King Saud University   Time: 2 Hours
Department of Mathematics   Full marks: 30
MATH 107 (Dr B. Halouani)

First Mid-Term
(Summer Semester, 1428-1429)

Question 1[3]:

Determine x and $y \in \mathbb{R}$ such that we have $A = 2BC - D$ for $A = \begin{pmatrix} x & 2x & y \\ 2 & 3x & x-y \end{pmatrix}$,

$B = \begin{pmatrix} 3 & 4 \\ 2 & -1 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} 36 & -2 & 11 \\ 16 & -12 & 7 \end{pmatrix}$.

Question 2[4]: For what value of $a \in \mathbb{R}$ does the system

$$\begin{cases} x + y + z & = & 3 \\ -x + y + z & = & 2 \\ 2y + (a^2 + 1)z & = & 6 \end{cases}$$

has a unique solution ?

Question 3[3]: Use Gauss-Jordan elimination to solve the system:

$$\begin{cases} 2x + y - z + w & = & 4 \\ x + z + w & = & 2 \end{cases}.$$

Question 4[4]:

Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ if it exists.

Question 5[6]: Find $Adj\,(A)$ and A^{-1} for the matrix $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

Question 6[4]

Evaluate $\det A$ if $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ -1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{pmatrix}$.

Question 7[6]

If $A^T = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix}$ find $A^2 - 3A$ and use it to find A^{-1}.