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References

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-*By :* Wayne W. Daniel

-Elementary Biostatistics with Applications from Saudi Arabia

By : Nancy Hasabelnaby

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Chapter 3: Some Basic Probability Concepts

3.1 General view of probability

Probability: The probability of some event is the likelihood (chance) that this event will occur.

An experiment: Is a description of some procedure that we do.

The universal set (Ω): Is the set of all possible outcomes,

An event: Is a set of outcomes in Ω which all have some specified characteristic.

Notes:

1. Ω (the universal set) is called sure event

2. ϕ (the empty set) is called impossible event

Example (3.1)

Consider a set of 6 balls numbered 1, 2, 3, 4, 5, and 6. If we put the sex balls into a bag and without looking at the balls, we choose one ball from the bag, then this is an **experiment** which is has 6 outcomes.

- Ω ={1, 2, 3, 4, 5 ,6 }
- Consider the following events
  - E_1=the event that an even number occurs={2, 4, 6}.
  - E_2=the event of getting number greater than 2={3,4, 5, 6}.
  - E_3=the event that an odd number occurs={1, 3, 5}.
  - E_4=the event that a negative number occurs={}= ϕ .

Equally likely outcomes:

The outcomes of an experiment are equally likely if they have the **same chance of occurrence.**

Probability of equally likely events

consider an experiment which has N equally likely outcomes, and let the numbers of outcomes in an event E given by ***n(E),*** then the probability of E is given by

$$\boxed{P(E) = \frac{n(E)}{n(\Omega)} = \frac{n(E)}{N}}$$

Notes

1. For any event A , $0 \le P(A) \le 1$ (why?)

   That is, probability is always between 0 and 1.

2. $P(\Omega)=1$, and $P(\phi)=0$ (why?)

1 means the event is a certainty, 0 means the event is impossible

Example (3.2)

In the ball experiment we have
$n(\Omega)=6$, $n(E_1)=3$, $n(E_2)=4$, $n(E_2)=3$

$P(E_1)=3/6=0.5$

$P(E_2)=4/6=0.667$

$P(E_3)=3/6=0.5$

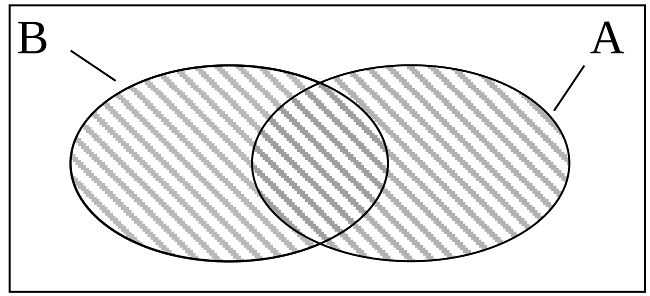
$P(E_4)=0$

Repaper that

- E_1=the event that an even number occurs={2, 4, 6}.
- E_2=the event of getting number greater than 2={3,4, 5, 6}.
- E_3=the event that an odd number occurs={1, 3, 5}.
- E_4=the event that a negative number occurs={}= .

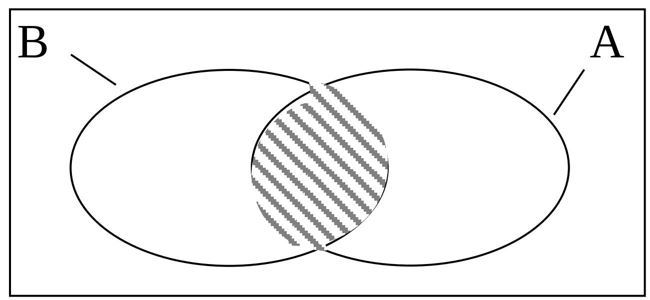
Relationships between events

❖ **Union** : $A \cup B$, consists of all those outcomes in A or in B or in both A and B



$A \cup B$

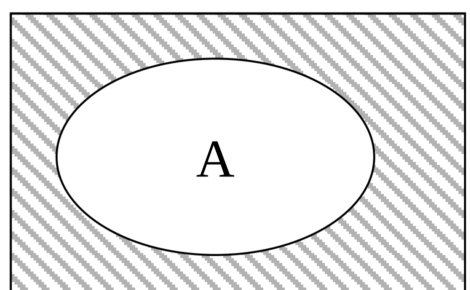
❖ **Intersection** : $A \cap B$, consists of all those outcomes in both A and B



$A \cap B$

❖ **Complement** : A^c (or A\`)

Consists of all outcomes that are in Ω but not in A



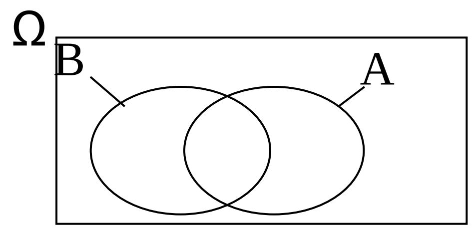
A^c

Notes:

1- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

and hence

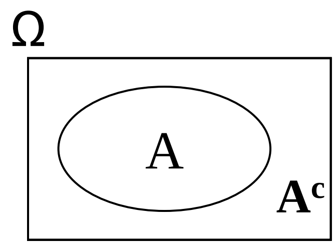
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



2. $n(A^c) = n(\Omega) - n(A)$

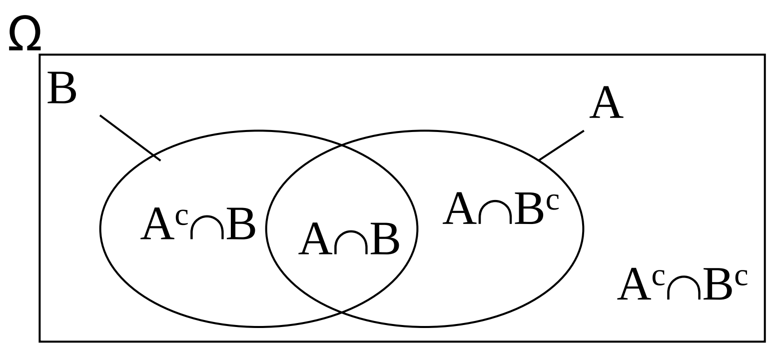
So that

$P(A^c) = 1 - P(A)$

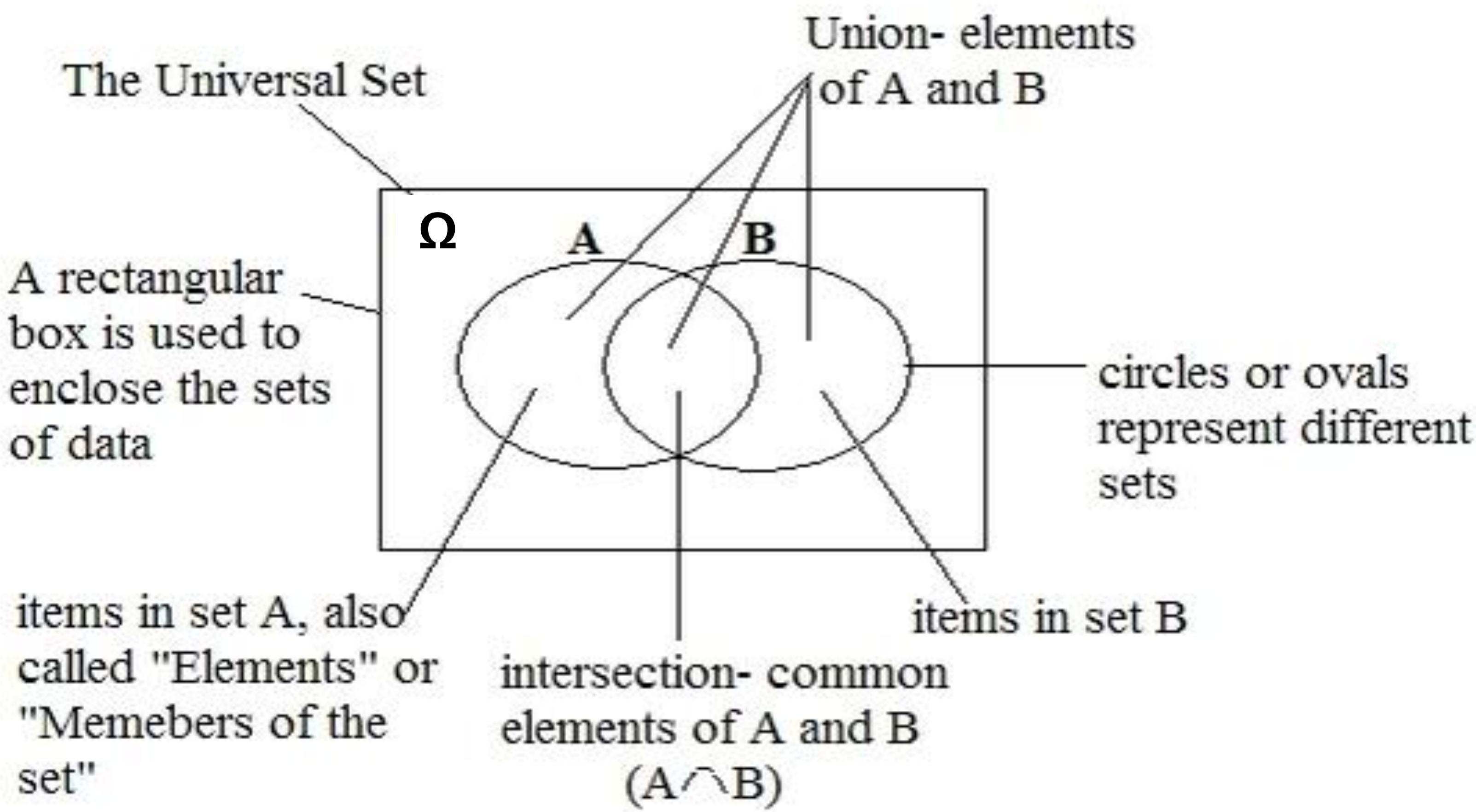


Sets (events) can be represented by

<u>Venn Diagram</u>



A Venn Diagram:



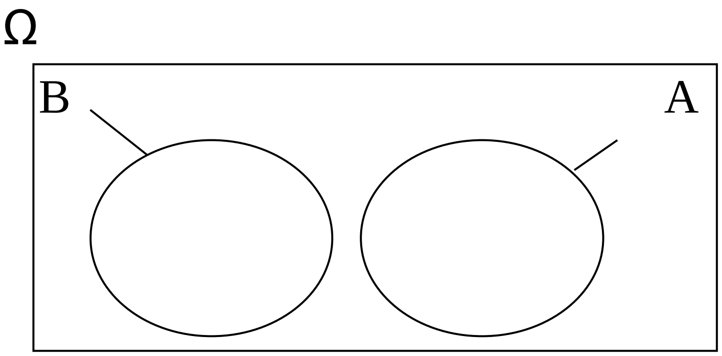
Disjoint events

Two events A and B are said to be disjoint (mutually exclusive) if $A \cap B = \phi$.

- In the case of disjoint events

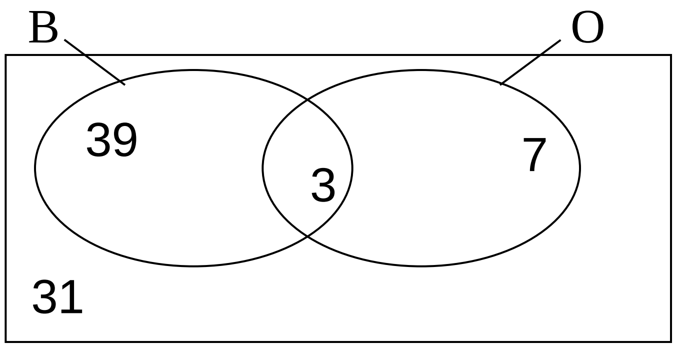
$P(A \cap B)=0$

$P(A \cup B)= P(A)+P(B)$



Example 3.3

From a population of 80 babies in a certain hospital in the last month, let the even B="is a boy", and O="is over weight" we have the following incomplete Venn diagram.



- It is a boy

P(B) =(3+39)/80=0.525

- It is a boy and overweight

P(B∩O)= 3/80=0.0357

- It is a boy or it is overweight

P(B U O)= (39+3+7)/80=0.6125

Conditional probability:

the conditional probability of A given B is equal to the probability of $A \cap B$ divided by the probability of B, providing the probability of B is not zero.

That is

$P(A \mid B)=P(A \cap B)/P(B) \quad , P(B) \neq 0$

Notes:

1. $P(A \mid B)$ is the probability of the event A if we know that the event B has occurred
2. $P(B \mid A)=P(A \cap B)/P(A) \quad , P(A) \neq 0$

Example

Referring to example 3.3 what is the probability that

- He is a boy knowing that he is over weight?

$P(B \mid O)= P(B \cap O)/P(O)= (3/80) / (10/80)=3/10=0.3$

- If we know that she is a girl, what is the probability that she is not overweight?

$P(O^c \mid B^c)= P(B^c \cap O^c) / P(B^c) = (31/80) / [(7+31)/80]= 31/38= 0.716$

Independent events

-Two events A and B are said to be independent if the occurrence of one of them has no effect on the occurrence of the other.

Multiplication rule for independent events

-If A and B are independent then

1-$P(A \cap B)=P(A)\,P(B)$

2-$P(A \mid B)= P(A)$ (Why?)

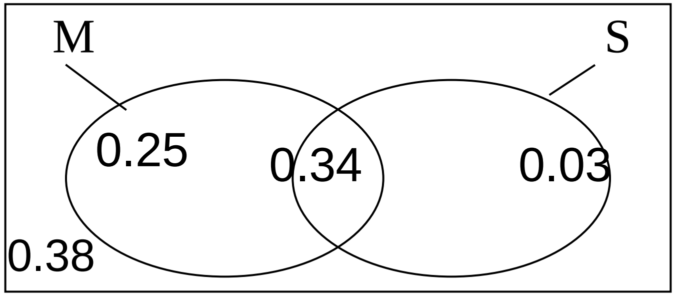
3- $P(B \mid A)= P(B)$ (Why?)

Example 3.4

In a population of people with a certain disease, let M="Men" and S="suffer from swollen leg "

We have the following incomplete Venn diagram

If we randomly choose one person



- Complete the Venn diagram

- Find the probability that this person

1- Is a man and suffer from swollen leg ?

$P(M \cap S) = 0.34$

2- Is a women?

$P(M^c) =$ 0.38+0.03= 0.41 (or $P(M^c) = 1 - P(M) = 1 - (0.25 + 0.34) = 0.41$)

3- Is a women that does not suffer from swollen leg ?

$P(M^c \cap S^c) =$ 0.38

4- Does not suffering from swollen leg?

$P(S^c) =$ 0.25+0.38= 0.63

Marginal prbability:

Definition: Given some variable that can be broken down into m categories designated by $A_1, A_2, \ldots, A_m$ and another jointly accurance variable that is broken down into n categories designated by $B_1, B_2, \ldots, B_n$, the *marginal probability* of A_i , called $P(A_i)$, is equal to the sum of the joint probabilities of A_i with all categories of B. That is

$P(A_i)=\sum P(A_i \cap B_j)$, for all values of j.

This will be clear in the following example

Example 3.5:

The following table shows 1000 nursing school applicants classified according to scores made on a college entrance examination and the quality of the high school form which they graduated, as rated by the group of educators.

		Quality of high school			
		Poor (p)	Average (A)	Superior (S)	total
Score	Low (L)	**105**	**60**	**55**	220
	Medium (M)	**70**	**175**	**145**	390
	High (H)	**25**	**65**	**300**	390
	total	200	300	500	1000

- Q1-How many marginal probabilities can be calculated from these data? State each probability notation and do calculations.
- 6 marginal probabilities, P(L), P(M), P(H), P(p), P(A), P(S).
- Q2-Calculate the probability that an applicant picked at random from this group:

1-Made a low score on the examination

P(L)= 220/1000=0.22

2- Graduated from superior high school.

P(S)= 500/1000=0.5

3- Made a low score on the examination given that he or she graduated from Superior high school

$P(L \mid S) = P(L \cap S) / P(S) = (55/1000) / (500/1000) = 55/500 = 0.11$

5- Made a high score or graduated from a superior high school.

$P(H \cup S) = P(H) + P(S) - P(H \cap S) = (390 + 500 - 300) / 1000 = 0.59$

- Calculate the following probabilities

1. $P(A) = 300/1000 = 0.3$
2. $P(S) = 500/1000 = 0.5$
3. $P(M) = 390/1000 = 0.39$
4. $P(M \cap P) = 70/1000 = 0.07$
5. $P(A \cup L) = (300+220-60)/1000 = 0.46$
6. $P(P \cap S) = 0$
7. $P(L \cup H) = (220+390)/1000 = 0.61$
8. $P(H/S) = 300/500 = 0.6$

		Quality of high school			
		P	A	S	total
Score	L	105	60	55	220
	M	70	175	145	390
	H	25	65	300	390
	total	200	300	500	1000

Chapter 4: Probability Distribution

4.1 Probability Distribution of Discrete Random Variables

- Random variable: is a variable that measured on population where each element must have an equal chance of being selected.
- let X be a discrete random variable, and suppose we are able to count the number of population where X=*x* , **then the value of *x* together with the probability P(X=*x*) are called probability distribution of the discrete random variable X.**

Example 4.1

Suppose we measure the number of complete days that a patient spends in the hospital after a particular type of operation in Dammam hospital in one year, obtaining the following results.

Number of days, *x*	Frequency
1	5
2	22
3	15
4	8
N	50

The probability of the event { X=x } is the relative frequency

P(X=x)= $\frac{n(X=x)}{n(S)} = \frac{n(X=x)}{N}$

That is: P(X=1)=5/50=0.1

P(X=2)=22/50=0.44

P(X=3)=15/50=0.3

P(X=4)=8/50=0.16

- What is the value of $\sum$P(X=x)?

Number of days, x	$P(X=x)$
1	0.1
2	0.44
3	0.3
4	0.16
Sum	1

The probability distribution must satisfy the conditions

`

1- $0 \le P(X = x) \le 1$

2- $\sum P(X = x) = 1$

The first condition must be satisfied since $P(X=x)$ is a probability, and the second condition must be satisfied since the events $\{X=x\}$ are mutually exclusive and there union is the sample space.

-**Population mean for a discrete random variable**: If we know the distribution function P(X=x) for each possible value x of a discrete random variable, then the population mean (or the expected value of the random variable X) is

$$\boxed{\mu = \sum x\, P(X = x)}$$

Example: The expected number of complete days that a patient spends in the hospital after a particular type of operation in Dammam hospital in one year (example 3.1) is

$\mu = \sum x\, P(X = x)$ =1(0.1)+2(0.44)+3(0.3)+4(0.16)=2.52

-Cumulative distributions : the cumulative distribution or the cumulative probability distribution of a random variable is $P(X \le x)$

It is obtained in a way similar to finding the cumulative relative frequency distribution for samples.

-referring to example 3.1

P(X≤1)=0.1

P(X≤2)=P(X=1)+P(X=2)=0.1+0.44=0.54

$P(X\leq 3)=P(X=1)+P(X=2)+P(X=3)=0.1+0.44+0.3=0.84$

$P(X\leq 4)=P(X=1)+P(X=2)+P(X=3)+ P(X=4)=0.1+0.44+0.3+0.16=1$

The cumulative probability distribution can be displayed in the following table

Number of days x	$P(X=x)$	$P(X\leq x)$
1	0.1	0.1
2	0.44	0.54
3	0.3	0.84
4	0.16	1
Sum	1	

-From the table find:

1-$P(X<3)=P(X\leq 2)=0.54$

2-$P(2\leq X\leq 4)=P(X=4)+P(X=3)+P(X=2)=0.9$

Or $P(2 \leq X\leq 4)=P(X\leq 4)-P(X<2)=1-0.1=0.9$

3-$P(X>2)= P(X=3)+P(X=4)=0.46$

Or $P(X>2)= 1-P(X\leq 2)=1-0.54=0.46$

In general we can use the following rules for integer number a and b

1- <u>$P(X \leq a)$ is a cumulative distribution probability</u>

2-$P(X < a)=P(X \leq a-1)$

3-$P(X \geq b)=1-P(X< b)=1-P(X \leq b-1)$

4-$P(X>b)= 1-P(X \leq b)$

5- $P(a\leq X\leq b)=P(X \leq b) – P(X<a)= P(X \leq b) – P(X\leq a-1)$

6- $P(a<X\leq b) = P(X \leq b) - P(X\leq a)$

7-$P(a \leq X<b) = P(X \leq b-1) - P(X\leq a-1)$

8- $P(a<X<b)=P(X \leq b-1)-P(X \leq a)$

4.2 Binomial Distribution

The binomial distribution is a discrete distribution that is used to model the following experiment

1-The experiment has a finite number of trials *n*.

2- Each single trial has only two possible (mutually exclusive)outcomes of interest such as recovers or doesn’t recover; lives or dies; needs an operation or doesn't need an operation. We will call having certain characteristic *success* and not having this characteristic *failure*.

3- The probability of a **success** is a constant **π** for each trial. The probability of a **failure** is **1- π**.

4- The trials are independent; that is the outcome of one trial has no effect on the outcome of any other trial.

Then the discrete random variable X=the number of successes in *n* trials has a Binomial(n,π) distribution for which the probability distribution function is given by

$$P(X=x)=\begin{cases} \binom{n}{x}\pi^{x}(1-\pi)^{n-x} & x=0,1,2,\ \dots,\ n \\ 0 & \text{otherwise} \end{cases}$$

Where $\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

Note

If the discrete random variable X has a binomial distribution , we write

X ~ Bin(n,π)

The mean and variance for the binomial distribution:

- The mean for a Binomial(n , π) random variable is $\boxed{\mu=\Sigma x\, P(X=x)=n\,\pi}$

The variance $\boxed{\sigma^2=n\,\pi\,(1-\pi)}$

Example 4.2

Suppose that the probability that Saudi man has a high blood pressure is 0.15. If we randomly select 6 Saudi men.

a- Find the probability distribution function for the number of men out of 6 with high blood pressure.

b- Find the probability that there are 4 men with high blood pressure?

c-Find the probability that all the 6 men have high blood pressure?

d-Find the probability that none of the 6 men have high blood pressure?

e- what is the probability that more than two men will have high blood pressure?

f-Find the expected number of high blood pressure.

Solution:

Let X= **the number of men out of 6 with high blood pressure.**

Then X has a binomial distribution (why ?).

Success= **The man has a high blood pressure**

Failure= **The man doesn't have a high blood pressure**

Probability of success= $\pi=0.15$ and hence *Probability of failure*= $1-\pi=0.85$

Number of trials= $n=6$

$\boxed{n=6\ ,\ \pi=0.15\ ,\ 1-\pi=0.85}$

- Then X has a Binomial distribution , ***X~ Bin (6,0.15)***

a - the probability distribution function is

$$P(X=x)=\binom{6}{x}0.15^x(0.85)^{6-x}$$
$$x=0,1,\ldots,6$$

---

b- the probability that 4 men will have high blood pressure

$P(X=4)=\binom{6}{4}0.15^4(0.85)^2=(15)(0.15)^4(0.85)^2=0.00549$

C- the probability that all the 6 men have high blood pressure

$P(X=6)=\binom{6}{6}0.15^6(0.85)^0=0.15^6=0.00001$

d-the probability that none of 6 men have high blood pressure is

P(X=0)= $\binom{6}{0} 0.15^0 (0.85)^6 = 0.85^6 = 0.37715$

e- the probability that more than two men will have high blood pressure is

P(X>2)=1-P(X≤2)=1-[P(X=0)+P(X=1)+P(X=2)]

=1-[$0.37715 + \binom{6}{1} 0.15^1 (0.85)^5 + \binom{6}{2} 0.15^2 (0.85)^4$]

=1-[$0.37715 + 0.39933 + 0.17618$] $= 1 - 0.95266 = 0.04734$

F- the <u>expected number of high blood pressure</u> is $\mu = n\pi = 6(0.15) = 0.9$

and <u>the variance</u> is $\sigma^2 = n\pi(1-\pi) = 6(0.15)(0.85) = 0.765$

4.3 The Poisson Distribution

The Poisson distribution is a discrete distribution that is used to model the random variable X that represents **the number of occurrences of some random event in the interval of time or space.**

The probability that X will occur (the probability distribution function) is given by:

$$P(X=x)=\begin{cases}\dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x=0,1,2,\ldots\ldots \\ 0 & \text{otherwise}\end{cases}$$

λ is the **average number** of occurrences of the random variable in the interval.

The mean $\mu=\lambda$

The variance $\sigma^2=\lambda$

If X has a Poisson distribution we write **X~ Poisson (λ)**

Examples of Poisson distribution:

- The number of patients in a waiting room in **an hour**.
- The number of serious injuries (الاصابات الخطيرة) in a particular factory in **a year**.
- The number of times a three year old child has an ear infection (عدوى الأذن) in **a year.**

- **Example 4.3:**

Suppose we are interested in the number of snake bite (لدغة الأفعى) cases seen in a particular Riyadh hospital *in a year*. Assume that **the average number** of snake bite cases at the hospital in a year is **6** .

1- What is the probability that in a randomly chosen year, the number of snake bites cases will be 7?

2- What is the probability that the number of cases will be less than 2 in 6 months?

3-What is the probability that the number of cases will be 13 in 2 year ?

4- What is Expected number of snake bites in a year? What is the variance of snake bites in a year?

Solution:

X= number of snake bite cases seen at this hospital *in a year. And the mean is 6*

Then X~ Poisson (6)

First note the following

- The average number of snake bite cases at the hospital in a *year* $=\lambda=6$

$$\boxed{X\sim \text{Poisson }(6)}$$

- The average number of snake bite cases at the hospital in *6 months* =

= the average number of snake bite cases at the hospital in (1/2) *year* $=(1/2)\lambda=3$

$$\boxed{Y\sim \text{Poisson }(3)}$$

- The average number of snake bite cases at the hospital in *2 years* $= 2\lambda=12$

$$\boxed{V\sim \text{Poisson }(12)}$$

1- The probability that the number of snake bites will be 7 in *a year*

$\lambda=6$

$$P(X=x)=\frac{e^{-6}6^{x}}{x!}, \qquad x=0,1,2,...$$

$$P(X=7)=\frac{e^{-6}6^{7}}{7!}=0.138$$

2- The probability that the number of cases will be less than 2 in 6 months

$\lambda^{*}=3$

$$P(Y=y)=\frac{e^{-3}3^{y}}{y!}, y=0,1,2...$$

$$P(Y<2)=P(Y=0)+P(Y=1)$$

$$=\frac{e^{-3}3^{0}}{0!}+\frac{e^{-3}3^{1}}{1!}=0.0498+0.1494 \quad =0.1992$$

Remember

If X~ Poisson (λ)

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$x=0,1,2,....$

3- The probability that the number of cases will be 13 in 2 years

$\lambda^{**}=12$

$$P(V=v)=\frac{e^{-12}12^{v}}{v!}$$

$$P(V=13)=\frac{e^{-12}12^{13}}{13!}=0.1056$$

4- the expected number of snake bites in a year: $\mu=\lambda=6$

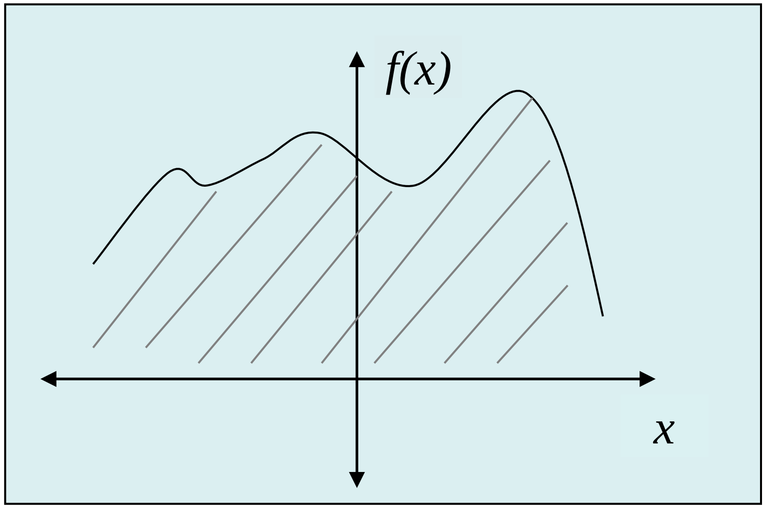
$\lambda=6$

the variance of snake bites in a year: $\sigma^{2}=\lambda=6$

4.4 Probability Distribution of Continuous Random Variable

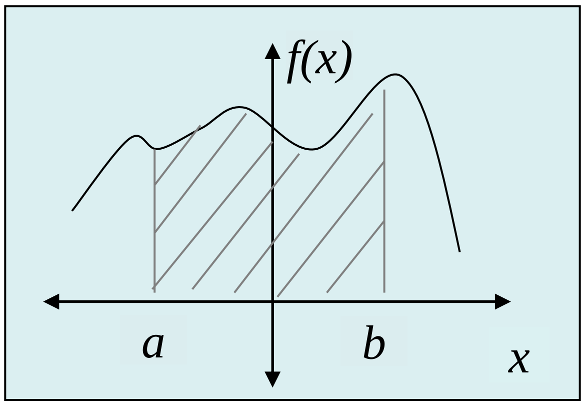
If X is a continuous random variable, then there exist a function *f(X)* called *probability density function* that has the following properties:

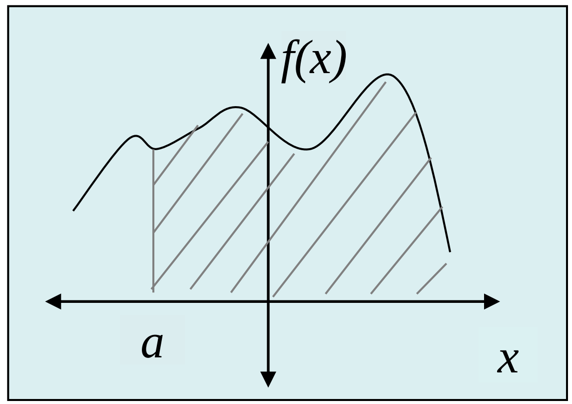
1- The area under the probability curve *f(x)* =1

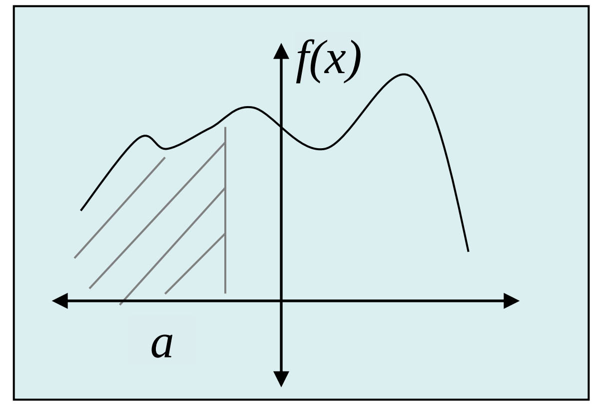


$$\text{area}= \int_{-\infty}^{\infty} f(x)dx = 1$$

2- Probability of interval events are given by areas under the probability curve







$P(a \le X \le b) = \int_a^b f(x)dx$ $P(X \ge a) = \int_a^\infty f(x)dx$ $P(X \le a) = \int_{-\infty}^a f(x)dx$

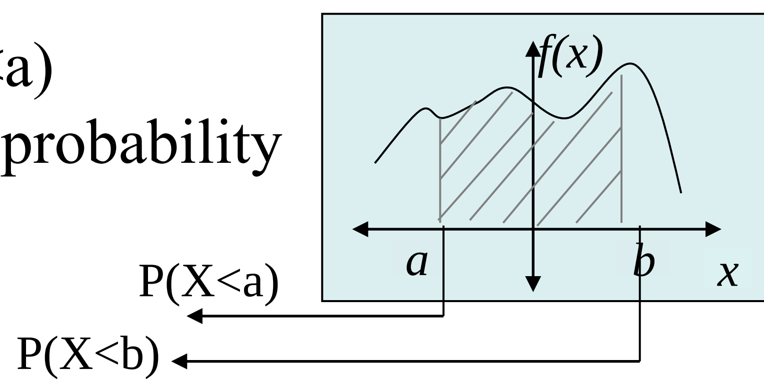
3- P(X=a)=0 (why?)

4-$P(X \ge a) = P(X > a)$ and $P(X \le a) = P(X < a)$

7- $P(X \le a) = P(X < a)$ is the cumulative probability

5- $P(X \ge a) = 1 - P(X \le a)$

6-$P(a < X < b) = P(X < b) - P(X < a)$



4.5 The Normal Distribution:

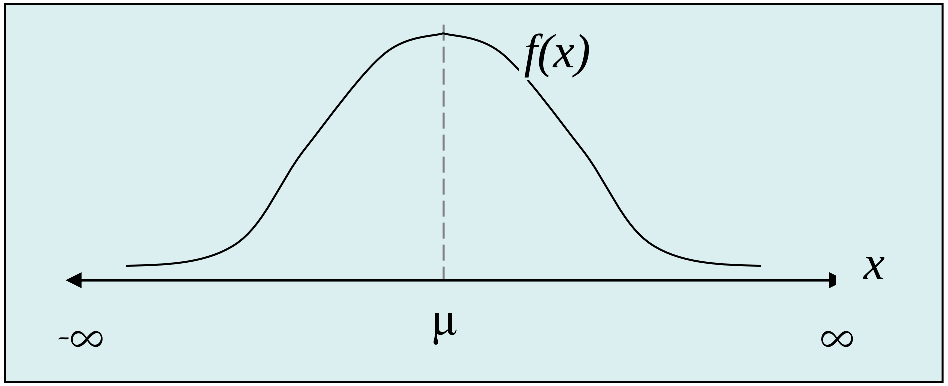
The normal distribution is one of the most important **continuous distribution** in statistics.

It has the following characteristics

1- X takes values from $-\infty$ to ∞.

2- The population mean is μ and the population variance is σ^2, and we write $X \sim N(\mu, \sigma^2)$.

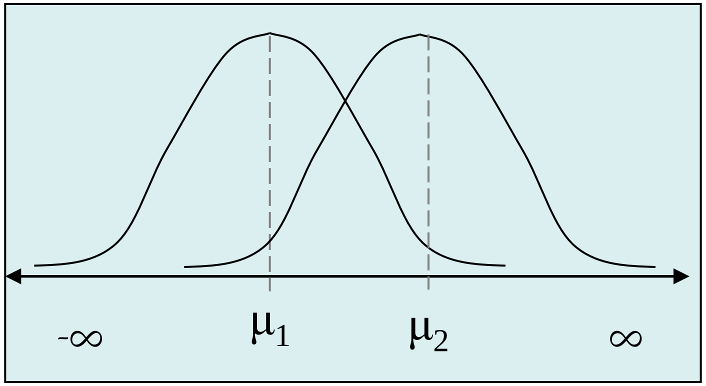
3- The graph of the density of a normal distribution has a bell shaped curve, that is symmetric about μ



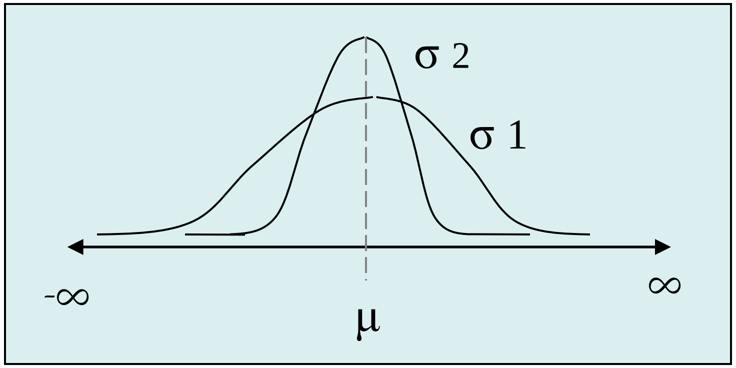
4- μ= mean=mode=median of the normal distribution.

5-The location of the distribution depends on μ (location parameter).

The shape of the distribution depends on σ (shape parameter).



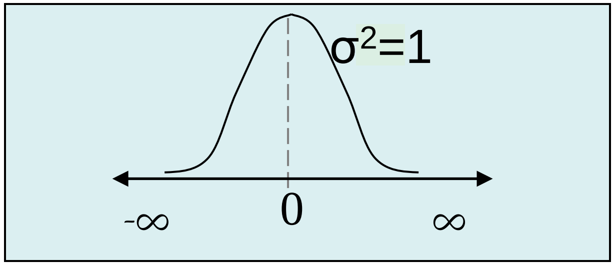
$\mu_1 < \mu_2$



$\sigma_1 > \sigma_2$

Standard normal distribution:

– The *standard normal distribution* is a normal distribution with mean $\mu=0$ and variance $\sigma^2=1$.



Result

– If $X \sim N(\mu, \sigma^2)$ then $\boxed{Z=\dfrac{X-\mu}{\sigma} \sim N(0, 1).}$

Notes

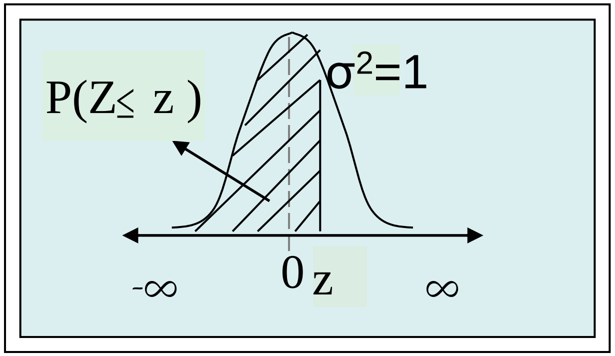
- The probability $A=P(Z \le z)$ is the area to the left of z under the standard normal curve.

-There is a Table gives values of <u>$P(Z \le z)$</u> for different values of z.

Calculating probabilities from Normal (0,1)

- $P(Z \leq z)$ From the table

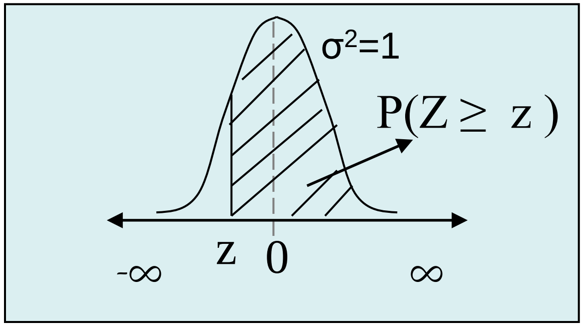
(the area under the curve to the left of z)



- $P(Z \geq z) = 1 - P(Z \leq z)$

From the table

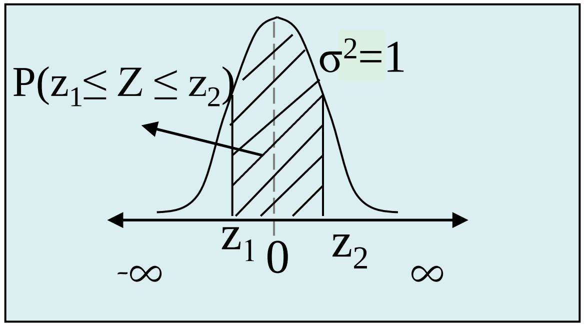
(the area under the curve to the right of z)



- $P(z_1 \leq Z \leq z_2) = P(Z \leq z_2) - P(Z \leq z_1)$

From the table

(the area under the curve between z_1 and z_2)



Notes:

- $P(Z \leq 0) = P(Z \geq 0) = 0.5$ (why?)
- $P(Z = z) = 0$ for any z.
- $P(Z \leq z) = P(Z < z)$ and $P(Z \geq z) = P(Z > z)$
- If $z \leq -3.49$ then $P(Z \leq z) = 0$, and if $z \geq 3.49$ then $P(Z \leq z) = 1$.

Example 4.1 :

- $P(Z \leq 1.5) = 0.9332$
- $P(-1.33 \leq Z \leq 2.42) = P(Z \leq 2.42) - P(Z < 1.33) =$

  $= 0.9922 - 0.0918 = 0.9004$
- $P(Z \geq 0.98) = 1 - P(Z \leq 0.98) = 1 - 0.8365 = 0.1635$

Z	0.00	0.01	…
:	$\Downarrow$		
1.5 $\Rightarrow$	0.933		
:			

Example 4.2 :

Suppose that the hemoglobin level for healthy adult males are approximately normally distributed with mean 16 and variance of 0.81. Find the probability that a randomly chosen healthy adult male has hemoglobin level

a) Less than 14. b) Greater than 15. C) Between 13 and 15

Solution

Let X= the hemoglobin level for healthy adult male, then

$X \sim N(\mu=16, \sigma^2=0.81)$.

a) Since $\mu=16$, $\sigma^2=0.81$, we have $\sigma=\sqrt{0.81}=0.9$

$P(X<14)= P(Z<\frac{14-\mu}{\sigma})= P(Z<\frac{14-16}{0.9})= P(Z< -2.22)=0.0132$

b) $P(X>15)= P(Z>\frac{15-\mu}{\sigma})= P(Z>\frac{15-16}{0.9})= P(Z> -1.11)= 1- P(Z\leq -1.11)=$

$= 1- 0.1335=0.8665$.

c) $P(13<X<15)= P(\frac{13-\mu}{\sigma}<Z<\frac{15-\mu}{\sigma})= P(Z<\frac{15-16}{0.9})- P(Z<\frac{13-16}{0.9})$

$= P(Z\leq -1.11) - P(Z \leq -3.33)$

$= 0.1335- 0= 0.1335$.

d) P(X=13)=0

Result(1)

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from N (μ, σ^2), then

1) $\overline{X} = \dfrac{\sum_{i=1}^{n} x_i}{n} \sim N(\mu, \sigma^2/n)$

2) $\boxed{Z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1).}$

Central Limit Theorem

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from any distribution with mean μ and variance σ^2, and if n is large ($n \geq 30$), then

$$\boxed{Z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \approx N(0,1).}$$

(that is, Z has approximately standard normal distribution)

Result (2)

If σ^2 is unknown in the central limit theorem, then s (the sample standard deviation) can be used instead of σ, that is

$$\boxed{Z = \frac{\bar{x} - \mu}{s / \sqrt{n}} \approx N(0, 1).}$$

Where $s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n(\bar{x})^2}{n-1}}$

Chapter 5: Statistical Inference

5.1 Introduction: There are two main purposes in statistics

-Organizing and summarizing data (descriptive statistics).

-Answer research questions about population parameter (statistical inference).

There are two general areas of statistical inference:

- **Hypothesis testing**: answering questions about population parameters.
- **Estimation**: approximating the actual values of population parameters.

there are two kinds of estimation:

- Point estimation.
- Interval estimation (confidence interval).

Here we will consider two types of population parameters

Population mean: μ
(for quantitative variable)

μ=The average (mean) value for some qualitative variable.

Examples:
-The mean life span for some bacteria
- The income mean for some bacteria
- The income mean of government employee in Saudi Arabia.

Population proportion π

$$\pi = \frac{\text{no. of element in the population with some charachtaristic}}{\text{Total no. of element in the population}}$$

Examples:
-The proportion of Saudi people who have some disease
- The proportion of smokers in Riyadh.
- The proportion of Children in Saudi Arabia.

5.2: Estimation of Population Mean: μ

1) Point Estimation:

- A point estimate is a single number used to estimate the corresponding population **parameter**.
- $\boxed{\bar{x} \quad \text{is a point estimate of } \mu}$

That is , the sample mean is a point estimate of the population mean.

---

2) Interval Estimation (Confidence Interval:C.I) of μ

- Definition: (1-α)100% Confidence Interval:

(1-α)100% Confidence Interval is an interval of numbers (L,U), defined by lower L and upper U limits that contains the population parameter with probability (1-α).

1-α: the confidence coefficient.

L: Lower limit of the confidence interval.

U : upper limit of the confidence interval.

A (1-α)100% CI for μ is

- If the distribution is normal
  - If σ is known
    - $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
  - If σ is unknown
    - n is large (n>30)
      - $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$
- If the distribution is not normal
  - n is large (n>30)
    - If σ is known
      - $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
    - If σ is unknown
      - $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$

Note: The C.I $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ means

(L , U)= $\left(\bar{x} - z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \quad \bar{x} + z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right)$

- Similarly for $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$, (L , U)= $\left(\bar{x} - z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \ \bar{x} + z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} . \right)$

- Interpretation of the CI: We are (1-α)100% confident that the (mean) of (variable) for the (population) is between L and U.

$\boxed{\mu}$

Example 5.1:

Let Z~N(0, 1)

$Z_{1-\frac{\alpha}{2}} = ???$

Here we have the probability (the area) and we want to find the exact value of z. hence we can use the table of standard normal but in the opposite direction.

a) α=0.05

$\alpha/2=0.025$

$1-\alpha/2=0.975$

From the standard normal table $\boxed{Z_{0.975} = 1.96}$

b) α=0.1

$\alpha/2=0.05$

$1-\alpha/2=0.95$

$\boxed{Z_{0.95} = 1.645}$

Z	…	0.06	…
:	:	⇑ ⇑	
1.9	⇐⇐	0.975	
:			

Example 5.2: On 123 patient of diabetic ketoacidosis (الحماض الكيتوني السكري) patient in Saudi Arabia , the mean blood glucose level was 26.2 with a standard deviation of 3.3 mm0l/l. Find the 90% confidence interval for the mean blood glucose level of such diabetic ketoacidosis patient.

Solution:

Variable: blood glucose level (in mmol/l)

Population: Diabetic ketoacodosis patient in Saudi Arabia.

Parameter: μ (the average blood glucose level)

n=123, $\bar{x} = 26.2$ s=3.3

- σ^2 unknown , n=123>30 (large) $\Rightarrow$ the 90% CI for μ is given by

$$\boxed{\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}}$$

90% = (1- α)100% $\Rightarrow$ *1- α=0.9*

α=0.1 $\Rightarrow$ *α/2=0.05* $\Rightarrow$ *1-α/2=0.95*

$Z_{0.95} = 1.645$

The 90% CI for μ is $\boxed{\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}}}$

Which is can be written as $(\bar{x} - z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}},\ \bar{x} + z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}})$

$$= (26.2 - (1645)\frac{3.3}{\sqrt{123}},\ 26.2 + (1645)\frac{3.3}{\sqrt{123}})$$

$$= (25.71,\ 26.69)$$

<u>Interpretation: We</u> are 90 % confident that the mean blood glucose level of diabetic ketoacidosis patient in Saudi Arabia is between 25.71 and 26.69

Exercises

Q1: Suppose that we are interested in making some statistical inferences about the mean μ of normal population with standard deviation 0.2 . Suppose that a random sample of size n = 49 from this population gave the sample mean4.5

The distribution of is

(a) N(0,1) (b) t(48) (c)N(μ,$(0.02857)^2$) (d)N(μ,2.0)

A good point estimate for μ is

(a) 4.5 (b) 2 (c) 2.5 (d) 7 (e) 1.125

Assumptions is

(a) Normal, σ known (b) Normal, σ unknown (c)not Normal, σ known

(d) not Normal, σ unknown

(4)A 95% confident interval for μ is

(a) (3.44,5.56) (b) (3.34,5.66) (c) (4.444, 4.556)

(d) (3.94,5.05) (e) (3.04,5.96)

Q2:An electronics company wanted to estimate in monthly operating expenses riyals (μ) . Assume that the population variance equals 0.584 .
Suppose that a random sample of size 49 is taken and found that the sample mean equals 5.47 . Find

Point estimate for μ
The distribution of the sample mean is
The assumptions ?
A 90% confident interval for μ.

Q3:The random variable X, representing the lifespan of a certain light bulb is distributed normally with mean of 400 hours ,and standard deviation of 10 hours.

-What is the probability that a particular light bulb will last for more than 380 hours ?
-What is the probability that a particular light bulb will last for exactly 399 hours ?
-What is the probability that a particular light bulb will last for between 380 and 420 hours ?
The mean is ……..
The variance is…..
The standard deviation ……

Q4: The tensile of a certain type of thread is approximately normally distributed with standard deviation of 6.8 Kg. A sample of 20 pieces of the thread has an average strength of 72.8 Kg. Then

A point estimate of the population mean of tensile strength μ is

(a)72.8 (b) 20 (c) 6.8 (d) 46.24 (e) none of these

A 98% Confident interval for mean of tensile strength μ,the lower bound equal to :

(a)68.45 (b) 69.26 (c) 71.44 (d) 69.68 (e) none of these

A 98% Confident interval for mean of tensile strength μ,the upper bound equal to :

(a)74.16 (b) 77.15 (c) 75.92 (d) 76.34 (e) none of these

5.3: Estimation of Population Proportion π

- **Recall that, the population proportion**

$$\pi = \frac{\text{no. of element in the population with some charachtaristic}}{\text{Total no. of element in the population}}$$

N

- **To estimate the population proportion we take a sample of size n from the population and find the sample proportion p**

$$p = \frac{\text{no. of element in the sample with some charachtristic}}{\text{Total no. of element in the sample}}$$

n

Result: when both nπ>5 and n(1- π)>5 then

$$p \approx \mathrm{N}(\pi, \pi(1-\pi)/n).$$

and hence

$$Z = \frac{p-\pi}{\sqrt{\pi(1-\pi)/n}} \approx \mathrm{N}(0,1).$$

Estimation for π

1) Point Estimation:

A point estimator of π (population proportion) is p (sample proportion)

1) Interval Estimation: If np>5 and n(1-p)>5,

The (1-α)100% Confidence Interval for π is given by

$$\boxed{p \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}}$$

Note:1) $p \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$ can be written as

$$\left(p - z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}, \; p + z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \right)$$

2) np= the number in the sample with the characteristic

n(1-p)= the number in the sample wich did not have the characteristic.

Example 5.2

In the study on the fear (خوف) of dental care in Riyadh, 22% of 347 adults said they would hesitate (تردد) to take a dental appointment due to fear. Find the point estimate and the 95% confidence interval for proportion of adults in Riyadh who hesitate to take dental appointments.

.Solution:

Variable: whether or not the person would hesitate to take a dental appointment out of fear.

Population: adults in Riyadh.

Parameter: π, the proportion who would hesitate to take an appointment.

n= 347 , p= 22%=0.22,

np=(347)(0.22)=76.34 >5 and n(1-p)=(437)(0.78)=270.66>5

1- point estimation of π is p=0.22

2- 95% CI for π is $p \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$

1-α=0.95 ⇒ α=0.05 ⇒ α/2=0.025 ⇒1- α/2=0.975

$$Z_{1-\alpha/2} = Z_{0.975} = 1.96$$

The 95 % CI for π is

$$\left(p - z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}, p + z_{1-\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \right)$$

$$= \left(0.22 - (1.96)\sqrt{\frac{0.22(0.78)}{347}}, 0.22 + (1.96)\sqrt{\frac{0.22(0.78)}{347}} \right)$$

$$= \left(0.22 - (1.96)(0.0222379), 0.22 + (1.96)(0.0222379)\right)$$

$$= (0.176, 0.264)$$

<u>Interpretation</u>: we are 95% confident that the true proportion of adult in Riyadh who hesitate to take a dental appointment is between 0.176 and 0.264 .

Exercises

Q1: A random sample of 200 students from a certain school showed that 15 students smoke. let π be the proportion of smokers in the school.

- Find a point estimate for π
- Find 95% confidence interval for π

Q2. A researcher was interested in making some statistical inferences about the proportion of females (π) among the students of a certain university. A random sample of 500 students showed that 150 students are female.

1. A good point estimate for π is

(A)	0.31	(B)	0.30	(C)	0.29	(D)	0.25	(E)	0.27

1. The lower limit of a 90% confidence interval for π is

(A)	0.2363	(B)	0.2463	(C)	0.2963	(D)	0.2063	(E)	0.2663

1. The upper limit of a 90% confidence interval for π is

(A)	0.3337	(B)	0.3137	(C)	0.3637	(D)	0.2937	(E)	0.3537

Q3. In a random sample of 500 homes in a certain city, it is found that 114 are heated by oil. Let π be the proportion of homes in this city that are heated by oil.

1. Find a point estimate for π.
2. Construct a 98% confidence interval for π.

Q4. In a study involved 1200 car drivers, it was found that 50 car drivers do not use seat belt.

- A point estimate for the proportion of car drivers who do not use seat belt is:
  (A) 50 (B) 0.0417 (C) 0.9583 (D) 1150 (E) None of these

- The lower limit of a 95% confidence interval of the proportion of car drivers not using seat belt is
  (A) 0.0322 (B) 0.0416 (C) 0 .0304 (D) –0.3500 (E) None of these

- The upper limit of a 95% confidence interval of the proportion of car drivers not using seat belt is
  (A) 0.0417 (B) 0.0530 (C) 0.0512 (D) 0.4333 (E) None of these

Q5. A study was conducted to make some inferences about the proportion of female employees (π) in a certain hospital. A random sample gave the following data:

•Calculate a point estimate (p) for the proportion of female employees (π).
•Construct a 90% confidence interval for p.

Sample size	250
Number of females	120