Name:......Answer......

ID:............

Section:............

Quiz1- First Semester 1443H – Math 244

Duration: 30 min

جامعة الملك سعود
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Question 1 [Marks: 1.5]:

Find the diagonal matrix $\mathbf{D}^3$ that satisfies $\mathbf{D} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

Answer:

$$D^3 = \begin{pmatrix} (-1)^3 & 0 & 0 \\ 0 & 2^{3/2} & 0 \\ 0 & 0 & 2^3 \end{pmatrix}$$

Question 2 [Marks: 1.5]:

Let $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ be a matrix and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be a unit matrix. Find x and y if

$A^2 + x\,A = -y\,I$.

Answer:

$$A^2 = A \cdot A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 9 & 4 \end{pmatrix}$$

$$A^2 + x\,A = \begin{pmatrix} 1 & 0 \\ 9 & 4 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 1+x & 0 \\ 9+3x & 4+2x \end{pmatrix} = -yI = \begin{pmatrix} -y & 0 \\ 0 & -y \end{pmatrix}$$

So $\begin{cases} 1+x = -y \\ 9+3x = 0 \\ 4+2x = -y \end{cases} \Leftrightarrow \begin{cases} x = -3 \\ y = 2 \end{cases}$

Question 3 [Marks: 1.5]:

Let $\mathbf{A} \in \mathbf{M}_{3\times 3}(\mathbb{R})$ with determinant $|A| = 2$. Find $\left|2(\boldsymbol{adj}(\boldsymbol{A}))^{-1} + \boldsymbol{A}\right|$.

Answer:

$$A^{-1} = \frac{1}{|A|} Adj(A) \quad So \quad (A^{-1})^{-1} = |A| \left[Adj(A)\right]^{-1}$$

$$\left[Adj(A)\right]^{-1} = \frac{1}{|A|} A$$

$$\left| 2\left[Adj(A)\right]^{-1} + A \right| = \left| \frac{2}{|A|} A + A \right| = |2A| = 2^3 |A| = 16$$

Question 4 [Marks: 1.5]**:**

Show that the matrix $\mathbf{A}=\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix}$ is row equivalent to the matrix $\mathbf{I}=\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$.

Answer:

$$\begin{array}{ccc} -2 & -2 & -1 \\ \end{array}$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & -1 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & -3 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & +2 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$