Second Semester	**First Exam**	**King Saud University**
(without calculators)	**Time allowed: 1 h and 30 m**	**College of Science**
Thursday 1-6-1437	**240 Math**	**Math. Department**

Q1: If $A=\begin{bmatrix}1 & -4 & 3\\ 0 & 2 & 5\\ 0 & 6 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 5 & 0\\ 6 & 0 & -3\end{bmatrix}$ and $C=\begin{bmatrix}2 & 1 & 5\\ 0 & -3 & 0\end{bmatrix}$, then find the following:

(a) B+3C (2 marks)

(b) $(-3A)^T$, BC^T (2 marks)

(c) tr(A^2) (2 marks)

Q2: (a) Put the following matrix in the reduced row echelon form (R.R.E.F.):

(4 marks)

$$A=\begin{bmatrix}1 & 1 & -2 & 3\\ 2 & 2 & -4 & 0\\ 4 & 6 & 4 & 0\end{bmatrix}$$

(b) Find the inverse of A in two different ways, where:

(5 marks)

$$A=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 3\\ 0 & 0 & 1\end{bmatrix}$$

Q3: (a) Find all values of k, if any, that satisfy the equation:

(2 marks)

$$\det\left(\begin{bmatrix}1 & 0 & 0\\ 3 & 5k & 5\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & k & 0\\ 2 & 0 & k\\ 2 & 1 & k\end{bmatrix}\right)=0$$

(b) Solve the following linear system:

(4 marks)

$$\begin{aligned} 2x_1 + 4x_2 - 2x_3 &= 2 \\ 3x_1 + 6x_2 - 5x_3 &= 9 \\ 4x_1 + 8x_2 - 4x_3 &= 4 \end{aligned}$$

Q4: (a) If $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find $\left(\left(A^T\right)^{-1}\right)^{36}$. (1 mark)

(b) Prove that if C and B are both inverses of the square matrix A, then B=C. (1 mark)

(c) Prove that if A is an invertible matrix, then

(1 mark)

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

(d) If $a_{ij} \in \mathbb{R}$ for all $i, j \in \{1,2,3,4,5,6\}$, then show that det(A)=0, where:

(1 mark)

$$A = \begin{bmatrix} 0 & 0 & a_{13} & a_{14} & 0 & 0 \\ 0 & 0 & a_{23} & a_{24} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & a_{63} & a_{64} & 0 & 0 \end{bmatrix}$$