Chapter 26

Capacitance

and

Dielectrics

Capacitors

- Capacitors are devices that store electric charge
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - energy-storing devices in electronic flashes

Definition of Capacitance

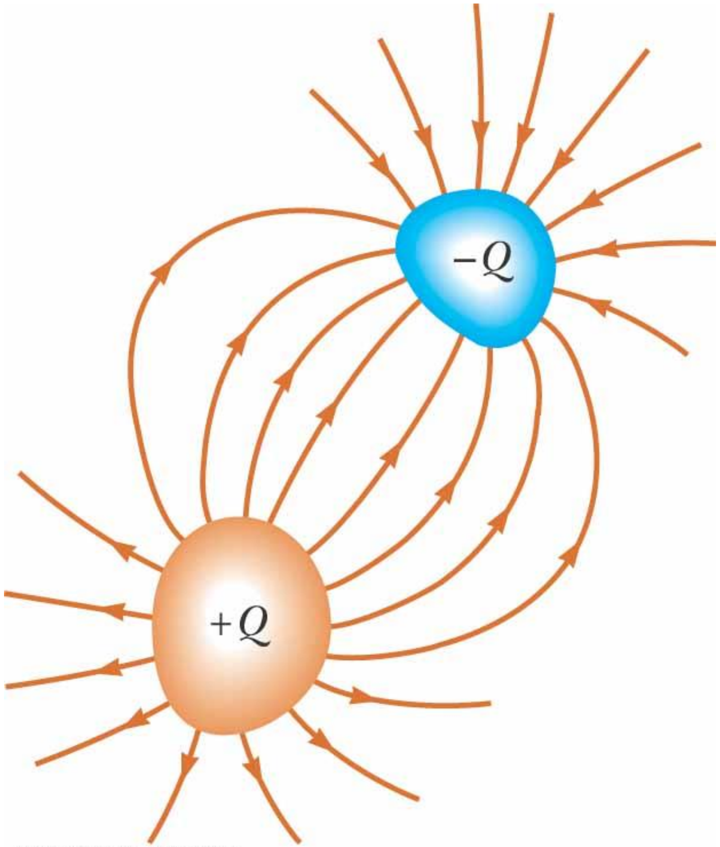
- The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors

$$C = \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F)

Makeup of a Capacitor

- A capacitor consists of two conductors
  - These conductors are called *plates*
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions
- A potential difference exists between the plates due to the charge

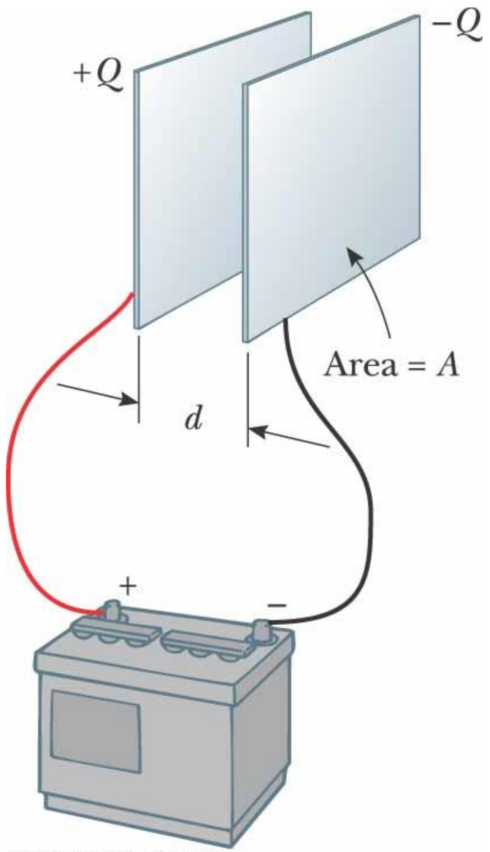


More About Capacitance

- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant
- The capacitance is a measure of the capacitor's ability to store charge
- The farad is a large unit, typically you will see microfarads (μF) and picofarads (pF)

Parallel Plate Capacitor

- Each plate is connected to a terminal of the battery
- If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires



Parallel Plate Capacitor, cont

- This field applies a force on electrons in the wire just outside of the plates
- The force causes the electrons to move onto the negative plate
- This continues until equilibrium is achieved
  - The plate, the wire and the terminal are all at the same potential
- At this point, there is no field present in the wire and the movement of the electrons ceases

Parallel Plate Capacitor, final

- The plate is now negatively charged
- A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged
- In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery

Capacitance – Isolated Sphere

- Assume a spherical charged conductor
- Assume $V = 0$ at infinity

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\varepsilon_o R$$

- Note, this is independent of the charge and the potential difference

Capacitance – Parallel Plates

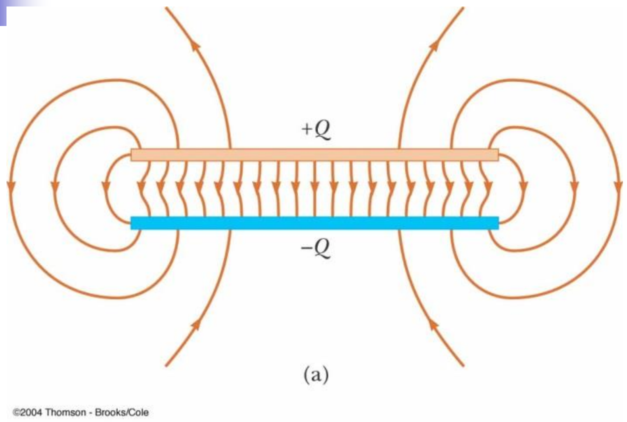
- The charge density on the plates is $\sigma = Q/A$
  - A is the area of each plate, which are equal
  - Q is the charge on each plate, equal with opposite signs
- The electric field is uniform between the plates and zero elsewhere

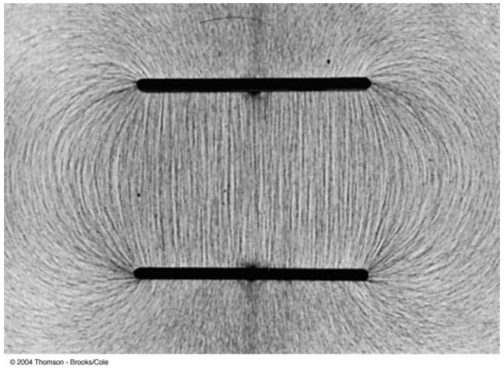
Capacitance – Parallel Plates, cont.

- The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_o A} = \frac{\varepsilon_o A}{d}$$

Parallel Plate Assumptions





- The assumption that the electric field is uniform is valid in the central region, but not at the ends of the plates
- If the separation between the plates is small compared with the length of the plates, the effect of the non-uniform field can be ignored

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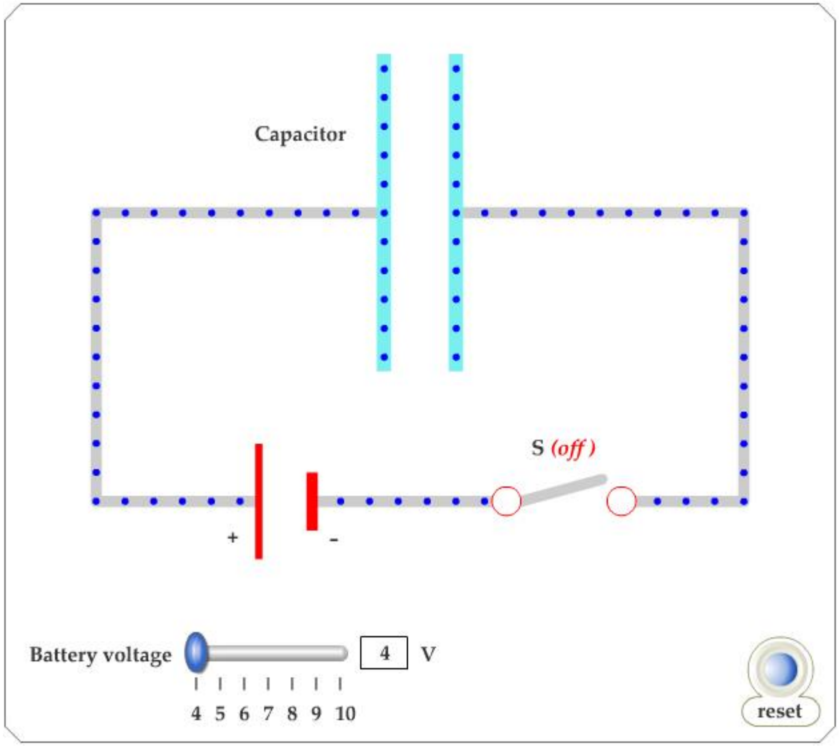
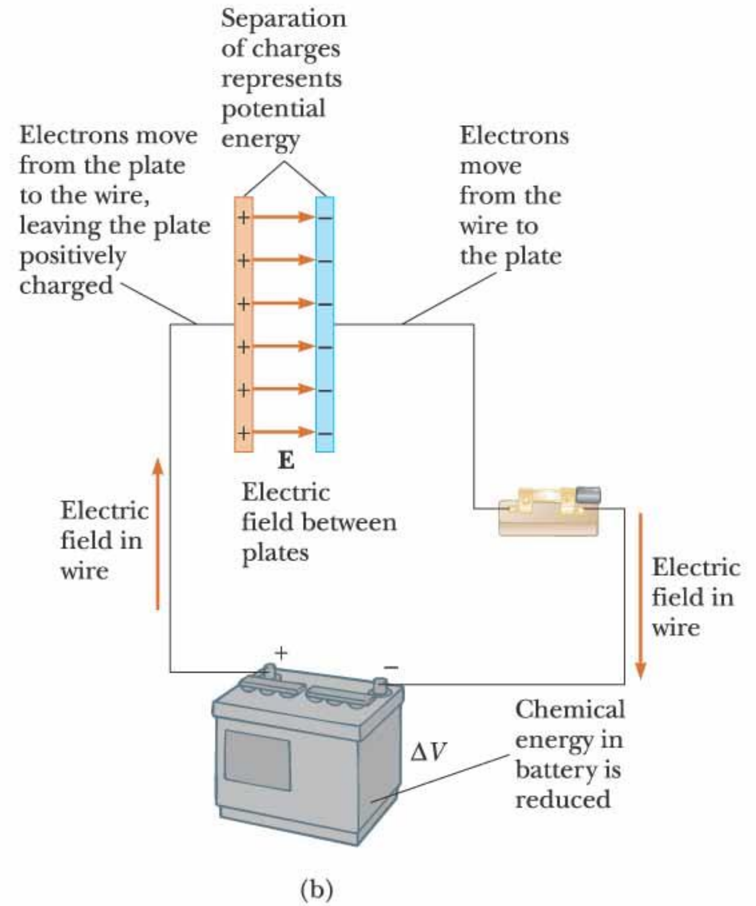


(SLIDESHOW MODE ONLY)

Energy in a Capacitor – Overview

- Consider the circuit to be a system
- Before the switch is closed, the energy is stored as chemical energy in the battery
- When the switch is closed, the energy is transformed from chemical to electric potential energy



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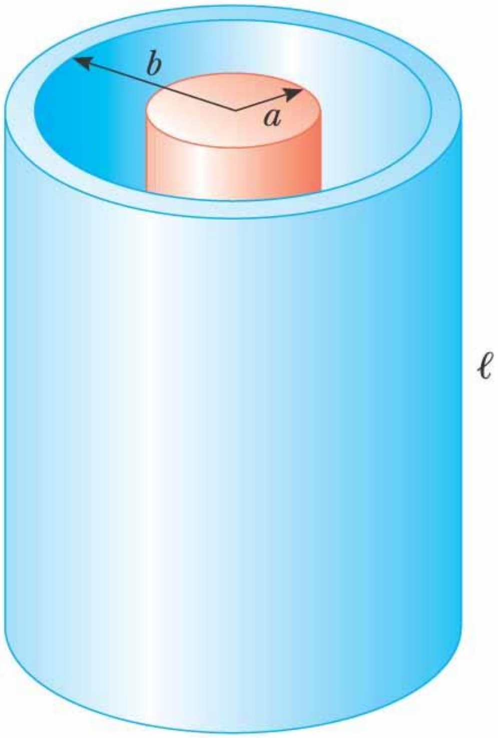
Energy in a Capacitor – Overview, cont

- The electric potential energy is related to the separation of the positive and negative charges on the plates
- A capacitor can be described as a device that stores energy as well as charge

Capacitance of a Cylindrical Capacitor

- From Gauss's Law, the field between the cylinders is
  $E = 2k_e\lambda / r$
- $\Delta V = -2k_e\lambda \ln(b/a)$
- The capacitance becomes

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln(b/a)}$$



(a)

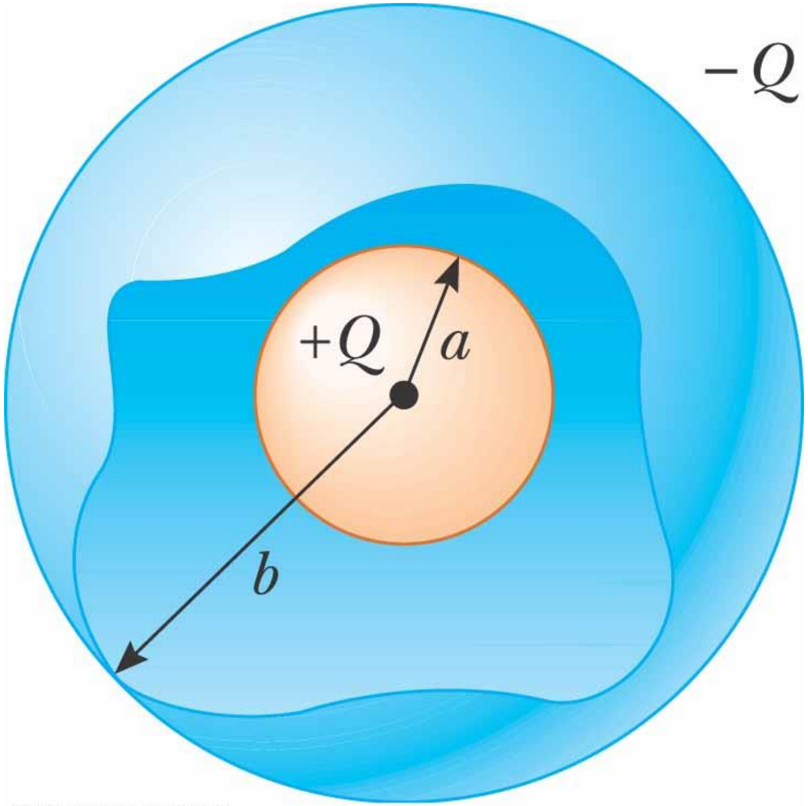
Capacitance of a Spherical Capacitor

- The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

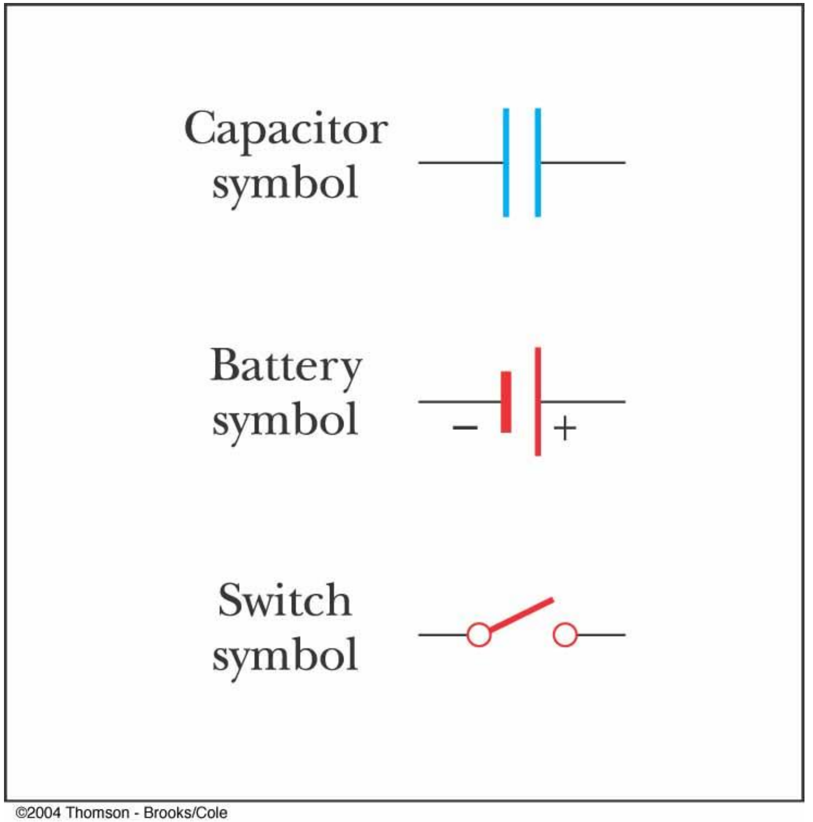
- The capacitance will be

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b - a\right)}$$



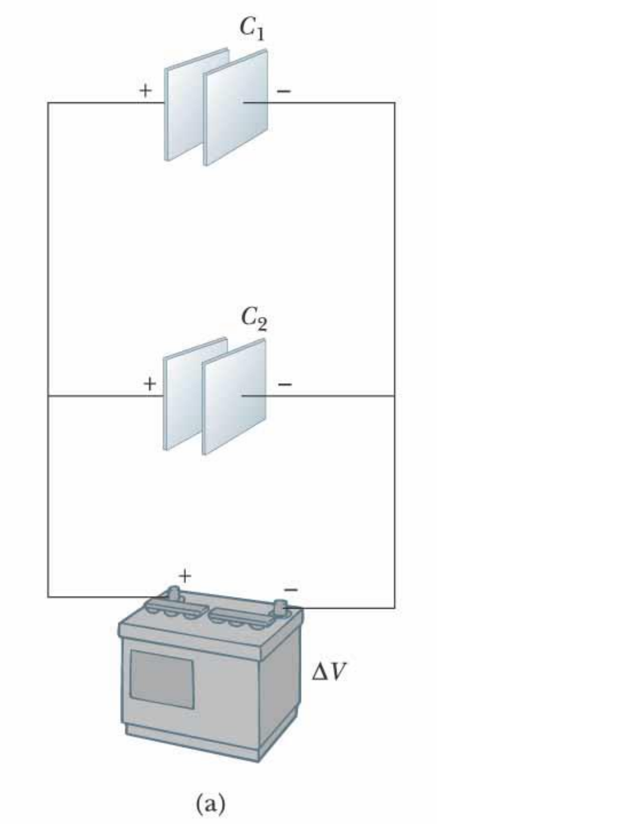
Circuit Symbols

- A circuit diagram is a simplified representation of an actual circuit
- Circuit symbols are used to represent the various elements
- Lines are used to represent wires
- The battery's positive terminal is indicated by the longer line



Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged

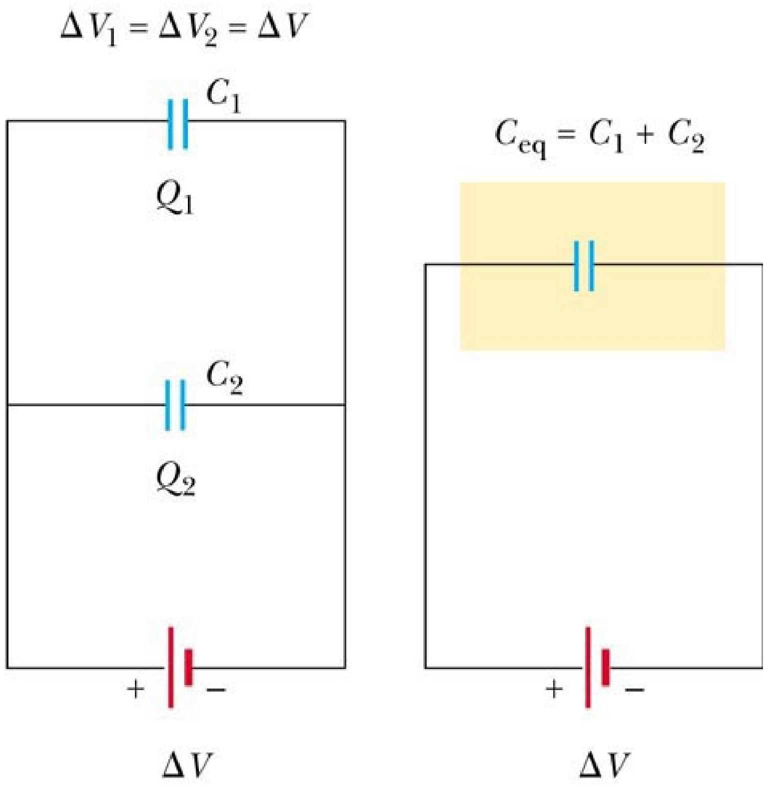


Capacitors in Parallel, 2

- The flow of charges ceases when the voltage across the capacitors equals that of the battery
- The capacitors reach their maximum charge when the flow of charge ceases
- The total charge is equal to the sum of the charges on the capacitors
  - $Q_{total} = Q_1 + Q_2$
- The potential difference across the capacitors is the same
  - And each is equal to the voltage of the battery

Capacitors in Parallel, 3

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}
  - The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors



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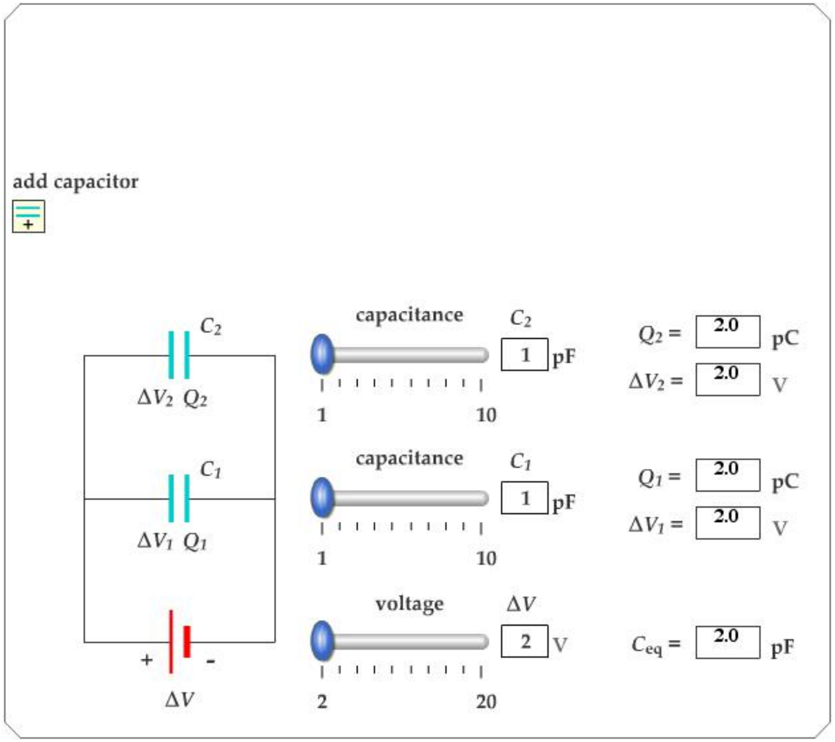


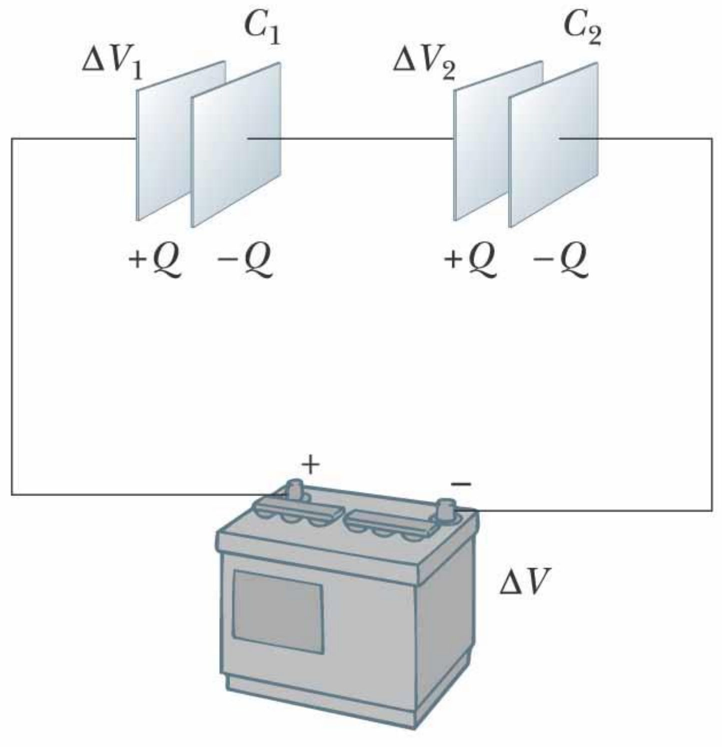
(SLIDESHOW MODE ONLY)

Capacitors in Parallel, final

- $C_{eq} = C_1 + C_2 + \dots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors
  - Essentially, the areas are combined

Capacitors in Series

- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery



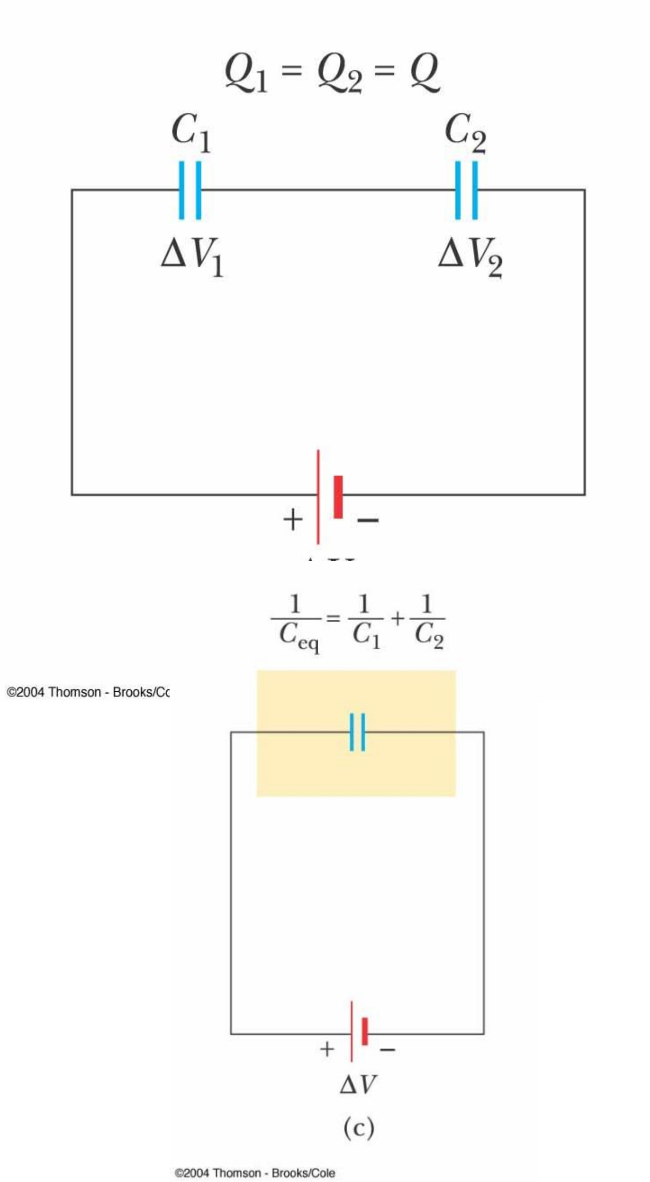
(a)

Capacitors in Series, 2

- As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge
- All of the right plates gain charges of $-Q$ and all the left plates have charges of $+Q$

Capacitors in Series, 3

- An equivalent capacitor can be found that performs the same function as the series combination
- The potential differences add up to the battery voltage



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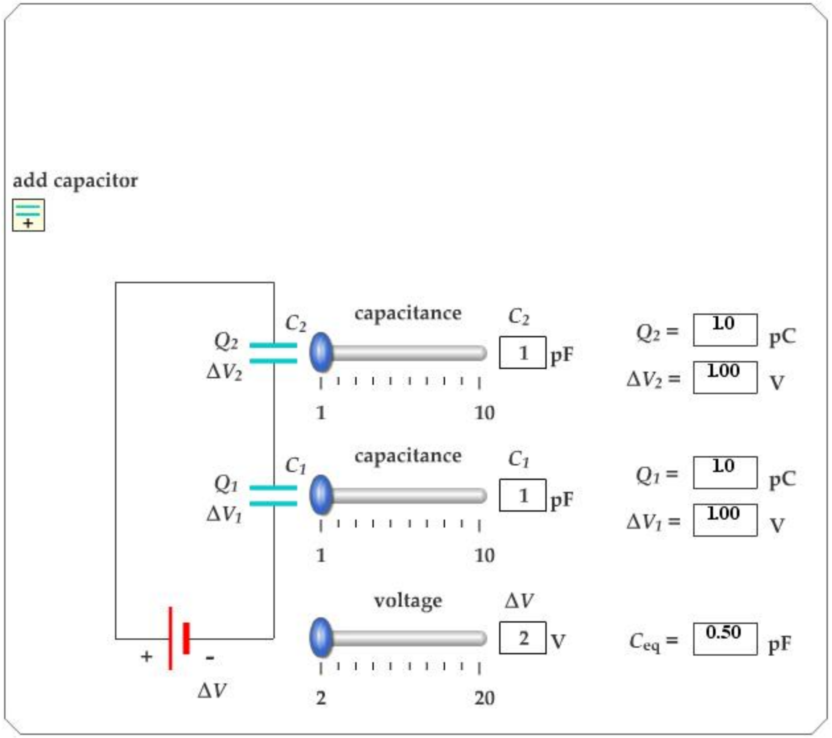


PLAY ANIMATION

(SLIDESHOW MODE ONLY)

Capacitors in Series, final

$$Q = Q_1 + Q_2 + \dots$$

$$\Delta V = V_1 + V_2 + \dots$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination

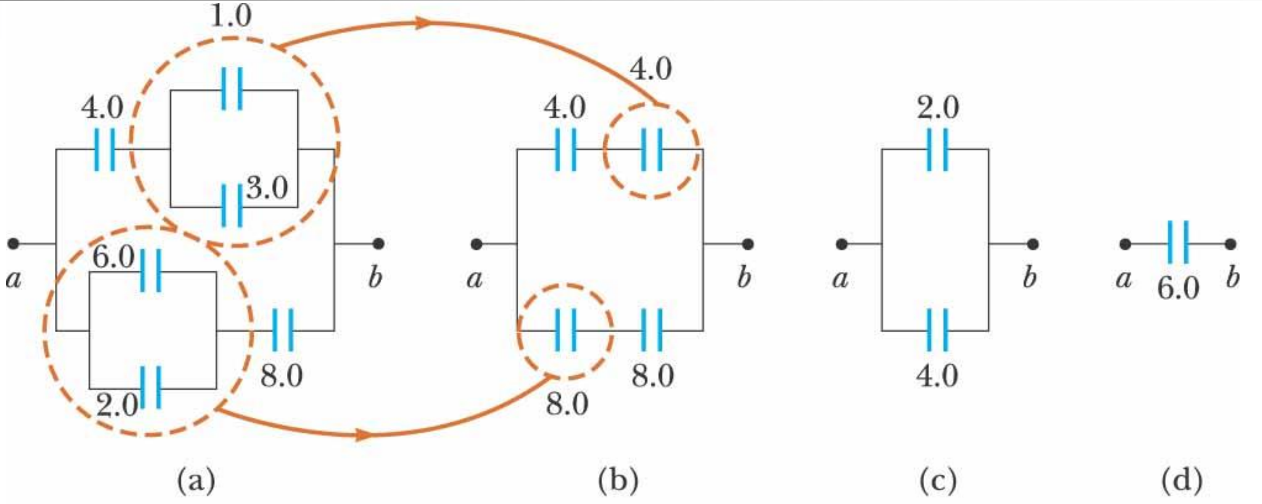
Problem-Solving Hints

- Be careful with the choice of units
  - In SI, capacitance is in farads, distance is in meters and the potential differences are in volts
  - Electric fields can be in V/m or N/C
- When two or more capacitors are connected *in parallel*, the potential differences across them are the same
  - The charge on each capacitor is proportional to its capacitance
  - The capacitors add directly to give the equivalent capacitance

Problem-Solving Hints, cont

- When two or more capacitors are connected *in series*, they carry the same charge, but the potential differences across them are not the same
  - The capacitances add as reciprocals and the equivalent capacitance is always less than the smallest individual capacitor

Equivalent Capacitance, Example



- The 1.0-μF and 3.0-μF capacitors are in parallel as are the 6.0-μF and 2.0-μF capacitors
- These parallel combinations are in series with the capacitors next to them
- The series combinations are in parallel and the final equivalent capacitance can be found

Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge *q* on it
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy, cont

- The work done in charging the capacitor appears as electric potential energy *U*:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

- This applies to a capacitor of any geometry
- The energy stored increases as the charge increases and as the potential difference increases
- In practice, there is a maximum voltage before discharge occurs between the plates

Energy, final

- The energy can be considered to be stored in the electric field
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as $U = ½ (\varepsilon_o A d) E^2$
- It can also be expressed in terms of the energy density (energy per unit volume)

  $u_E = ½\ \varepsilon_o E^2$

Some Uses of Capacitors

- Defibrillators
  - When fibrillation occurs, the heart produces a rapid, irregular pattern of beats
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern
- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse

Capacitors with Dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, plastic, and waxed paper
- For a parallel-plate capacitor, $C = \kappa C_o = \kappa \varepsilon_o (A/d)$
  - The capacitance is multiplied by the factor κ when the dielectric completely fills the region between the plates

Dielectrics, cont

- In theory, *d* could be made very small to create a very large capacitance
- In practice, there is a limit to *d*
  - *d* is limited by the electric discharge that could occur though the dielectric medium separating the plates
- For a given *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material

Dielectrics, final

- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching
    - This decreases d and increases C

Table 26.1

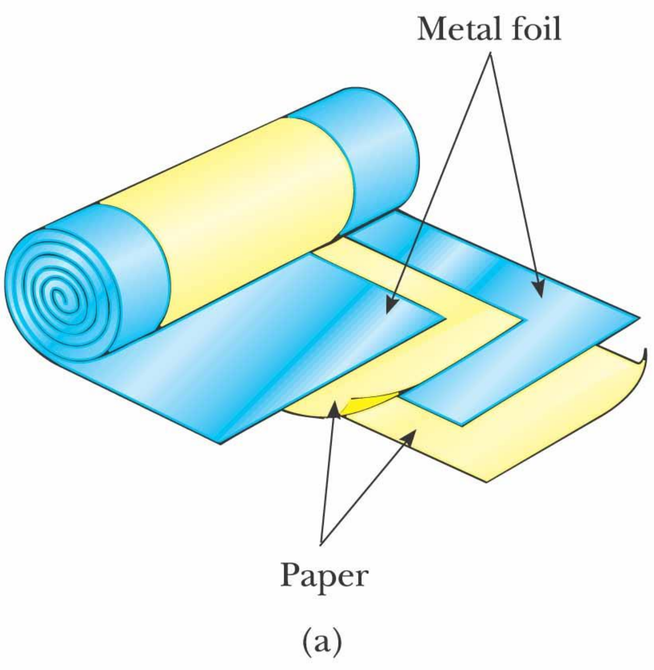
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

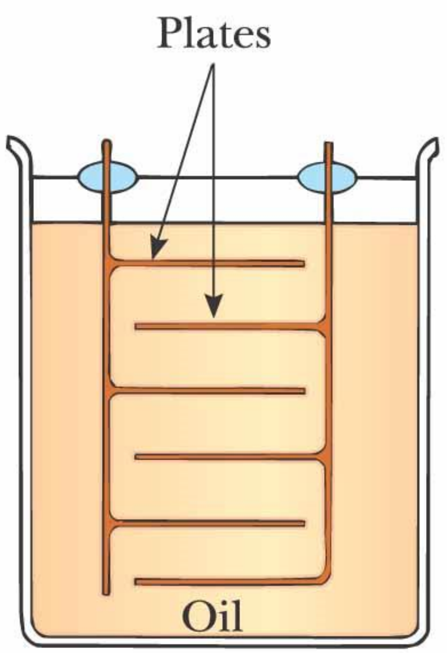
Types of Capacitors – Tubular

- Metallic foil may be interlaced with thin sheets of paper or Mylar
- The layers are rolled into a cylinder to form a small package for the capacitor



Types of Capacitors – Oil Filled

- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil



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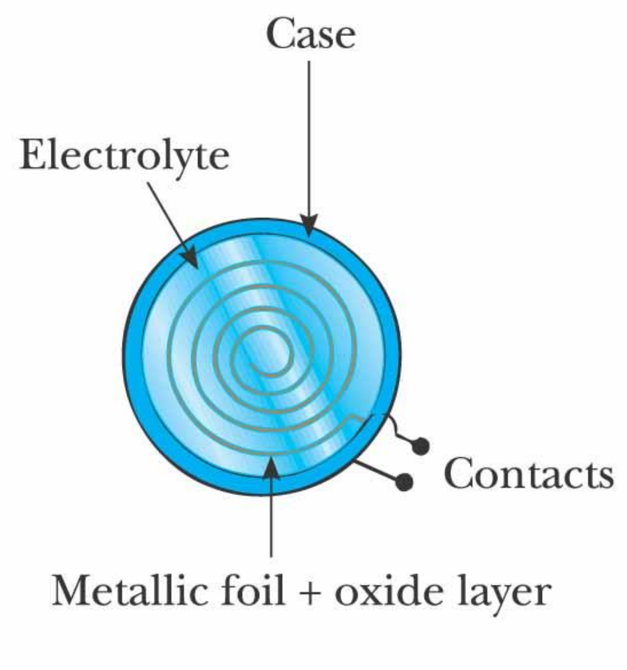
Types of Capacitors – Electrolytic

- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution



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Types of Capacitors – Variable

- Variable capacitors consist of two interwoven sets of metallic plates
- One plate is fixed and the other is movable
- These capacitors generally vary between 10 and 500 pF
- Used in radio tuning circuits