Kingdom of Saudi Arabia
Ministry of Education
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المملكة العربية السعودية
وزارة التعليم
جامعة الملك سعود
كلية العلوم
قسم الرياضيات

MIDTERM 2 EXAM

SEMESTER	SECOND TERM	YEAR	2017/2016
	COURSE	ACTU 466	
DATE	19/04/2017	DURATION	1H 30 MNS

	رقم الشعبة:		إســم الطالب(ة):
	توقيع الطالب(ة):		الرقم الجامعي للطالب(ة):

INSTRUCTIONS

1) Please check that your exam contains **08 pages** total (including the first page!!), and **05 questions**.
2) **Answer all questions.**
3) No books, No notes and no phones are allowed.
4) A standard no programmable calculator is allowed.
5) Table for most used distributions is included.
6) Z-table is included.
7) Modifications for distributions table is included.

Question	**1**	**2**	**3**	**4**	**5**	
Total score	**4**	**10**	**3**	**3**	**5**	
Score						

1) (2+1+1=4 marks) Suppose a random variable X has a Pareto distribution with parameters $\alpha = 2$ and θ:

$$F_X(x) = 1 - \frac{\theta^2}{(x+\theta)^2} \quad \text{for } x \geq 0.$$

a) Show that $E(X \wedge t) = \theta\left(1 - \frac{\theta}{t+\theta}\right)$ for any positive real number t.

b) Deduce $E(X)$.

c) Find the distribution of cX for a positive constant c.

2) (2+2+2+2+2=10 marks) An insurance policy is subject to an ordinary deductible of d. The cdf of the loss amount X is given in **Exercise 1**.

a) Compute the cdf and pdf for Y^L.

b) Compute the cdf and pdf for Y^P.

d) Compute the mean of Y^L and Y^P.

e) Compute the loss elimination ratio.

f) Deduce the loss elimination ratio after a uniform inflation of 100r %.

3) (3 marks) Consider two insurance contracts. One has a policy limit of u. The second has a coinsurance α. Losses in both contracts follow the same distribution in **Exercise 1** with parameter θ.

Find the relationship between u, α and θ so that the expected loss per cost is the same for the two contracts.

4) (1+2=3 marks) Individual losses have a Pareto distribution with parameter θ (as in **Exercise 1**). The number of losses when there is no deductible has a negative binomial distribution with parameters r and p.

a) Determine the expected number of cost-per payments when a deductible d is applied.

b) Determine the expected total cost-per payment.

5) (1+2+2=5 marks) The number of losses follows a geometric distribution with parameter p.

a) Solve the equation $\frac{1-p}{p} = t$ where t is known.

Using the data $X_1, X_2, \dots X_n$ of the number of losses for the last n years, compute the estimate for the parameter p, by applying:

b) Method of moments,

c) Maximum likelihood method.

		CDF	PDF
Ordinary deductible $Y^L = (x-d)_+$	Y^L	$\begin{cases} 0 & ; y < 0 \\ F_X(y+d) & ; y \ge 0 \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ F_X(d) & ; y = 0 \\ f_X(y+d) & ; y \ge 0 \end{cases}$
$Y^P = (Y^L \mid X > d)$	Y^P	$\begin{cases} 0 & ; y < 0 \\ \dfrac{F_X(y+d) - F_X(d)}{1 - F_X(d)} & ; y \ge 0 \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ \dfrac{f_X(y+d)}{1 - F_X(d)} & ; y \ge 0 \end{cases}$
Franchise deductible $Y^L = \begin{cases} 0 & ; X \le d \\ X & ; X \ge d \end{cases}$ $Y^P = (Y^L \mid X > d)$	Y^L	$\begin{cases} 0 & ; y < 0 \\ F_X(d) & ; 0 \le y < d \\ F_X(y) & ; y \ge d \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ F_X(d) & ; y = 0 \\ 0 & ; 0 < y < d \\ f_X(y) & ; y \ge d \end{cases}$
	Y^P	$\begin{cases} 0 & ; y < d \\ \dfrac{F_X(y) - F_X(d)}{1 - F_X(d)} & ; y \ge d \end{cases}$	$\begin{cases} 0 & ; y < d \\ \dfrac{f_X(y)}{1 - F_X(d)} & ; y \ge d \end{cases}$
Policy limit $Y = X \wedge u$		$\begin{cases} F_X(y) & ; 0 \le y < u \\ 1 & ; y \ge u \end{cases}$	$\begin{cases} f_X(y) & ; 0 \le y < u \\ 1 - F_X(u) & ; y = u \\ 0 & ; y > u \\ 0 & ; y < 0 \end{cases}$
coinsurance $Y = \alpha X$		$= F_X\left(\dfrac{y}{\alpha}\right)$	$= \dfrac{1}{\alpha} f_X\left(\dfrac{y}{\alpha}\right)$
Inflation effect with coefficient r $Y = (1+r)X$		$= F_X\left(\dfrac{y}{1+r}\right)$	$= \dfrac{1}{1+r} f_X\left(\dfrac{y}{1+r}\right)$
Combination $Y^L = \alpha((1+r)X \wedge u - (1+r)X \wedge d)$ $\begin{cases} 0 & ; x < \dfrac{d}{1+r} \\ \alpha((1+r)x - d); & \dfrac{d}{1+r} \le x < \dfrac{u}{1+r} \\ \alpha(u-d) & ; x \ge \dfrac{u}{1+r} \end{cases}$	Y^L	$\begin{cases} 0 & ; y < 0 \\ F_X\left(\dfrac{y/\alpha + d}{1+r}\right) & ; 0 \le y < \alpha(u-d) \\ 1 & ; y \ge \alpha(u-d) \end{cases}$	$\begin{cases} \dfrac{1}{\alpha(1+r)} f_X\left(\dfrac{y/\alpha + d}{1+r}\right) & ; 0 \le y < \alpha(u-d \\ 1 - F_X\left(\dfrac{u}{1+r}\right) & ; y = \alpha(u-d) \\ F_X\left(\dfrac{d}{1+r}\right) & ; y = 0 \\ 0 & ; y < 0 \;\; y > \alpha(u-d) \end{cases}$
	Y^P	$\begin{cases} 0 & ; y < 0 \\ \dfrac{F_X\left(\dfrac{\frac{y}{\alpha} + d}{1+r}\right) - F_X\left(\dfrac{d}{1+r}\right)}{1 - F_X\left(\dfrac{d}{1+r}\right)} & ; 0 \le y < \alpha(u - \\ 1 & ; y \ge \alpha(u-d) \end{cases}$	$\begin{cases} 0 & ; y < 0 \\ \dfrac{\frac{1}{\alpha(1+r)} f_X\left(\frac{\frac{y}{\alpha}+d}{1+r}\right)}{1 - F_X\left(\frac{d}{1+r}\right)} & ; 0 \le y < \alpha(u-d) \\ 0 & ; y \ge \alpha(u-d) \end{cases}$

Table A The most frequently used discrete and continuous distributions

Distribution	Density & support	Moments & cumulants	Mgf
Binomial(n,p) $(0<p<1,\ n\in\mathbb{N})$	$\binom{n}{x}p^x(1-p)^{n-x}$ $x=0,1,\dots,n$	$E=np, Var=np(1-p)$, $\gamma=\frac{np(1-p)(1-2p)}{\sigma^3}$	$(1-p+pe^t)^n$
Bernoulli(p)	$\equiv$ Binomial$(1,p)$		
Poisson(λ) $(\lambda>0)$	$e^{-\lambda}\frac{\lambda^x}{x!}, x=0,1,\dots$	$E=Var=\lambda$, $\gamma=1/\sqrt{\lambda}$, $\kappa_j=\lambda,\ j=1,2,\dots$	$\exp[\lambda(e^t-1)]$
Negative binomial(r,p) $(r>0,\ 0<p<1)$	$\binom{r+x-1}{x}p^r(1-p)^x$ $x=0,1,2,\dots$	$E=r(1-p)/p$ $Var=E/p$, $\gamma=\frac{(2-p)}{p\sigma}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Geometric(p)	$\equiv$ Negative binomial$(1,p)$		
Uniform(a,b) $(a<b)$	$\frac{1}{b-a}; a<x<b$	$E=(a+b)/2$, $Var=(b-a)^2/12$, $\gamma=0$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
$N(\mu,\sigma^2)$ $(\sigma>0)$	$\frac{1}{\sigma\sqrt{2\pi}}\exp\frac{-(x-\mu)^2}{2\sigma^2}$	$E=\mu$, $Var=\sigma^2$, $\gamma=0$ $(\kappa_j=0,\ j\geq 3)$	$\exp(\mu t+\frac{1}{2}\sigma^2t^2)$
Gamma(α,β) $(\alpha,\beta>0)$	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}, x>0$	$E=\alpha/\beta$, $Var=\alpha/\beta^2$, $\gamma=2/\sqrt{\alpha}$	$\left(\frac{\beta}{\beta-t}\right)^\alpha$ $(t<\beta)$
Exponential(β)	$\equiv$ gamma$(1,\beta)$		
$\chi^2(k)$ $(k\in\mathbb{N})$	$\equiv$ gamma$(k/2,1/2)$		
Inverse Gaussian(α,β) $(\alpha>0,\beta>0)$	$\frac{\alpha x^{-3/2}}{\sqrt{2\pi\beta}}\exp\left(\frac{-(\alpha-\beta x)^2}{2\beta x}\right)$	$E=\alpha/\beta$, $Var=\alpha/\beta^2$, $\gamma=3/\sqrt{\alpha}$	$e^{\alpha(1-\sqrt{1-2t/\beta})}$ $(t\leq\beta/2)$

$$F(x)=\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}+\sqrt{\beta x}\right)+e^{2\alpha}\Phi\left(\frac{-\alpha}{\sqrt{\beta x}}-\sqrt{\beta x}\right),\quad x>0$$