Displacement*
Speed	*Torque*
Volume	*Weight*

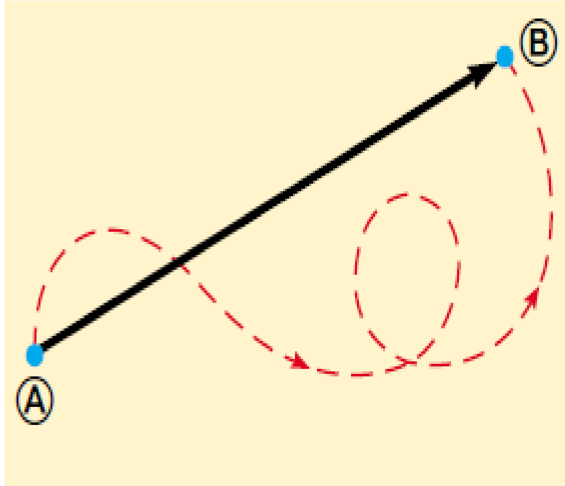
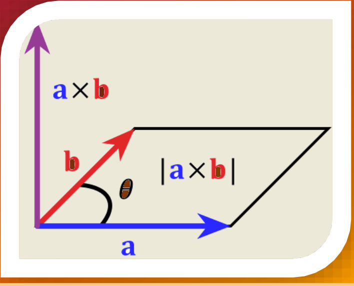
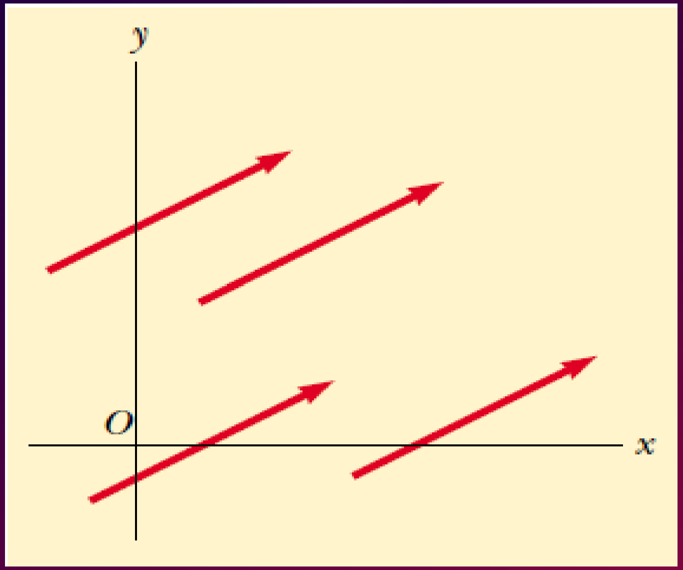


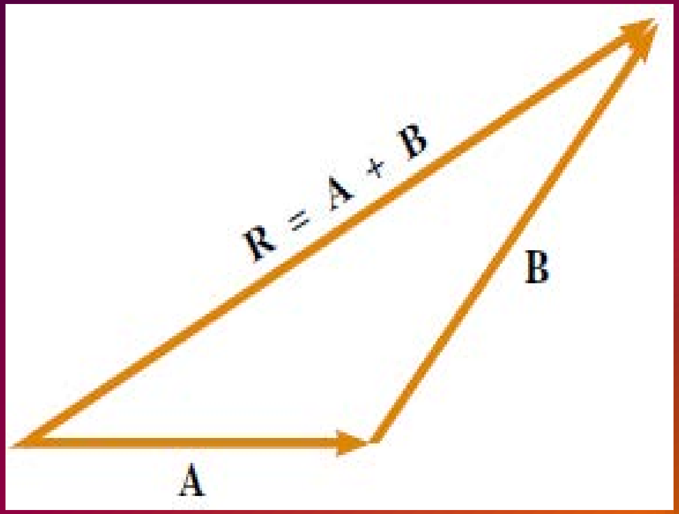
Figure 3.4 As a particle moves from Ⓐ to Ⓑ along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from Ⓐ to Ⓑ.

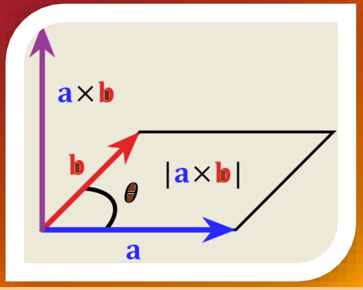


3.3 Some Properties of Vectors (1)

- *Equality of Two Vectors*: **A** = **B** if $A = B$ and if **A** and **B** point in the same direction along parallel lines
- *Adding Vectors*: To add vector **B** to vector **A**, first draw vector **A** on graph paper, and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure. The resultant vector **R** = **A** + **B** is the vector drawn from the tail of **A** to the tip of **B**.
- It is also possible to add vectors using Unit vectors. We shall discuss this feature later in this lecturer.





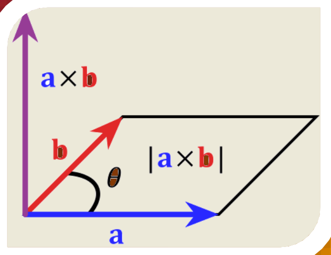


3.3 Some Properties of Vectors (2)

- *commutative law of addition* : **A** + **B** = **B** + **A**
- *associative law of addition:* **A** + (**B** + **C**) = (**A** + **B**) + **C**
- *Negative of a Vector*: The negative of the vector **A** is defined as the vector that when added to **A** gives zero for the vector sum. That is:

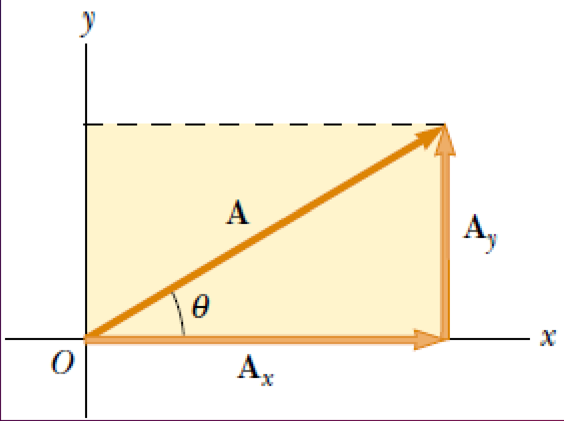
A + (-**A**) = 0. The vectors **A** and -**A** have the same magnitude but point in *opposite directions*

- *Subtracting Vectors:* We define the operation **A** - **B** as vector -**B** added to vector **A**: **A** - **B** = **A** + (-**B**)
- *Multiplying a Vector by a Scalar:* m**A** has *same* direction of **A** with mA in magnitude. -m**A** has opposite direction of **A** with mA in magnitude.
- *For example*, the vector 5**A** is five times as long as **A** and points in the same direction as **A**



3.4 Components of a Vector and Unit Vectors

▶ Consider a vector **A** lying in the xy plane and making an arbitrary angle θ with the positive x axis, as shown in Figure. This vector can be expressed as the sum of two other vectors $\mathbf{A}_x$ and $\mathbf{A}_y$.



$\mathbf{A}_x$ and $\mathbf{A}_y$ are the *VECTOR COMPONENTS* of The vector **A**

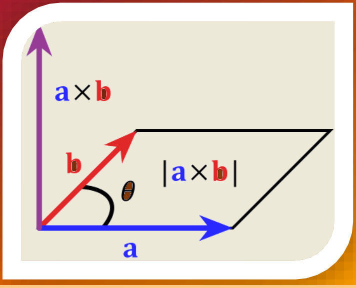
A_x and A_y are the *COMPONENTS* of The vector **A**

$$A_x = A\cos\theta \qquad (3.8)$$

$$A_y = A\sin\theta \qquad (3.9)$$

$$\tan\theta = \frac{A_y}{A_x} \qquad (3.10)$$

$$A = \sqrt{A_x{}^2 + A_y{}^2} \qquad (3.11)$$



3.4 Unit Vectors

- A unit vector is a dimensionless vector having a magnitude of exactly 1.

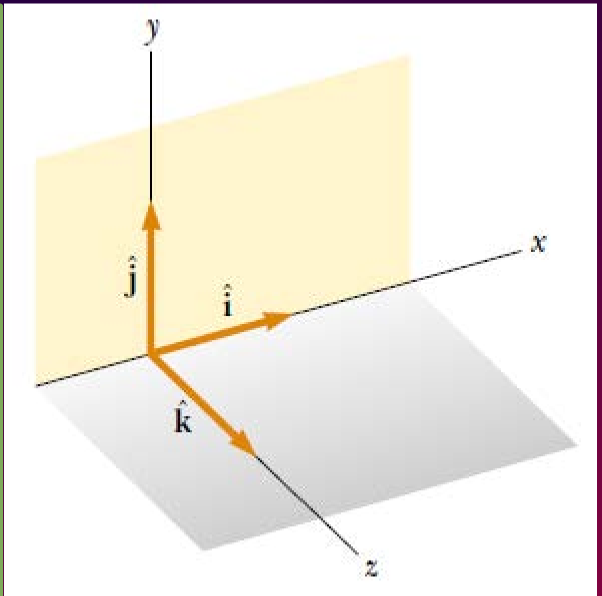
on x: we use: $\hat{i}$

on y: we use: $\hat{j}$

on z: we use: $\hat{k}$

$$\left|\hat{i}\right| = \left|\hat{j}\right| = \left|\hat{k}\right| = 1$$

We express a vector using unit vetors as: $A = A_x \hat{i} + A_y \hat{j}$



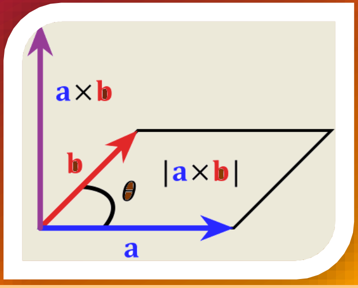
Vector addition is easier with unit vectors:

$$R = A + B = R_x \hat{i} + R_y \hat{j}$$

$$\therefore R = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j})$$

$$\Rightarrow R = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\therefore R_x = A_x + B_x \qquad R_y = A_y + B_y$$



3.4 Unit Vectors

- To find magnitude of the Resultant vector R and the angle it makes with +tive x-axis; we just do same as we did before:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \qquad (3.16)$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_x + B_x}{A_y + B_y} \qquad (3.17)$$

in 3D:

$$A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad (3.18)$$

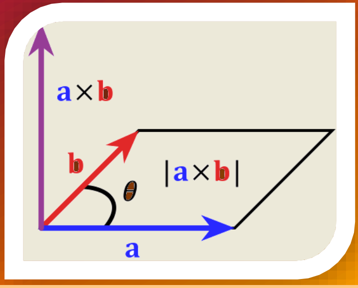
$$B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \qquad (3.19)$$

The sum of A and B is :

$$R = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \qquad (3.20)$$

magnetude of the R:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2 + (A_z + B_z)^2}$$



Example 3.3 The Sum of Two Vectors

- To Find the sum of two vectors A and B lying in the xy plane and given by:

$$A = (2.0\hat{i} + 2.0\hat{j})\,m \qquad \text{and} \qquad B = (2.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}})\,\mathrm{m}$$

Solution:

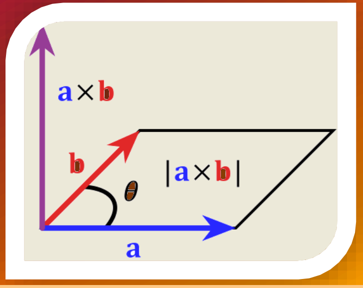
$$\because R = A + B = (2.0\hat{i} + 2.0\hat{j}) + (2.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}}) = (4.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}})\,\mathrm{m}$$

$$\rightarrow |\mathrm{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(4)^2 + (-2)^2} = 4.5\,m$$

$$\theta = \tan^{-1}\frac{-2}{4} = \tan^{-1}(-0.5) = -27^o$$

- But because R is in the $4^{th.}$ quadrant; this angle is not the actual angle based on our convention. We need to add 360^o to this angle:

$$\therefore \theta = -27^o + 360^o = 333^o$$



Example 3.3 The Resultant Displacement

A particle undergoes three consecutive displacements:

$$d_1 = (15\hat{i} + 30\hat{j} + 12\hat{k})\ \text{cm}\,,\quad d_2 = (23\hat{i} - 14\hat{j} - 5\hat{k})\ \text{cm}$$

and $d_3 = (-13\hat{i} + 15\hat{j})\ \text{cm}$

Find the components of the resultant displacement and its magnitude.

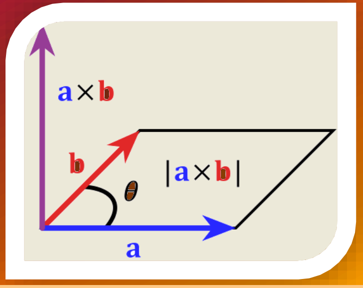
Solution:

$$\because R = d_1 + d_2 + d_3$$

$$\therefore R = (15\hat{i} + 30\hat{j} + 12\hat{k}) + (23\hat{i} - 14\hat{j} - 5\hat{k}) + (-13\hat{i} + 15\hat{j})$$

$$\Rightarrow R = (25\hat{i} + 31\hat{j} + 7\hat{k})\ \text{cm}$$

$$|\mathrm{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25)^2 + (31)^2 + (7)^2} = 40cm$$



Example 3.5 Taking a hike (a)

- A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(a) *Determine the components of the hiker's displacement for each day.*

Components of her first day:

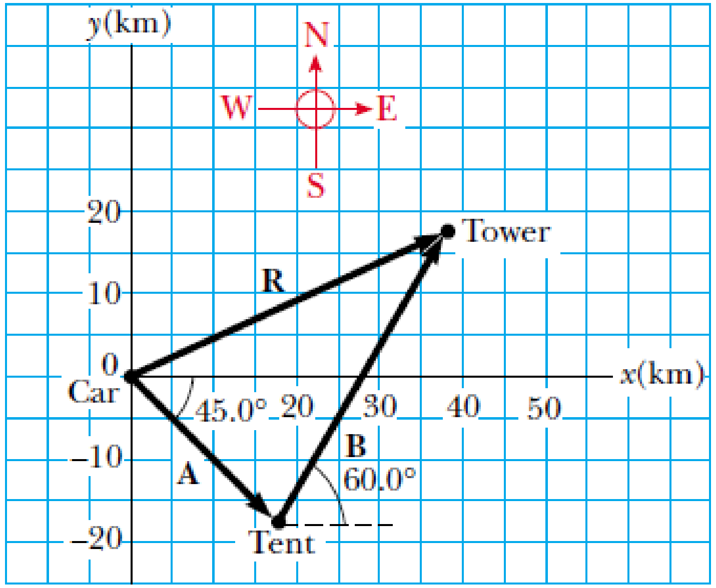
$$A_x = A\cos(-45^o) = 25\cos(-45^o) = 17.7 \text{ km}$$

$$A_y = A\sin(-45^o) = 25\sin(-45^o) = -17.7 \text{ km}$$

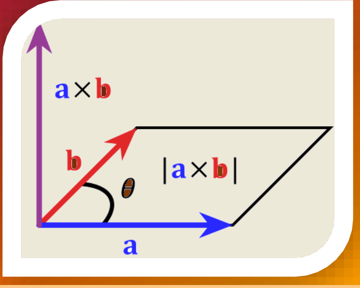
Components of her 2nd day:

$$B_x = B\cos(60^o) = 25\cos(60^o) = 20 \text{ km}$$

$$B_y = B\sin(60^o) = 25\sin(60^o) = 34.6 \text{ km}$$



Please note that: cos(315°) = cos(-45°) and sin (315°) = sin (-45°)
315° is the angle between +x and the vector A (counterclockwise)



Example 3.5 Taking a hike (b)

- (b) *Determine the components of the hiker's resultant displacement R for the trip. Find an expression for R in terms of unit vectors..*

Solution:

$\because$ R=A+B

$$\therefore R_x = A_x + B_x = 17.7 + 20 = 37.7\,km$$

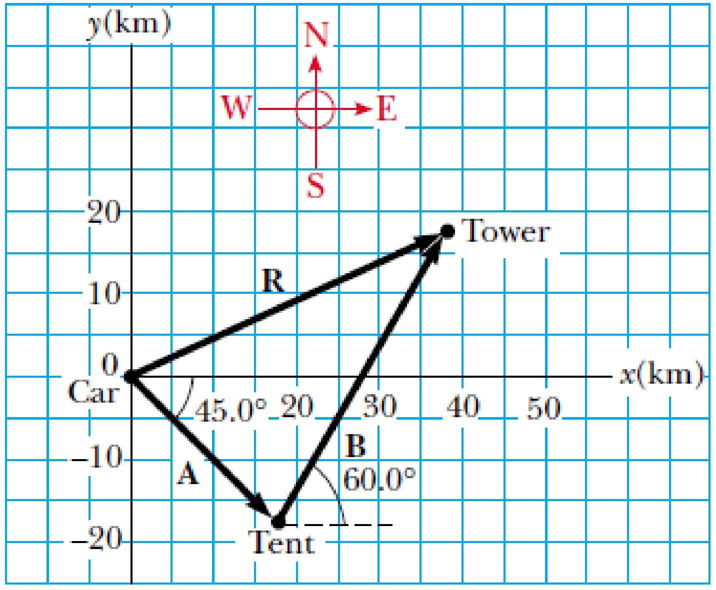
$$R_y = A_y + B_y = -17.7 + 34.6 = 16.9\,km$$

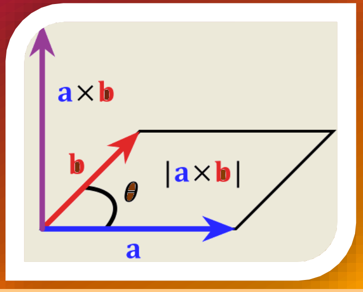
using unit vector notation:

$$R = (37.7\hat{i} + 16.9\hat{j})\,km$$

$$\therefore |R| = \sqrt{(37.7)^2 + (169)^2} = 41.3\,km$$

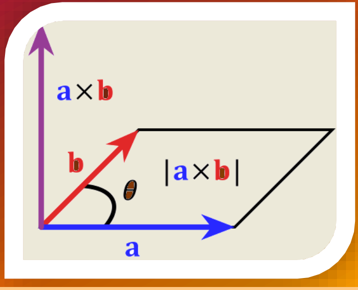
$$\theta = \tan^{-1}\frac{16.9}{37.7} = 24.1^o$$





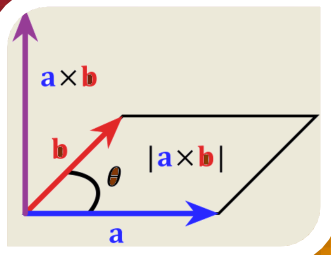
Lecture Summary

- *Scalar quantities*: are those that have only a numerical value and no associated direction.
- *Vector quantities*: have both magnitude and direction and obey the laws of vector addition.
- *The magnitude of a vector* is always a positive number.
- When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity.
- We can add two vectors **A** and **B** graphically. In this method, the resultant vector **R = A + B** runs from the tail of **A** to the tip of **B**.
- A 2nd method involves components of the vectors. $\mathbf{A}_x$ of the vector **A** is equal to the projection of **A** along the x axis of a coordinate system, where $\mathbf{A}_x = \mathbf{A} \cos \theta$. $\mathbf{A}_y$ of **A** is the projection of **A** along the y axis, where $\mathbf{A}_y = \mathbf{A} \sin \theta$

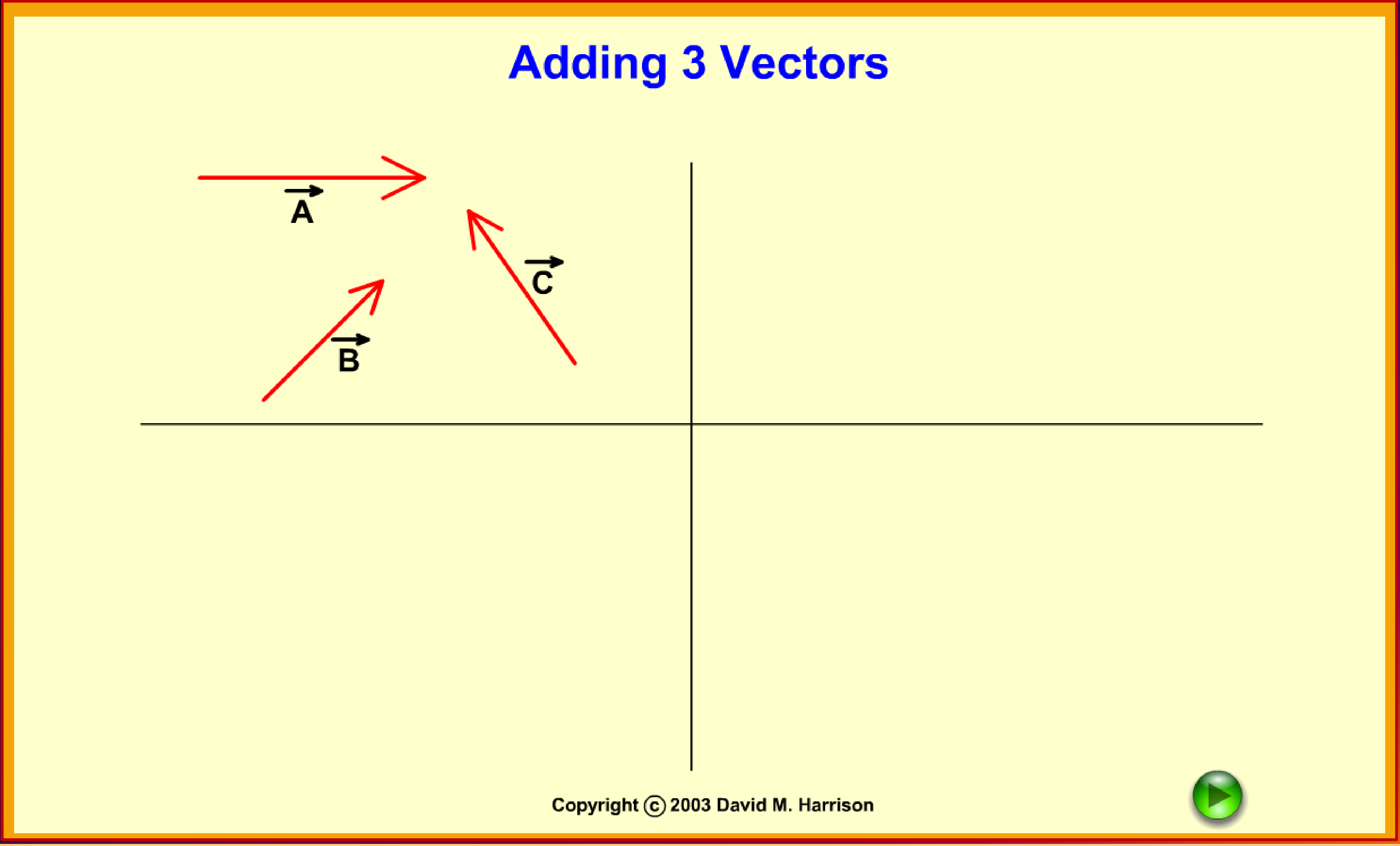


Lecture Summary (continued)

- Be sure you can determine which trigonometric functions you should use in all situations, especially when θ is defined as something other than the counterclockwise angle from the positive x axis.
- If a vector **A** has an x component $\mathbf{A}_x$ and a y component $\mathbf{A}_y$, the vector can be expressed in unit–vector form as $A = A_x \hat{i} + A_y \hat{j}$
- We can find the resultant of two or more vectors by resolving all vectors into their x and y components, adding their resultant x and y components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. We can find the angle that the resultant vector makes with respect to the x axis by using a suitable trigonometric function.



Adding 3 Vectors (Activity)



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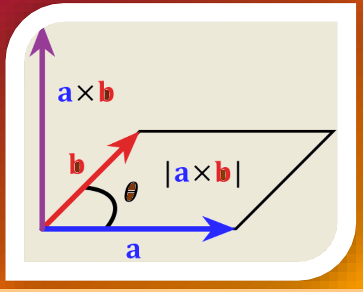
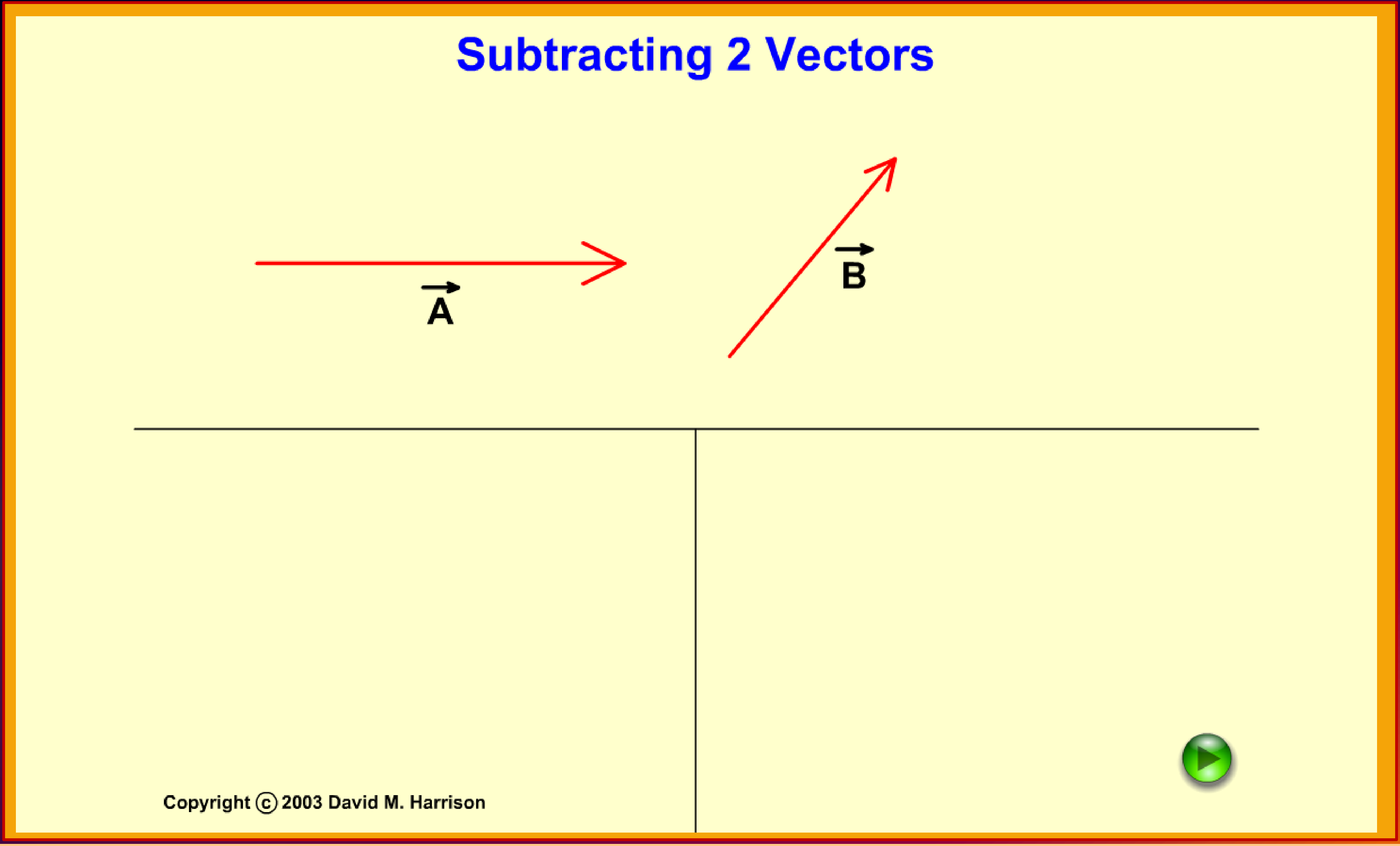


Vector Subtraction(Activity)



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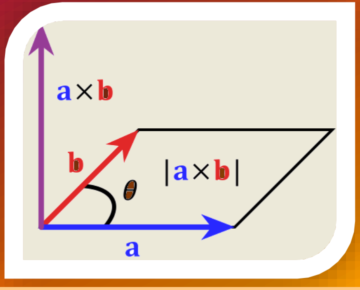
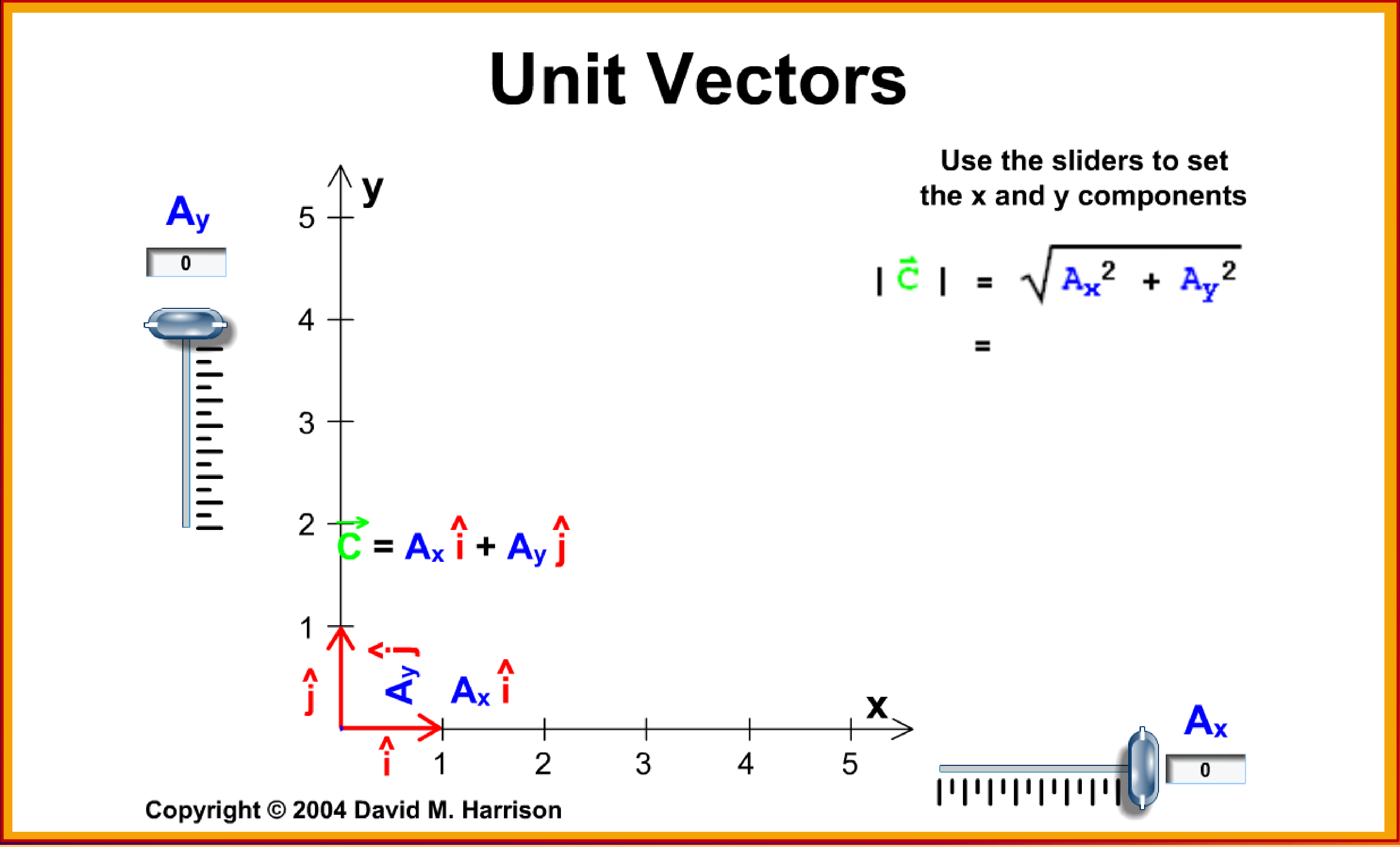
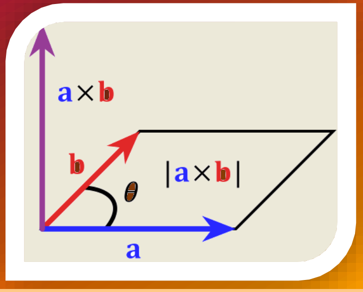


Unit Vectors (Activity)





Quiz

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