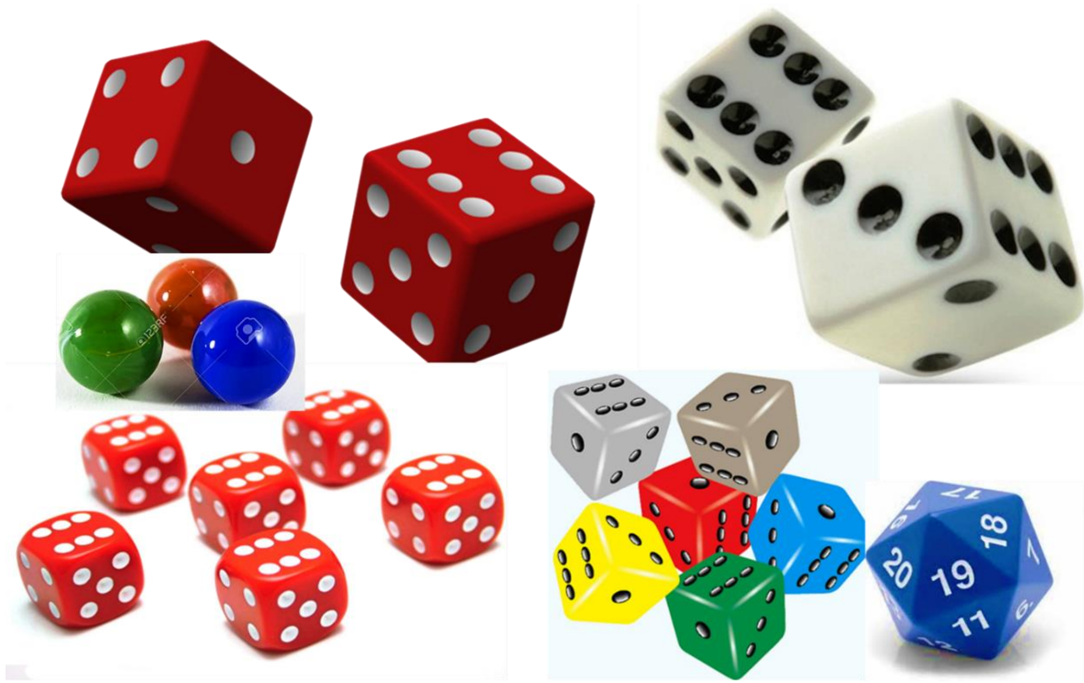
Probability Sample Distribution



OPTO442

Lecture Five

Learning Outcomes

King Saud University

Outcomes

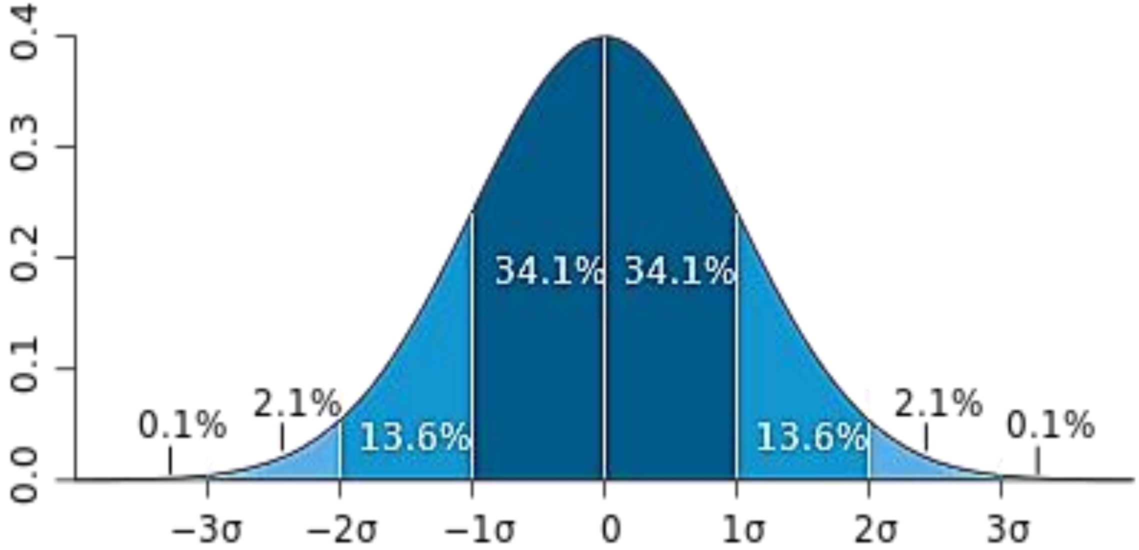
- How to identify the nature of probability distribution?
- How to differentiate between probability and cumulative distribution function?
- What is meant by two events are mutually exclusive?
- What is meant by two events are disjoined?
- How to differentiate between continuous and discrete probability distribution?

Probability Distributions

- To understand probability distributions, it is important to understand the meaning of variables and random variables.
- A probability function maps the possible values of x against their respective probabilities of occurrence, $P(x)$.
- $P(x)$ is a number from 0 to 1.0 or can be between 0 to 100%.
- The area under a probability function is always 1 (or 100%).

Probability Distributions

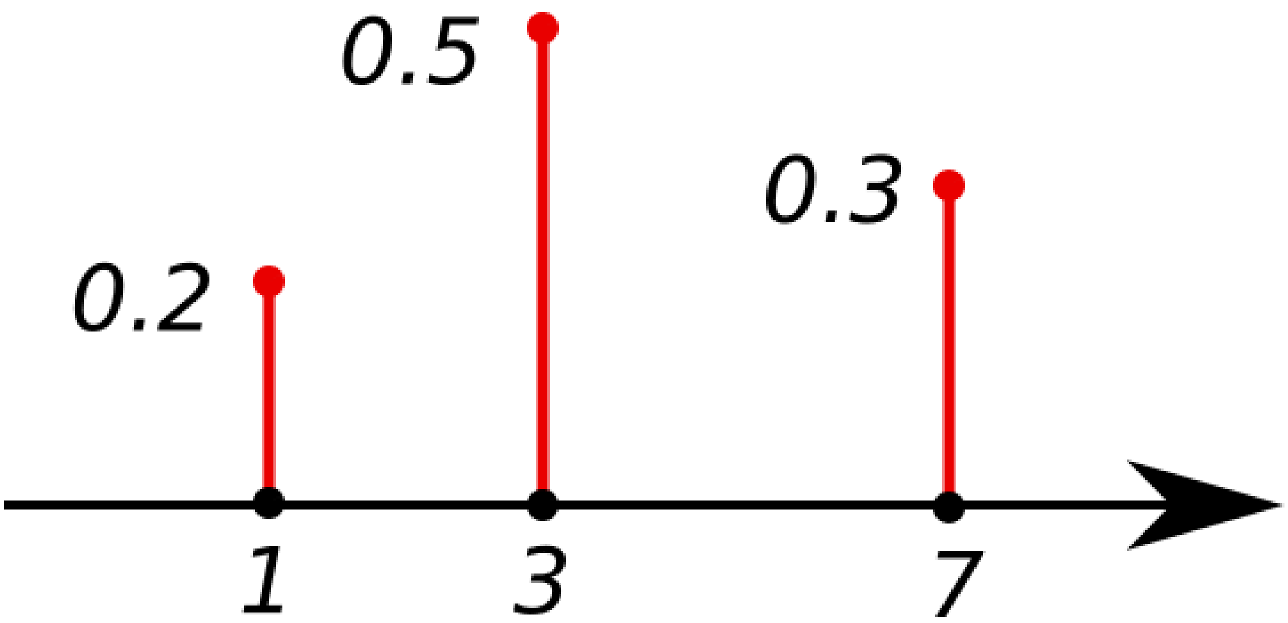
- The probability density function (pdf) of normal distribution is the most important continuous random distribution.



- The probabilities of intervals of values correspond to the area under the curve.

Probability Distributions

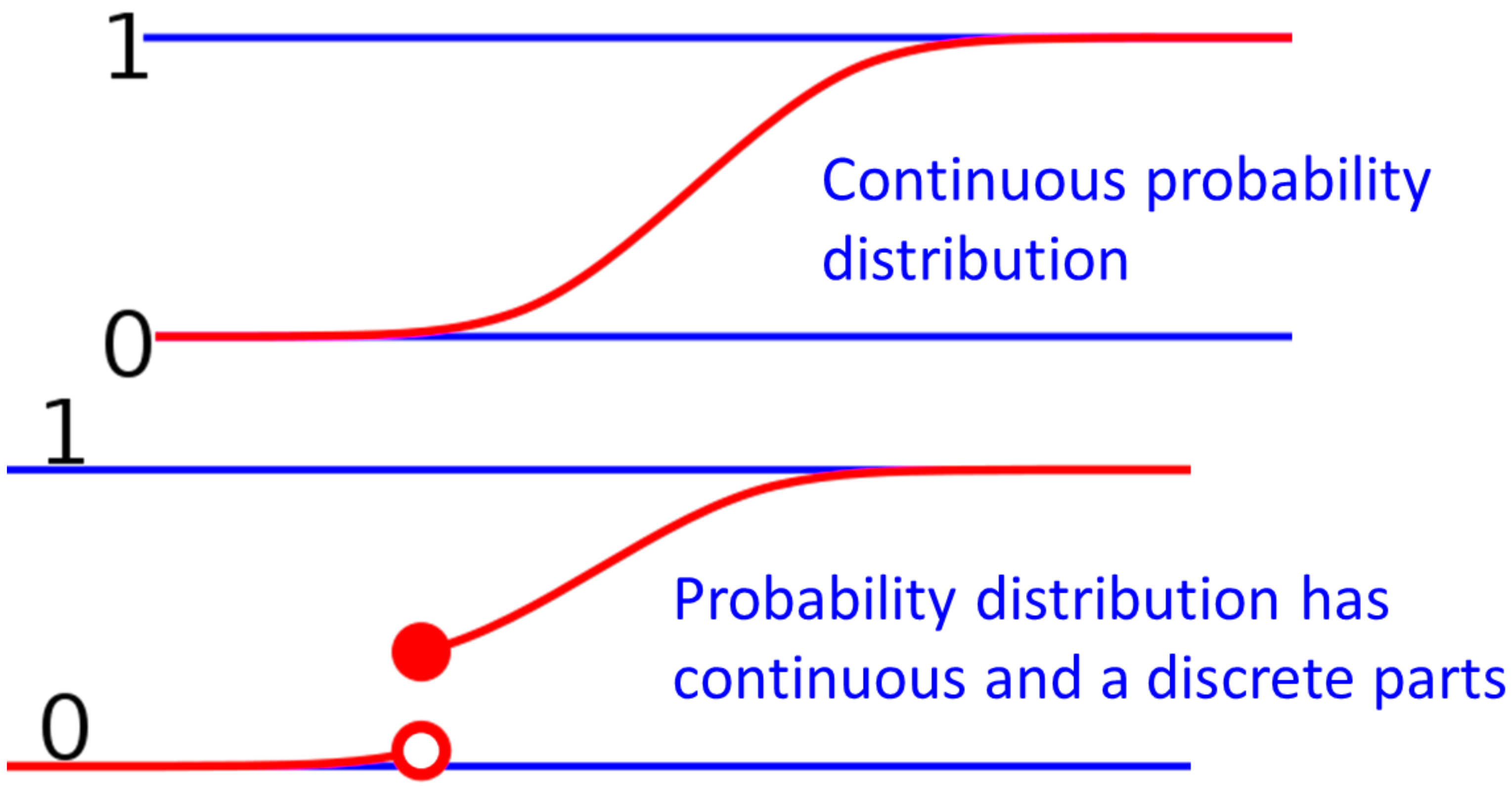
- The probabilities of the singletons {1}, {3}, {7} are 0.2, 0.5, 0.3, respectively.



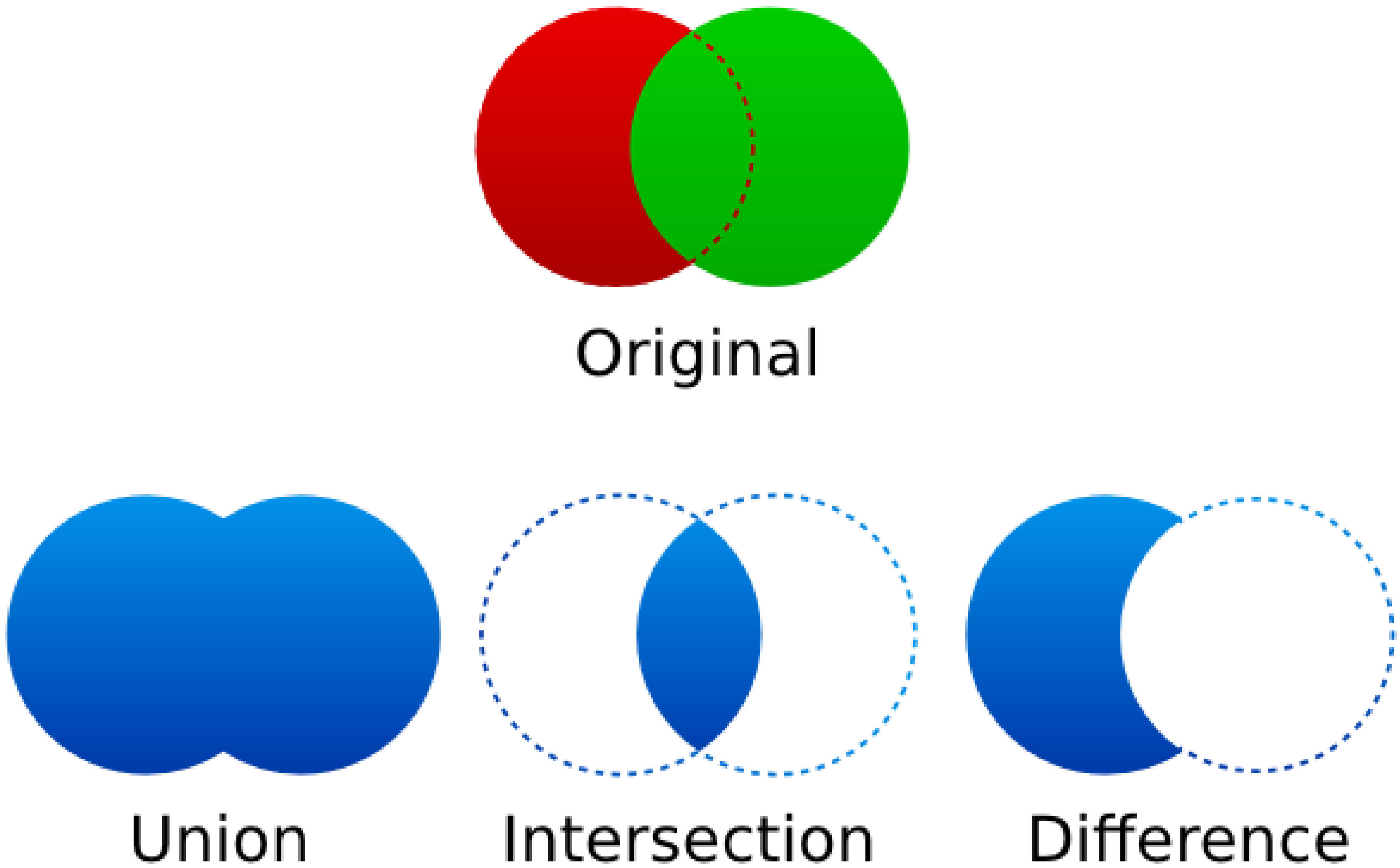
- A set not containing any of these points has probability zero.

Probability Distributions

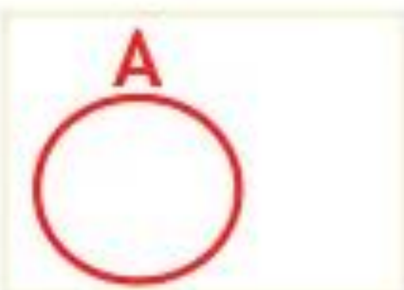
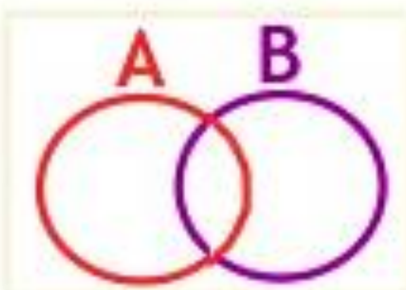
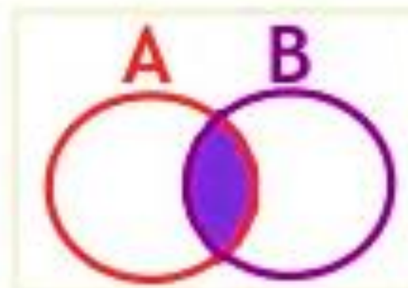
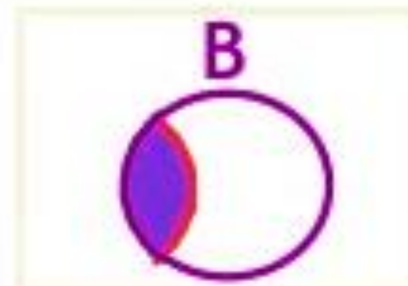
- The probability distribution can be continuous and/or discrete.



Probability Distributions



Probability Distributions

Marginal	Union	Joint	Conditional
$P(A)$	$P(A \cup B)$	$P(A \cap B)$	$P(A \mid B)$
The probability of **A** occurring	The probability of **A or B** occurring	The probability of **A and B** occurring	The probability of **A occurring given that B** has occurred
			

Probability Distributions

- The symbol for union is $\cup$ and the symbol for intersection is $\cap$.
- The union of sets is the set of elements that is in the first set "or" the second set.
- The intersection of sets is the set of elements that are in the first set "and" the second set.
- A = {2,4,6,8,10} and B = {1,2,3,4,5}
- $A \cup B$ = {1,2,3,4,5,6,8,10} (Union)
- $A \cap B$ = {2,4} (Intersection)

Probability Distributions

- “OR” or “Unions”
- Two events are mutually exclusive if they cannot occur at the same time.
- In other words, the two mutually exclusive events are disjoint.
- If two events are disjoint, then the probability of them both occurring at the same time is 0.

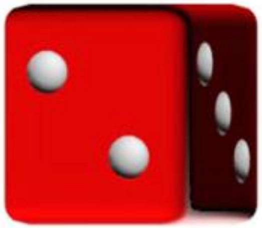
$$P(A \text{ and } B) = 0$$

Probability Distributions

- If two events are mutually exclusive, then the probability of either occurring is the sum of the probabilities of each occurring.

$$P(A \text{ or } B) = P(A) + P(B)$$

- "AND" or Intersections



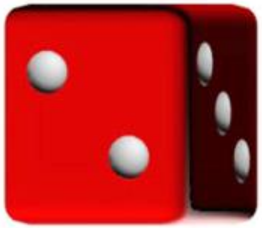
- Two events are independent if the occurrence of one does not change the probability of the other occurring.

- *e.g*. Rolling a 2 on a die and flipping a head on a coin.

Probability Distributions

- Rolling either one of them does not affect the probability of the other.

- If events are independent, then the probability of them both occurring is the product of the probabilities of each occurring.



$$P(A \text{ and } B) = P(A)\,P(B)$$

- If the occurrence of one event does affect the probability of the other occurring, then the events are dependent.

Probability Distributions

King Saud University

If *A* and *B* denotes two possible outcomes then:

$$P(\text{not } A) = 1 - P(A)$$

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ and } B) = P(A)P(B)$$

Exercise

- The blood group type for a randomly selected KSU students in CAMS are shown in the Table.

Blood Group	Males	Females	Total
O	20	20	40
A	17	18	35
B	8	7	15
AB	5	5	10
Total	50	50	100

Exercise

- A subject was randomly selected what is the probability to be a male?
- A subject was randomly selected what is the probability to be a female?
- A subject was randomly selected what is the probability to be a male with O type?
- A subject was randomly selected what is the probability to be a female with AB?
- A subject was randomly selected what is the probability to be a male with A type?

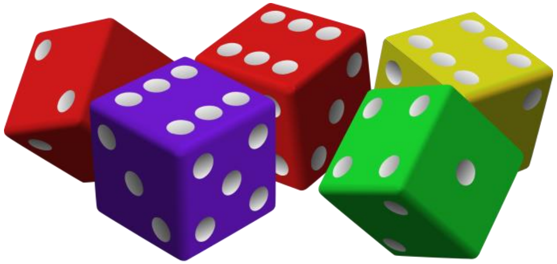
Exercise

Blood Group	Males	Females	Total
O	20	20	40
A	17	18	35
B	8	7	15
AB	5	5	10
Total	50	50	100

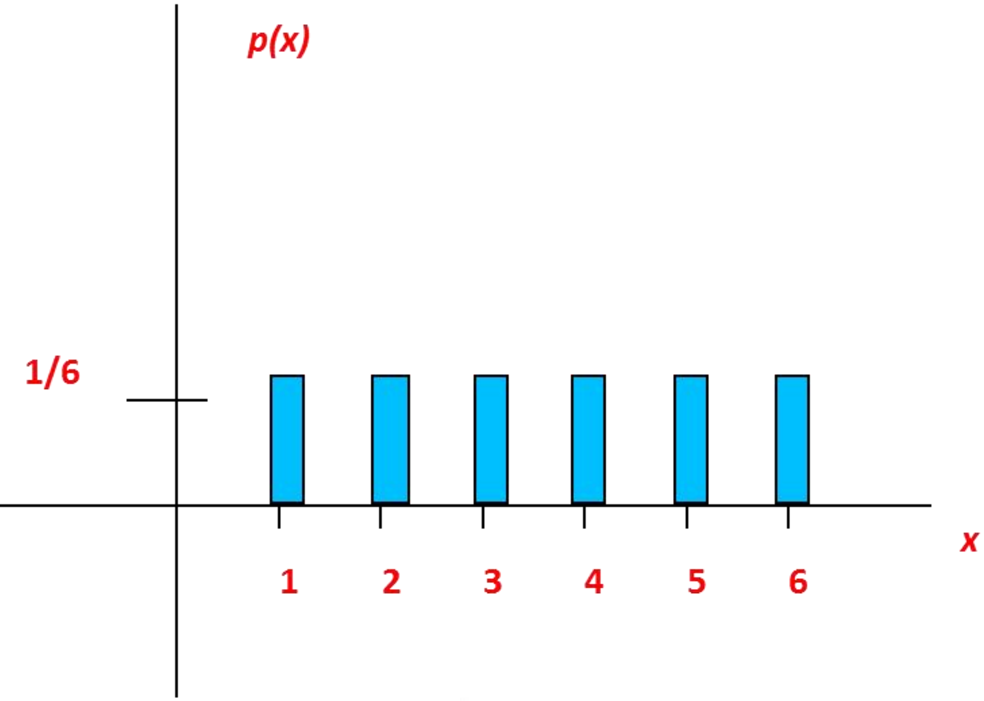
- P(Male) = 50/100 = 0.5
- P(Female) = 50/100 = 0.5
- P(Male | O) = 20/50 = 0.4
- P(Female| AB) = 5/50 = 0.1
- P(Male | A) = 17/50 = 0.34

Example: Roll of a die

x	$P(x)$
1	$P(x = 1) = 1/6$
2	$P(x = 2) = 1/6$
3	$P(x = 3) = 1/6$
4	$P(x = 4) = 1/6$
5	$P(x = 5) = 1/6$
6	$P(x = 6) = 1/6$
Total	1.0



Example: Roll of a die

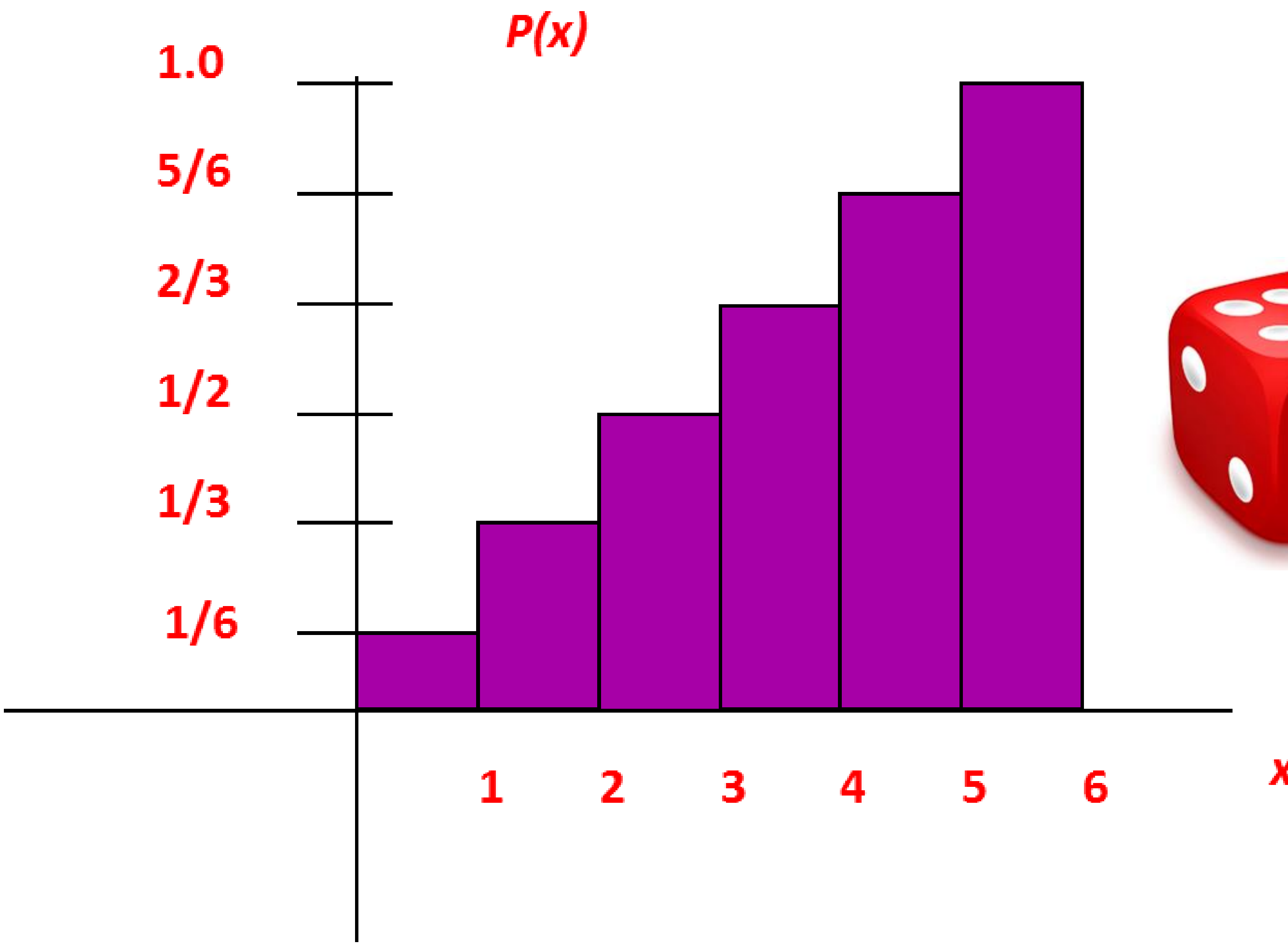


$$\sum_{\text{all } x} P(x) = 1$$

Cumulative Distribution Function

x	$P(x)$
1	$P(x \leq 1) = 1/6$
2	$P(x \leq 2) = 2/6$
3	$P(x \leq 3) = 3/6$
4	$P(x \leq 4) = 4/6$
5	$P(x \leq 5) = 5/6$
6	$P(x \leq 6) = 6/6$

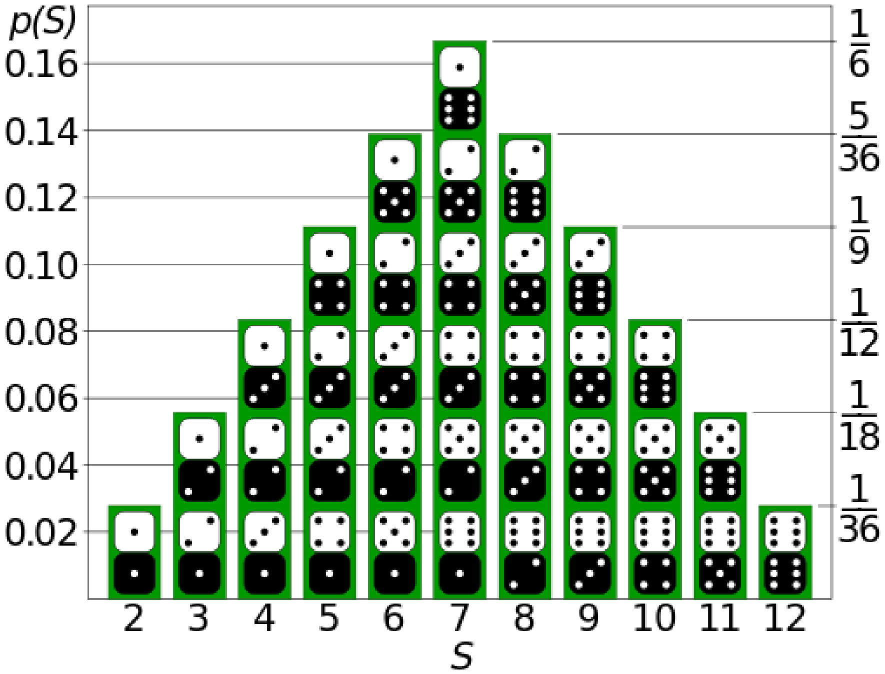
Cumulative Distribution Function



Probability Distributions

King Saud University

- The probability distribution for the sum *S* of counts from two dice is shown below.



- $P(11) = 1/18$
- $P(S > 9) = 1/12 + 1/18 + 1/36 = 1/6$

Exercise

- If you toss a die, what's the probability that you roll a 3 or less?

a) 1/6

b) 1/3

1/2

d) 5/6

e) 0.1

Exercise

- Two dice are rolled and the sum of the face values is six.
- What is the probability that at least one of the dice came up a 3?

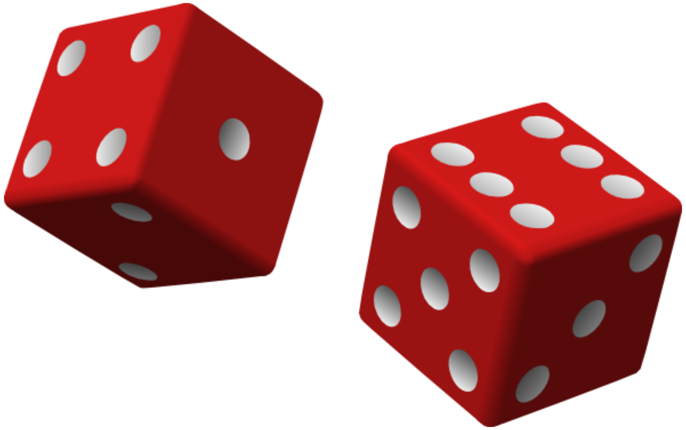
1/5

b) 2/3

c) 1/2

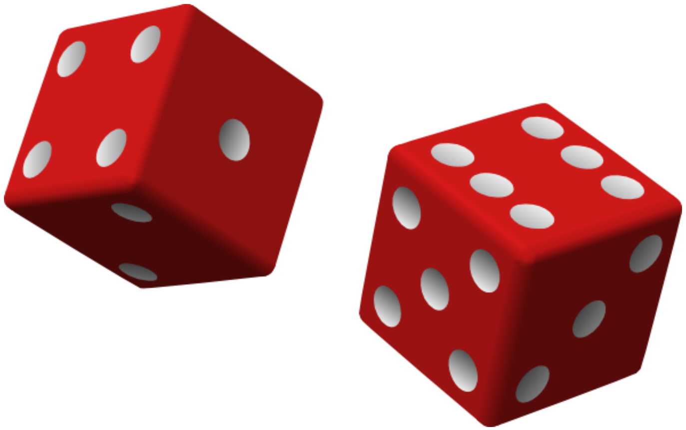
d) 5/6

e) 1.0



Exercise

- Explanation for previous exercise's answer.
- Five possibilities as follows:
- 1 and 5
- 5 and 1
- 2 and 4
- 4 and 2
- 3 and 3
- One of these five has a 3.
- Therefore, $P(x) = 1/5$



Exercise

- The number of patients seen in a hospital in any given hour is a random variable represented by *x*. The probability distribution for *x* is shown the Table.

x	10	11	12	13	14
P(x)	0.4	0.2	0.2	0.1	0.1

- Find the probability that in a given hour:

a) Exactly 14 patients arrived.

b) At least 12 patients arrived.

c) At most 11 patients arrived.

Exercise

x	10	11	12	13	14
$P(x)$	0.4	0.2	0.2	0.1	0.1

a) Exactly 14 patients arrived

$P(x = 14) = 0.1$

b) At least 12 patients arrived

$P(x \geq 12) = 0.2 + 0.1 + 0.1 = 0.4$

c) At most 11 patients arrived

$P(x \leq 11) = 0.4 + 0.2 = 0.6$

Exercise

Income	Very	Pretty	Not too	Total
Above Average	164	233	26	423
Average	293	473	117	883
Below Average	132	383	172	687
Total	589	1089	315	1993

Exercise

Income	Very	Pretty	Not too	Total
Above Average	164	233	26	423
Average	293	473	117	883
Below Average	132	383	172	687
Total	589	1089	315	1993

- Let A = above average income, B = very happy:

$P(A) = 423/1993 = 0.212$

$P(\text{not } A) = 1 - P(A) = 1 - 0.212 = 0.788$

Exercise

Income	Very	Pretty	Not too	Total
Above Average	164	233	26	423
Average	293	473	117	883
Below Average	132	383	172	687
Total	589	1089	315	1993

P(B) = 589/1993 = 0.296

P(B given A) = 164/423 = 0.388

P(A and B) = P(A)P(B given A) =

0.212 × 0.388 = 0.082

i.e. = 164/1993

Exercise

- A random sample of 3 people has been asked whether they favor (*F*) or oppose (*O*) legalization of a new law.

- *y* = (*F*) number who "favor" (0, 1, 2, or 3)
- For possible samples of size $n = 3$, the vote recorded are shown in the Table.

Sample	*OOO*	*OOF*	*OFO*	*FOO*	*OFF*	*FOF*	*FFO*	*FFF*	Total
y	0	1	1	1	2	2	2	3	8

Exercise

- If population equally split between *F* (favor) and *O* (oppose) these eight samples are equally likely.

- In practice, probability distributions are often estimated from sample data and then have the form of frequency distributions.

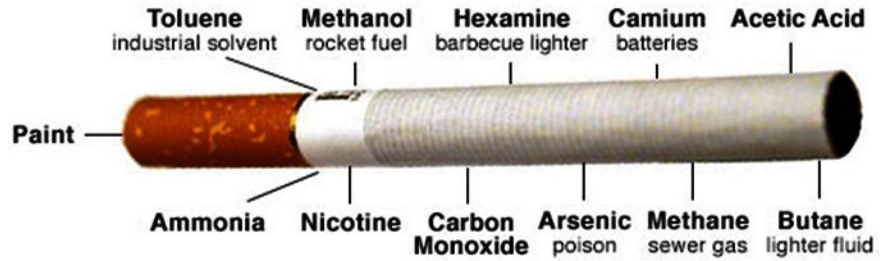
Y = F	**0**	**1**	**2**	**3**
P(y)	1/8	3/8	3/8	1/8

Exercise

- A case-control study of smoking and dryness of eye has been conducted.
- In a statistics class, 12 students have normal eyes and 9 have dry eyes.

If one of the students is randomly selected, find the probability of getting one have a dry eye.

- *P(dry eye) =* $9/21 = 0.429$



Exercise

- Based on data from the National Health Examination, the probability of a randomly selected adult being 1.8 meter or shorter is 0.86.
- Find the probability of randomly selecting an adult and getting someone taller than 1.8 meter.
- *P(>1.8 m)* = 1 – 0.86 = 0.14