Solution of the final examination	Math 106	Semester I	1439-1440

Exercise 1 :

1.

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	$2\sqrt{2}$	1	2.8284
1	1	3	4	12
2	2	4	2	8
3	3	$\sqrt{35}$	4	23.6643
4	4	$6\sqrt{2}$	1	8.4853
				54.978

1,5

$$\int_0^4 \sqrt{x^3+8}dx \approx 18.326.$$ 0,5

2. $\ln F(x) = e^x \ln(2+\sin(x)),$ 0,5

$$F'(x) = F(x)\left(e^x \ln(2+\sin(x)) + \frac{e^x\cos(x)}{2+\sin(x)}\right).$$ 1,5

Exercise 2 :

1. $\displaystyle\int (3^x + 3^{-x} + 2)dx = \frac{1}{\ln 3}3^x - \frac{1}{\ln 3}3^{-x} + 2x + c.$ 1+2

2. $\displaystyle\int \frac{dx}{\sqrt{2^{2x}-1}} \overset{t=2^x}{=} \frac{1}{\ln 2}\int \frac{dt}{t\sqrt{t^2-1}} = \frac{1}{\ln 2}\sec^{-1}(2^x) + c.$ 2+1

3. $\displaystyle\int \frac{dx}{\sqrt{x}\sqrt{1+x}} \overset{x=t^2}{=} \int \frac{2dt}{\sqrt{1+t^2}} = 2\sinh^{-1}(\sqrt{x}) + c.$ 2+1

Exercise 3 :

1. $\displaystyle\int \frac{dx}{x\sqrt{4-x^6}} \overset{x^3=2t}{=} \frac{1}{6}\int \frac{dt}{t\sqrt{1-t^2}} = -\frac{1}{6}\text{sech}^{-1}(\frac{x^3}{2}) + c.$ 2+1

2.

$$\begin{aligned}
\int \frac{dx}{(x^2-1)^{\frac{3}{2}}} &\overset{x=\sec(\theta)}{=} \int \frac{\cos(\theta)}{\sin^2(\theta)}d\theta && 1\\
&= -\csc(\theta) + c && 1\\
&= -\frac{x}{\sqrt{x^2-1}} + c. && 1
\end{aligned}$$

3.

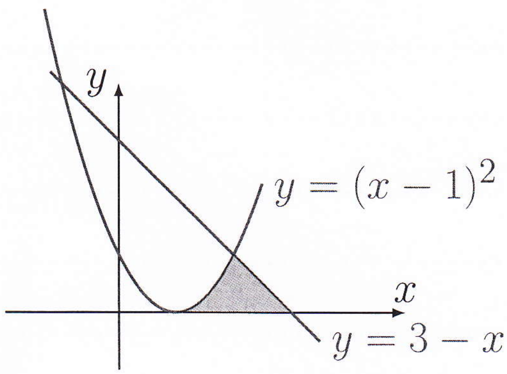
$$\int \frac{4x^2}{(x-1)^2(x+1)}dx = \int \frac{3}{x-1}dx + \int \frac{2}{(x-1)^2}dx + \int \frac{1}{x+1}dx$$

1,5 + 1,5 $$= 3\ln|x-1| + \ln|x+1| - \frac{2}{x-1} + c.$$

Exercise 4 :

1. $\displaystyle\int (1+2x)e^{-x}dx = -(3+2x)e^{-x} + c$, the the integral 1,5

$\displaystyle\int_0^{+\infty} (1+2x)e^{-x}dx$ converges and its value is 3. 1,5

2. 

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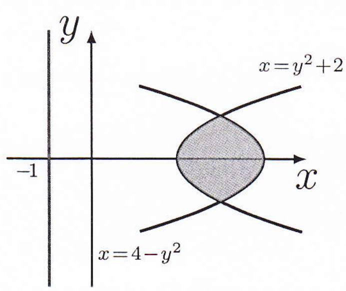
$$A = \int_1^2 (x-1)^2 dx + \int_2^3 (3-x)dx = \frac{5}{6}.$$ 1,5 + 0,5

3. 

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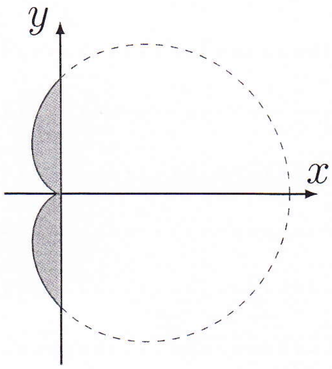
$$V = \pi \int_{-1}^{1} \left((1+4-y^2)^2 - (1+y^2+2)^2 \right) dy.$$ 2

Exercise 5 :

1.

$$L = \int_0^1 \sqrt{t^4 + t^7}dt = \int_0^1 t^2\sqrt{1+t^3}dt = \frac{2}{9}(2\sqrt{2}-1). \quad 1+1+1$$



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2.

$$A = \frac{2}{2}\int_{\frac{\pi}{2}}^{\pi} 9(1+\cos(\theta))^2 d\theta = 9(\frac{3}{4}\pi - 2). \quad 1+1$$

3.

$$A = 2\pi \int_0^{\frac{\pi}{2}} 8\cos^2(\theta)\sqrt{64\cos^2(\theta) + 64\sin^2(\theta)}d\theta = 32\pi^2.$$

1+1+1