KING SAUD UNIVERSITY
DEPARTMENT OF MATHEMATICS
TIME: 1H 30 mn, FULL MARKS: 40, 07/06/1432
MATH 204

Question 1. a) [7] Show that $f(x) = \cosh 2x$, $g(x) = e^{-2x}$ are solutions of the differential equation: $y'' - 4y = 0$, and show that $\{\cosh 2x,\ e^{-2x}\}$ is a fundamental set of solutions of the differential equation: $y'' - 4y = 0$.

b) [8] If the function $y_1 = x$ is a solution of the differential equation

$$x^2y'' - (x^2 + 2x)y' + (x + 2)y = 0, \qquad x > 0.$$

Use the method of reduction of order to find the second solution y_2, and hence find the general solution.

Question 2. [8] Use the method of variation of parameters to find the general solution of the nonhomogeneous differential equation:

$$y'' - 2y' + 2y = e^x \tan\ x.$$

(**Hint**: $\int \sec \alpha x dx = \frac{1}{\alpha} \ln|\sec \alpha x + \tan \alpha x|$).

Question 3. [8] Determine only the form of the particular solution of the differential equation

$$y''' - 2y'' + y' = 7e^{-x} \sin 2x.$$

Question 4. [9] Solve the system of differential equations

$$\begin{cases} \frac{dx}{dt} + 2y = -x \\ 2\frac{dx}{dt} + \frac{dy}{dt} = -y. \end{cases}$$