question 27 Page 17

$$R.H.S = \begin{vmatrix} a_1+b_1 & a_1-b_1 & c_1 \\ a_2+b_2 & a_2-b_2 & c_2 \\ a_3+b_3 & a_3-b_3 & c_3 \end{vmatrix}$$

$$\underset{=}{-C_1+C_2} \begin{vmatrix} a_1+b_1 & -2b_1 & c_1 \\ a_2+b_2 & -2b_2 & c_2 \\ a_3+b_3 & -2b_3 & c_3 \end{vmatrix}$$

$$= -2 \begin{vmatrix} a_1+b_1 & b_1 & c_1 \\ a_2+b_2 & b_2 & c_2 \\ a_3+b_3 & b_3 & c_3 \end{vmatrix}$$

$$\underset{=}{-C_2+C_1} -2 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = L.H.S$$

question 2 Page 17

$$|A| = \begin{vmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} \underset{\substack{R_1+R_3 \\ R_1+R_4}}{\overset{R_1+R_2}{=\!=\!=}} \begin{vmatrix} -1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 2 & 0 & 2 \\ 0 & 2 & 2 & 2 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 2 \end{vmatrix} = (-1)\left[-2 \begin{vmatrix} 2 & 2 \\ 2 & 2 \end{vmatrix} + 2 \begin{vmatrix} 2 & 0 \\ 2 & 2 \end{vmatrix} \right]$$

$$= -8$$

Question 6 Page 17

we have $A^3 + 4A^2 - 2A + 2I = 0$

$\iff A^3 + 4A^2 - 2A = -2I$

$\iff \frac{-1}{2}A^3 - 2A^2 + A = I$

$\iff A\underbrace{\left(\frac{-1}{2}A^2 - 2A + I\right)}_{A^{-1}} = I$

So, A is invertible.

Question 26 Page 17

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= t\begin{vmatrix} \frac{1}{t}a_1+b_1 & \frac{1}{t}a_2+b_2 & \frac{1}{t}a_3+b_3 \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overset{-R_1+R_2}{=} t\begin{vmatrix} \frac{1}{t}a_1+b_1 & \frac{1}{t}a_2+b_2 & \frac{1}{t}a_3+b_3 \\ (t-\frac{1}{t})a_1 & (t-\frac{1}{t})a_2 & (t-\frac{1}{t})a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= t(t-\tfrac{1}{t})\begin{vmatrix} \frac{1}{t}a_1+b_1 & \frac{1}{t}a_2+b_2 & \frac{1}{t}a_3+b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overset{R_{12}}{=} -t(t-\tfrac{1}{t})\begin{vmatrix} a_1 & a_2 & a_3 \\ \frac{1}{t}a_1+b_1 & \frac{1}{t}a_2+b_2 & \frac{1}{t}a_3+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overset{-\frac{1}{t}R_1+R_2}{=} \underbrace{-t(t-\tfrac{1}{t})}_{\Downarrow \\ (1-t^2)}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = R\cdot H\cdot S$$

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Question 24 Page 17

(a) we will use cofactors of the first row:

$$\begin{vmatrix} 0 & 0 & a_{13} \\ 0 & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{13}\begin{vmatrix} 0 & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = -a_{13}a_{22}a_{31}$$

(b) we will use the cofactors of the first row:

$$\begin{vmatrix} 0 & 0 & 0 & a_{14} \\ 0 & 0 & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{42} & a_{44} \end{vmatrix} = -a_{14}\begin{vmatrix} 0 & 0 & a_{23} \\ 0 & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{42} \end{vmatrix}$$

$$= -a_{14}\left(a_{23}\begin{vmatrix} 0 & a_{32} \\ a_{41} & a_{42} \end{vmatrix}\right)$$

$$= a_{14}a_{23}a_{32}a_{41}$$

Question 25 Page 17

we will use Columns elementry operations

$$L.H.S = \begin{vmatrix} a_1 & b_1 & a_1+b_1+c_1 \\ a_2 & b_2 & a_2+b_2+c_2 \\ a_3 & b_3 & a_3+b_3+c_3 \end{vmatrix} \quad \underset{\Longleftrightarrow}{-C_2+C_3}$$

$$= \begin{vmatrix} a_1 & b_1 & b_1+c_1 \\ a_2 & b_2 & b_2+c_2 \\ a_3 & b_3 & b_3+c_3 \end{vmatrix} \quad \underset{\Longleftrightarrow}{-C_2+C_3}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = R.H.S$$

Ex 1 P. 18 Solutions

(1) $AB + AC - D = 0$

$\Leftrightarrow \quad A(B+C) - D = 0$

So, $|A|(|B| + |C|) - |D| = 0 \quad \cdots (*)$

Now

$|B| = -2 \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0$

$|C| = 1 \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -1$

, and by assumption $|D| = 6$

Therefore, by $(*)$ $|A| = \frac{-6}{-1} = 6$.

(2) since $RS + R - 2I = 0$, $R(S+I) = 2I$.

$\Rightarrow \quad \frac{1}{2} R(S+I) = I$

$\Rightarrow \quad R[\frac{1}{2}(S+I)] = I$

Hence, $\frac{1}{2}(S+I) = R^{-1}$

$\Rightarrow \quad \frac{1}{2}\left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) = R^{-1}$

$\Leftrightarrow$ complete !!

(I) a $(A-B)(C-A) + (C-B)(A-C) + (C-A)^2$

$= AC - BC - A^2 + BA$
$+ CA - C^2 - BA + BC$
$+ C^2 - AC - CA + A^2$

$= 0$

b Notice that size (A) = size $(B) = 3$.

As, $|2A^{-1}| = |A^3 (B^{-1})^T| = -4$

$\Leftrightarrow 2^3 \cdot \frac{1}{|A|} = -4$ and $|A^3| \cdot |(B^{-1})^T| = -4$

$\Leftrightarrow \boxed{|A| = \frac{8}{-4} = -2}$ and $(-8) \cdot \frac{1}{|B|} = -4 \Rightarrow \boxed{|B| = 2}$

c

Since $B = \begin{bmatrix} -5 & 3 \\ 1 & 0 \end{bmatrix}$, $B^T = \begin{bmatrix} -5 & 1 \\ 3 & 0 \end{bmatrix}$ $\Rightarrow -2B^T = \begin{bmatrix} 10 & -2 \\ -6 & 0 \end{bmatrix}$

By assumption,

$$A^3 = \begin{bmatrix} 18 & -2 \\ -6 & 1 \end{bmatrix} - \begin{bmatrix} 10 & -2 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

Question II Page 19

(a) False

Let $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Notice that

$A \neq 0$ and $B \neq 0$, but $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

(b) True

Let A and B be symetric matrices. Then $A^T = A$ and $B^T = B$. Now, $(A+B)^T = A^T + B^T$
$= A + B$

Therefore, $A+B$ is symetric.

Question I Page 19

(3) we will use rows elementry operation to turn the assumed matrix to Lower tringular matrix.

The assumed matrix $\begin{bmatrix} x^2 & 0 & x^2-4 \\ -1 & 3 & 2y-6 \\ 1 & 7 & 2x-5y \end{bmatrix}$ to be Lower tringular matrix should:

$x^2 - 4 = 0$ and $2y - 6 = 0$

$\Rightarrow \boxed{x = \pm 2}$ and $\boxed{y = 3}$