Second Semester	**Second Exam**	**King Saud University**
(without calculators)	**Time allowed: 1 h and 30 m**	**College of Science**
Thursday 28-7-1437	**240 Math**	**Math. Department**

Q1: (a) Let V be any set which has two operations are defined: addition and scalar multiplication. State the 10 axioms that should be satisfied by all scalars and all objects in V that make V a vector space. (5 marks)

(b) Let $V=M_{nn}$ and W is the set of all symmetric matrices of degree n. Prove that W is a subspace of V. (3 marks)

Q2: (a) Use the Wronskian to show that 1+x, 1-x, x^2 are linearly independent.

(3 marks)

(b) show that the vectors (1,2,1), (2,1,2), (1,1,0) form a basis for $\mathbb{R}^3$. (3 marks)

Q3: (a) Let B={(1,2),(2,5)} and B'={(1,1),(2,0)} be two bases of $\mathbb{R}^2$. Find the transition matrix from B' to B. (3 marks).

(b) Find a basis for the column space of the matrix:

$$A=\begin{bmatrix} 1 & 2 & 6 & -1 \\ 2 & 4 & 4 & 6 \\ 3 & 6 & 10 & 5 \end{bmatrix}$$

and **<u>deduce</u>** dim(null(A^T)) without solving any linear system. (3 marks)

Q4: Show that $A=\begin{bmatrix} 1 & 0 & 3 \\ 1 & 4 & 5 \\ 0 & 0 & -1 \end{bmatrix}$ is diagonalizable and find a matrix P that diagonalizes A. (5 marks)