⑤ ML for incomplete data:

$\boxed{5.1}$ Censored data:

• **left censored data:** An observation less then a value d; is recorded as d. أي قيمة أقل من 3 تكتب 3

$$(1,4,10,11,10,5,3,2) \xrightarrow{d=3} (3,4,10,11,10,5,3,3)$$

• **Right censored:** An observation greater than a value u; is recorded as u.

*In policy limit contract data is right censored

The MLM for censored data is done in the same way as for mixed data.

Example: A ground-up loss variable X has a policy limit of 30. let 6 payment 20, 25, 27, 28, 30, 30 موجودة في الفترة $(30, \infty)$

Let $X \sim Exp(\lambda)$ **Find the MLE of λ?**

$$Y^L = X \wedge 30$$

$$L(\lambda) = \lambda e^{-20\lambda} \lambda e^{-25\lambda} \lambda e^{-27\lambda} \lambda e^{-28\lambda} \left(e^{-30\lambda}\right)^2 \quad (1 - F_X(30))^2$$

$$= \lambda^4 e^{-100\lambda} \left(e^{-30\lambda}\right)^2$$

$$\ell(\lambda) = 4 \ln(\lambda) - 100\lambda + (-60\lambda)$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{4}{\lambda} - 160 = 0 \Rightarrow \hat{\lambda} = \frac{4}{160} = \frac{1}{40} = 0.025$$

5.2 Truncated data:

- **Left truncated:** An observation less than a value d, is <u>not</u> recorded.

- **Right truncated:** An observation greater than a value u, is <u>not</u> recorded.

$$B = \{\text{deleted observation}\} \leftarrow$$ القيم الي أصغر من d وأكبر من u

$$L(\theta) = \prod_{i=1}^{n} f_{X|B'}(x_i)$$

Example: A ground up loss X has a deductible of 7 applied. A random sample of 6 insurance policies (<u>after</u> deductible is applied) is given:

3, 6, 7, 8, 10, 12 ← data of $X-d$

Suppose $X \sim Exp(\theta)$. Find the MLE for θ?

$X|_{X \geq d}$ $\quad Y^P = (Y^L \mid Y^L > 0)$

(1) $$f_{Y^P}(y) = \frac{f_X(y+d)}{S_X(d)} \quad ; \; y \geq 0$$

$$L(\theta) = \frac{\theta e^{-10\theta}\, \theta e^{-13\theta}\, \theta e^{-14\theta}\, \theta e^{-15\theta}\, \theta e^{-17\theta}\, \theta e^{-19\theta}}{(e^{-7\theta})^6}$$

$$= \theta^6 e^{-46\theta}$$

$$\ell(\theta) = 6\ln(\theta) - 46\theta$$

$$\frac{\partial \ell}{\partial \theta} = \frac{6}{\theta} - 46 = 0 \Rightarrow \hat{\theta} = \frac{6}{46} = 0.13$$

Or ② 10, 13, 14, 15, 17, 19

$f_{X|X\geq 7} = ?$

Franchise : $Y^L = \begin{cases} X & , X \geq d \\ 0 & , X < d \end{cases}$

$$Y^P = (Y^L \mid Y^L > 0) = (X \mid X \geq d)$$

$f_{X|X\geq d}(y) = f_{Y^P}(y) = \dfrac{f_X(y)}{S_X(d)}$; $y \geq d$

$L(\theta) = \ldots$ ← same ①

Example: For a dental policy, you are given:

- $X \sim Exp(\theta)$
- Losses under 50 are not reported.
- For each loss over 50, there is a deductible of 50 and a policy limit of 400

A random sample: $50, 150, 200, 350^+, 350^+$

where + indicate that the original loss exceeds 400.

$L(\theta) = \prod_{i=1}^{5} f_{Y^P}(y_i)$

$$f_{Y^P}(y) = \begin{cases} 0 & , y < 0 ;\ y > u-d \\ \dfrac{f_X(y+d)}{S_X(d)} & 0 \leq y < u-d \\ \dfrac{F_X(u) - F_X(d)}{S_X(d)} = \dfrac{S_X(u)}{S_X(d)} & y = u-d \end{cases}$$

$$\frac{S_x(u)}{S_x(d)}$$

$$L(\theta) = \frac{\theta e^{-1000\theta}}{e^{-500\theta}} \cdot \frac{\theta e^{-2000\theta}}{e^{-500\theta}} \cdot \frac{\theta e^{-2500\theta}}{e^{-500\theta}} \cdot \left(\frac{e^{-4000\theta}}{e^{-500\theta}}\right)^2$$

$$= \theta^3 e^{-11000\theta}$$

$$\ell(\theta) = 3\ln(\theta) - 11000\theta$$

$$\frac{\partial \ell}{\partial \theta} = \frac{3}{\theta} - 1100 = 0 \Rightarrow \hat{\theta} = \frac{3}{1100} = 0.0027$$

Example: A random sample of 6 payment (after a deductible of 7 is applied) is given:

3, 6, 7, 8, 10, 12

a. if $X \sim Unif(0, \theta)$ Find the MLE for the mean of X

$$E(X) = \frac{\theta}{2}$$

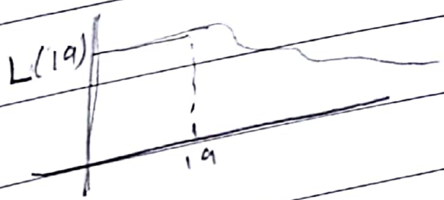
$12 + 7$

$\theta \geq 19 \leftarrow$ [illegible]

$$f_{Y^P}(y) = \frac{f_X(y+d)}{S_X(d)}$$

$$L(\theta) = \frac{\left(\frac{1}{\theta}\right)^6}{\left(1 - \frac{7}{\theta}\right)^6} = \frac{1}{(\theta - 7)^6}$$

$$\frac{\partial L}{\partial \theta} = -\frac{6(\theta - 7)^5}{(\theta - 7)^{12}} = \frac{-6}{(\theta - 7)^7} < 0$$



$$\hat{\theta} = 19 \rightarrow \text{Estimate of } E(X) = \frac{19}{2}$$

Scanned by CamScanner

b. if lost per payment $y \sim Unif(0, \theta)$ Find the MLE for the mean of y?

$$E_y = \frac{\theta}{2}$$

$$L(\theta) = \left(\frac{1}{\theta}\right)^6 \downarrow \quad , \quad \theta \geq 12$$

$$\hat{\theta} = 12 \quad , \quad E(y) \simeq \frac{12}{2} = 6$$