Relating continuous, discrete and mthly insurances.

- using the UDD assumption:

we have under UDD assumption that:

for $0 \leq s < 1$ and integer y

$$ {}_sp_y \, \mu_{y+s} = q_y \quad \text{then} $$

$$ \bar{A}_x = \int_0^\infty e^{-\delta t} \, {}_tp_x \, \mu_{x+t} \, dt $$

$$ = \sum_{k=0}^{\infty} \int_k^{k+1} e^{-\delta t} \, {}_tp_x \, \mu_{x+t} \, dt $$

$$ = \sum_{k=0}^{\infty} \int_0^1 e^{-\delta(k+s)} \, {}_{k+s}p_x \, \mu_{x+k+s} \, ds $$

$t = k+s \Rightarrow dt = ds$

$t = k \Rightarrow s = 0$

$t = k+1 \Rightarrow s = 1$

$$ = \sum_{k=0}^{\infty} e^{-\delta k} \, {}_kp_x \int_0^1 e^{-\delta s} \, {}_sp_{x+k} \, \mu_{x+k+s} \, ds $$

$$ = \sum_{k=0}^{\infty} {}_kp_x \, v^{k+1} \int_0^1 e^{(1-s)\delta} \underbrace{{}_sp_{x+k} \, \mu_{x+k+s}}_{q_{x+k} \ (\text{UDD})} \, ds $$

$$ = \sum_{k=0}^{\infty} {}_kp_x \, q_{x+k} \, v^{k+1} \int_0^1 e^{(1-s)\delta} \, ds $$

$$ = A_x \, \frac{e^{\delta} - 1}{\delta} $$

as we have $e^{\delta} = 1+i \Rightarrow$ under the UDD weget. ②

$$\bar{A}_x = \frac{i}{\delta} A_x$$

- Using the claims accelaration approach consider for example, A_x and $A_x^{(4)}$ and suppose that the insured life, (x), dies in the year of age $x+k_x$ to $x+k_x+1$. Under the end year of death benefit (valued by A_x), the sum insured is paid at time K_x+1, in this case the benefit will be paid either at $K_x+\frac{1}{4}$, $K_x+\frac{2}{4}$, $K_x+3/4$ or K_x+1. depending on the quarter year in which the death occurred, in this case the average the benefit is paid at $K_x+5/8$ which is $3/8$ years earlier than the end of year of death benefit.

Similarly, suppose the benefit is paid in the end of the month of ~~death~~ death, Assume the death occurs uniformly over the year, $\Rightarrow$ average benefit is paid $K_x+13/24$ which is $11/24$ years. earlier in general for 1 umte mthy benfit and we assume the death is uniformly distributed. over year age, the average time of payment of the death benefit is $(m+1)/2m$ in the year of death. ③

$\Rightarrow$

$$A_x^{(m)} \simeq q_x\, v^{\frac{m+1}{2m}} + {}_{1|}q_x\, v^{1+\frac{m+1}{2m}} + {}_{2|}q_x\, v^{2+\frac{m+1}{2m}} + \cdots$$

$$\simeq \sum_{k=0}^{\infty} {}_{k|}q_x\, v^{k+\frac{m+1}{2m}}$$

$$\simeq (1+i)^{\frac{m-1}{2m}} \sum_{k=0}^{\infty} {}_{k|}q_x\, v^{k+1}$$

$$A_x^{(m)} \simeq (1+i)^{\frac{m-1}{2m}}\, A_x$$

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* using

In general we have 2 methods of adjustment.

- Using the UDD assumption, the APV of an mthly insurance can be obtained by multiplying the corresponding discrete (annual) insurance with the adjustment factor $i/i^{(m)}$.

$$\frac{i^{(m)}}{m} + 1 = (1+i)^{\frac{1}{m}}$$

- Using the accelerated claim approach, The APV of an mthly insurance can be obtained by multiplying the corresponding discrete (annual) insurance with the adjustment factor $(1+i)^{\frac{m-1}{2m}}$.

Policy	UDD	Accelerated claims
Level benefit whole life	$A_x^{(m)} = \frac{i}{i^{(m)}} A_x$	$A_x^{(m)} = (1+i)^{\frac{m-1}{2m}} A_x$
Level benefit n-year term life	$A^{1\,(m)}_{x:\overline{n}\rceil} = \frac{i}{i^{(m)}} A^1_{x:\overline{n}\rceil}$	$A^{1\,(m)}_{x:\overline{n}\rceil} = (1+i)^{\frac{m-1}{2m}} A^1_{x:\overline{n}\rceil}$
Annually increasing whole life	$(IA^{m})_x = \frac{i}{i^{(m)}} (IA)_x$	$(IA^{(m)})_x = (1+i)^{\frac{m-1}{2m}} (IA)_x$

Annually increasing n-year term	$(IA^{(m)})^1_{x:\overline{n}\lvert} = \frac{i}{i^{(m)}} (IA)^1_{x:\overline{n}\lvert}$	$(I\hat{A})^{\hat{1}}_{x:\overline{n}\lvert} = (1+i)^{\frac{m-1}{2m}} (IA)^1_{x:\overline{n}\lvert}$

If we used $m \to \infty$ then

$\frac{i}{i^{(m)}} \to \frac{i}{\delta}$ and $(1+i)^{\frac{m-1}{2m}} \to (1+i)^{1/2}$ in this case

we will have:

Policy	UDD	Accelarated claims
Level benefit whole life	$\bar{A}_x = \frac{i}{\delta} A_x$	$\bar{A}_x = (1+i)^{1/2} A_x$
Level benefit n-year life term	$\bar{A}^1_{x:\overline{n}\lvert} = \frac{i}{\delta} A^1_{x:\overline{n}\lvert}$	$\bar{A}^1_{x:\overline{n}\lvert} = (1+i)^{1/2} A^1_{x:\overline{n}\lvert}$
Annually increasing whole life	$(I\bar{A})_x = \frac{i}{\delta} (IA)_x$	$(I\bar{A})_x = (1+i)^{1/2} (IA)_x$
Annually increasing n-year term	$(I\bar{A})_{x:\overline{n}\lvert} = \frac{i}{\delta} (IA)^1_{x:\overline{n}\lvert}$	$(I\bar{A})^1_{x:\overline{n}\lvert} = (1+i)^{1/2} (IA)^1_{x:\overline{n}\lvert}$