SOCIETY OF ACTUARIES

EXAM MFE MODELS FOR FINANCIAL ECONOMICS

EXAM MFE SAMPLE SOLUTIONS
INTRODUCTORY DERIVATIVES

These questions are from McDonald Chapters 1, 2, 3, and 5 only. Questions 1-64 are former Exam FM Sample Exam Questions. Some of the questions have been slightly modified to match the format of Exam MFE. Questions 65-75 are newly added.

These questions are representative of the types of questions that might be asked of candidates sitting for Exam MFE. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

February 8, 2017 changes:

Question 31 has been deleted and Question 69 has been edited.

MFE-02-17

1. Solution: D

If the call is at-the-money, the put option with the same cost will have a higher strike price. A purchased collar requires that the put have a lower strike price.

2. Solution: C

$66.59 – 18.64 = 500 – K\exp(–0.06)$ and so $K = (500 – 66.59 + 18.64)/\exp(–0.06) = 480$.

3. Solution: D

The accumulated cost of the hedge is (84.30 – 74.80)exp(0.06) = 10.09.

Let x be the market price in one year.

If $x < 0.12$ the put is in the money and the payoff is $10,000(0.12 – x) = 1,200 – 10,000x$. The sale of the jalapenos has a payoff of $10,000x – 1,000$ for a profit of $1,200 – 10,000x + 10,000x – 1,000 – 10.09 = 190$.

From 0.12 to 0.14 neither option has a payoff and the profit is $10,000x – 1,000 – 10.09 = 10,000x – 1,010$. The range is 190 to 390.

If $x > 0.14$ the call is in the money and the payoff is $–10,000(x – 0.14) = 1,400 – 10,000x$. The profit is $1,400 – 10,000x + 10,000x – 1,000 – 10.09 = 390$.

The range is 190 to 390.

4. DELETED

5. Solution: E

Consider buying the put and selling the call. Let x be the index price in one year. If $x > 1025$, the payoff is $1025 – x$. After buying the index for x you have $1,025 – 2x$ which is not the goal. It is not necessary to check buying the call and selling the put as that is the only other option. But as a check, if $x > 1025$, the payoff is $x – 1025$ and after buying the stock you have spent 1025. If $x < 1025$, the payoff is again $x – 1025$.

One way to get the cost is to note that the forward price is 1,000(1.05) = 1,050. You want to pay 25 less and so must spend 25/1.05 = 23.81 today.

6. Solution: E

In general, an investor should be compensated for time and risk. A forward contract has no investment, so the extra 5 represents the risk premium. Those who buy the stock expect to earn both the risk premium and the time value of their purchase and thus the expected stock value is greater than 100 + 5 = 105.

7. Solution: C

All four of answers A-D are methods of acquiring the stock. Only the prepaid forward has the payment at time 0 and the delivery at time T.

8. Solution: B

Only straddles use at-the-money options and buying is correct for this speculation.

9. Solution: D

To see that D does not produce the desired outcome, begin with the case where the stock price is S and is below 90. The payoff is $S + 0 + (110 – S) – 2(100 – S) = 2S – 90$ which is not constant and so cannot produce the given diagram. On the other hand, for example, answer E has a payoff of $S + (90 – S) + 0 – 2(0) = 90$. The cost is 100 + 0.24 + 2.17 – 2(6.80) = 88.81. With interest it is 93.36. The profit is 90 – 93.36 = –3.36 which matches the diagram.

10. Solution: D

Answer A is true because forward contracts have no initial premium.

Answer B is true because both payoffs and profits of long forwards are opposite to short forwards.

Answer C is true because to invest in the stock, one must borrow 100 at t = 0, and then pay back 110 = 100(1 + 0.1) at t = 1, which is like buying a forward at t = 1 for 110.

Answer D is false because repeating the calculation shown for Answer C, but with 10% as a continuously compounded rate, the stock investor must now pay back 100exp(0.1) = 110.52 at t = 1; this is more expensive than buying a forward at t = 1 for 110.00.

Answer E is true because the calculation would be the same as shown above for Answer C but now the stock investor gets an additional dividend of 3.00 at t = 0.5, which the forward investor does not receive (due to not owning the stock until t = 1.

11. Solution: C

The future value of the cost of the options is 9.12(1.08) = 9.85, 6.22(1.08) = 6.72, and 4.08(1.08) = 4.41 respectively.

If $S<35$ no call is in the money and the profits are –9.85, –6.72, & –4.41. The condition isn't met.

If $35 < S < 40$ the 35-strike call returns $S – 35$ and the profit is $S – 44.85$. For the 45-strike call to have a lower profit than the 35-strike call, we need $–4.41 < S – 44.85$ or $S > 40.44$. This is inconsistent with the assumption.

If $40 < S < 45$ the same condition applies for comparing the 35- and 45-strike calls and so $S > 40.44$ is needed. The 40-strike call has a profit of $S – 40 – 6.72 = S – 46.72$. For the 45-strike to exceed the 40-strike, we need $–4.41 > S – 46.72$ or $S < 42.31$.

There is no need to consider $S > 45$.

12. Solution: B

Let S be the price of the index in six months.

The put premium has future value (at t = 0.5) of 74.20[1 + 0.02] = 75.68.

The 6-month profit on a long put position is max(1,000 – S, 0) – 75.68.

The 6-month profit on a short put position is 75.68 – max(1,000 – S, 0).

0 = 75.68 – max(1,000 – S, 0).

75.68 = max(1,000 – S, 0).

75.68 = 1,000 – S. S = 924.32.

13. Solution: D

Buying a call in conjunction with a short position is a form of insurance called a cap. Answers (A) and (B) are incorrect because a floor is the purchase of a put to insure against a long position. Answer (E) is incorrect because writing a covered call is the sale of a call along with a long position in the stock, so that the investor is selling rather than buying insurance.

The profit is the payoff at time 2 less the future value of the initial cost. The stock payoff is –75 and the option payoff is 75 – 60 = 15 for a total of –60. The future value of the initial cost is (–50 + 10)(1.03)(1.03) = –42.44. the profit is –60 – (–42.44) = –17.56.

14. Solution: A

Let C be the price for the 40-strike call option. Then, C + 3.35 is the price for the 35-strike call option. Similarly, let P be the price for the 40-strike put option. Then, P – x is the price for the 35-strike put option, where x is the desired quantity. Using put-call parity, the equations for the 35-strike and 40-strike options are, respectively,

$$(C+3.35)+35e^{-0.02}-40=P-x$$

$$C+40e^{-0.02}-40=P.$$

Subtracting the first equation from the second, $5e^{-0.02}-3.35=x, \quad x=1.55.$

15. Solution: C

The initial cost to establish this position is $5(2.78) – 3(6.13) = –4.49$. Thus, you are receiving 4.49 up front. This grows to $4.49\exp[0.25(0.08)] = 4.58$ after 3 months. Then, if S is the value of the stock at time 0.25, the profit is

$5\max(S – 40,0) – 3\max(S – 35,0) + 4.58$. The following cases are relevant:

$S < 35$: Profit = 0 – 0 + 4.58 = 4.58.

$35 < S < 40$: Profit = $0 – 3(S – 35) + 4.58 = –3S + 109.58$. Minimum of –10.42 is at $S = 40$ and maximum of 4.58 is at $S = 35$.

$S > 40$: Profit is $5(S – 40) – 3(S – 35) + 4.58 = 2S – 90.42$. Minimum of –-10.42 is at $S = 40$ and maximum if infinity.

Thus the minimum profit is –10.42 for a maximum loss of 10.42 and the maximum profit is infinity.

16. Solution: D

The straddle consists of buying a 40-strike call and buying a 40-strike put. This costs 2.78 + 1.99 = 4.77 and grows to $4.77\exp(0.02) = 4.87$ at three months. The strangle consists of buying a 35-strike put and a 45-strike call. This costs 0.44 + 0.97 = 1.41 and grows to $1.41\exp(0.02)$ = 1.44 at three months. Let S be the stock price in three months.

For $S < 40$, the straddle has a profit of $40 – S – 4.87 = 35.13 – S$.

For $S > 40$, the straddle has a profit of $S – 40 – 4.87 = S – 44.87$

For $S < 35$, the strangle has a profit of $35 – S – 1.44 = 33.56 – S$.

For $35 < S < 45$, the strangle has a profit of –1.44.

For $S > 45$, the strangle has a profit of $S – 45 – 1.44 = S – 46.44$.

For $S < 35$ the strangle underperforms the straddle.

For $35 < S < 40$, the strangle outperforms the straddle if $–1.44 > 35.13 – S$ or $S > 36.57$. At this point only Answer D can be correct.

As a check, for $40 < S < 45$, the strangle outperforms the straddle if $–1.44 > S – 44.87$ or $S < 43.43$.

For $S > 45$, the strangle outperforms the straddle if $S – 46.44 > S – 44.87$, which is not possible.

17. Solution: B

Strategy I –Yes. It is a bear spread using calls, and bear spreads perform better when the prices of the underlying asset goes down.

Strategy II – Yes. It is also a bear spread – it just uses puts instead of calls.

Strategy III – No. It is a box spread, which has no price risk; thus, the payoff is the same (1,000 – 950 = 50), no matter the price of the underlying asset.

18. DELETED

19. DELETED

20. Solution: B

We need the future value of the current stock price minus the future value of each of the 12 dividends, where the valuation date is time 3. Thus, the forward price is

$$
\begin{aligned}
&200e^{0.04(3)} - 1.50[e^{0.04(2.75)} + e^{0.04(2.5)}1.01 + \cdots + e^{0.04(0.25)}1.01^{10} + 1.01^{11}] \\
&= 200e^{0.12} - 1.50e^{0.11}[1 + (e^{-0.01}1.01) + \cdots + (e^{-0.01}1.01)^{10} + (e^{-0.01}1.01)^{11}] \\
&= 200e^{0.12} - 1.50e^{0.11}\frac{1-(e^{-0.01}1.01)^{12}}{1-e^{-0.01}1.01} = 225.50 - 1.67442(11.99672) = 205.41.
\end{aligned}
$$

21. Solution: E

The fair value of the forward contract is given by $S_0e^{(r-d)T} = 110e^{(0.05-0.02)0.5} = 111.66.$

This is 0.34 less than the observed price. Thus, one could exploit this arbitrage opportunity by selling the observed forward at 112 and buying a synthetic forward at 111.66, making 112 – 111.66 = 0.34 profit.

22. DELETED

23. DELETED

24. Solution: D

(A) is a reason because hedging reduces the risk of loss, which is a primary function of derivatives.

(B) is a reason because derivatives can be used the hedge some risks that could result in bankruptcy.

(C) is a reason because derivatives can provide a lower-cost way to effect a financial transaction.

(D) is not a reason because derivatives are often used to avoid these types of restrictions.

(E) is a reason because an insurance contract can be thought of as a hedge against the risk of loss.

25. Solution: C

(A) is accurate because both types of individuals are involved in the risk-sharing process.

(B) is accurate because this is the primary reason reinsurance companies exist.

(C) is not accurate because reinsurance companies share risk by issuing rather than investing in catastrophe bonds. They are ceding this excess risk to the bondholder.

(D) is accurate because it is diversifiable risk that is reduced or eliminated when risks are shared.

(E) is accurate because this is a fundamental idea underlying risk management and derivatives.

26. Solution: B

I is true. The forward seller has unlimited exposure if the underlying asset's price increases.

II is true. The call issuer has unlimited exposure if the underlying asset's price rises.

III is false. The maximum loss on selling a put is FV(put premium) – strike price.

27. DELETED

28. DELETED

29. Solution: A

The current price of the stock and the time of future settlement are not relevant, so let both be 1. Then the following payments are required:

Outright purchase, payment at time 0, amount of payment = 1.

Fully leveraged purchase, payment at time 1, amount of payment = exp(r).

Prepaid forward contract, payment at time 0, amount of payment = exp(-d).

Forward contract, payment at time 1, amount of payment = exp(r-d).

Since $r > d > 0$, $\exp(-d) < 1 < \exp(r-d) < \exp(r)$.

The correct ranking is given by (A).

30. Solution: C

(A) is a distinction. Daily marking to market is done for futures, not forwards.

(B) is a distinction. Futures are more liquid; in fact, if you use the same broker to buy and sell, your position is effectively cancelled.

(C) is not a distinction. Forwards are more customized, and futures are more standardized.

(D) is a distinction. With daily settlement, credit risk is less with futures (v. forwards).

(E) is a distinction. Futures markets, like stock exchanges, have daily price limits.

31. DELETED

32. Solution: E

The notional value of this short futures position is 1500(20)(250) = 7.5 million. The initial margin requirement is 5% of 7.5 million, or 375,000, and the maintenance margin requirement is 90% of 375,000, or 337,500. Judy has a short position, so when the index decreases/increases, her margin account would increase/decrease.

At the first marking-to-market, when the index has fallen to 1498, the margin account is:

$375{,}000 \times \exp(0.04/365) + (1500 - 1498)(20)(250) = 375{,}041.10 + 10{,}000 = 385{,}041.10.$

For Judy not to get a margin call at the second marking-to-market, the value of the index, X, would have to rise so that the account balance decreases to 337,500:

$385{,}041.10 \times \exp(0.04/365) + (1498 - X)(20)(250) = 337{,}500$

$X = 1507.52.$

33: Solution: E

Option I is American-style, and thus, it can be exercised at any time during the 6-month period. Since it is a put, the payoff is greatest when the stock price is smallest (18). The payoff is 20 – 18 = 2.

Option II is Bermuda-style, and can be exercised at any time during the 2nd 3-month period. Since it is a call, the payoff is greatest when the stock price S is largest (28). The payoff is 28 – 25 = 3.

Option III is European-style, and thus, it can be exercised only at maturity. Since it is a 30-strike put, the payoff equation is 30 – 26 = 4.

The ranking is III > II > I.

34. DELETED

35. Solution: C

If the index declined to $45 and the customer exercised the put (buying 100 shares in the market and selling it to the writer for the $50 strike price), the customer would make $500 ($5000 proceeds of sale – $4500 cost = $500). However, this would be offset by the $500 premium paid for the option. The net result would be that the customer would break even.

36. DELETED

37. Solution: B

When there are discrete dividends, the pricing formula is $S(1 + i)$ – AV(dividends), where S is the current stock price. Thus,

$$75 = S(1.06) - [1.5(1.06)^{0.5} + 1.5 = S(1.06) - 3.0443$$
$$S = 78.0433/1.06 = 73.626.$$

38. Solution: C

For stocks without dividends and in the absence of transaction costs, the stock's forward price is the future value of its spot price based on the risk-free interest rate; otherwise there would be an arbitrage opportunity. Because the risk-free interest rate is positive, the forward price must be greater than the spot price of 75.

Because these investors are risk-averse (i.e. they prefer not to take risks if the average rate of return is the same) they need to receive on the average a greater return than the risk-free interest rate on the shares they invest in this stock. In other words, they need to receive a risk premium (incentive) for taking on risk. The forward price only includes the risk-free interest rate and not the risk premium, so the forward price is less than the expected value of the future stock price, namely 90.

39. Solution: E

By definition, one way to use a 3:1 ratio spread is to buy 1 call and sell 3 calls at a different strike price, with the same 1-year maturity. (This can also be done using all puts.)

40. Solution: C

If F is the forward price, K is the strike price, C is the call option price, P is the put option price, v is the annual discounting factor at the risk-free rate, and t is the number of years, then we have the put-call parity formula $C - P = v^t\left(F - K\right)$.

Using the data for each commodity (with C being the call price zinc and K the common strike price), we have

$$700 - 550 = \frac{1}{(1.06)^2}\left(1400 - K\right)$$
$$C - 550 = \frac{1}{(1.06)^2}\left(1600 - K\right).$$

Now we could solve for K in the top equation and then use this to solve the second equation for C, but a more efficient method is to subtract the top equation from the bottom equation to cancel out K. Therefore,

$$C - 700 = \frac{200}{1.1236} \Rightarrow C = 878.00.$$

41. Solution: E

The forwards do not have any premium. Due to put-call parity, the net premium of the remaining strategies will increase with an increasing strike price. With a 1% interest rate, the net premium for E will be positive.

42. Solution: D

Purchasing the stock results in paying K today and receiving S in t years, so the profit at expiration from this transaction is $S - Ke^{rt}$.

Selling the call results in receiving the premium C today and paying max(0, S – K) in t years. Because $S > K$, the profit from this transaction at expiration is $Ce^{rt} - S + K$.

The overall profit is the sum, $Ce^{rt} + K - Ke^{rt} = Ce^{rt} + K(1 - e^{rt})$.

43. Solution: C

A written collar consists of a short put option and a long call option. The initial cost of the position is –44 – 2.47 + 3.86 = –42.61.

The payoff and profit table is:

	$S_T \le 40$	$40 < S_T \le 50$	$S_T > 50$
Short stock	$-S_T$	$-S_T$	$-S_T$
Short Put	$-(40 - S_T)$	0	0
Long Call	0	0	$(S_T - 50)$
Total Payoff	-40	$-S_T$	-50
Total Profit	$-40 + 42.61e^{0.05} = 4.79$	$-S_T + 44.79$	$-50 + 44.79 = -5.21$

As shown in the table, the maximum profit is 4.79.

44. Solution: E

At expiration the price is 50 and both options are “out-of-the-money” eliminating answers (A) and (B). With a strike price of 45 and a minimum stock price of 46, option A is never in the money, eliminating answer (D). With a strike price of 55, option B will be in the money at the time the stock price is 58, eliminating answer (C) and verifying answer (E).

45. Solution: C

The change in the futures contract in the three month period is

$$200[Se^{0.75(0.02-0.04)} - 1100e^{1.0(0.02-0.04)}] = -100$$

$$197.022S - 215,643,708 = -100$$

$$S = 1094.01$$

46. Solution: B

Answer (A) is false because naked writing involves selling, not buying options.

Answers (C) and (D) are false because it is an American option that can be exercised at any time.

Answer (E) is false because being in-the-money means there is a payoff, not necessarily a profit.

47. Solution: B

Writing a covered call requires shorting the call option along with simultaneous ownership in the stock (i.e., the underlying asset).

48. Solution: E

The payoff for the 45-strike call is 12 = max(0, S – 45), so 12 = S – 45 and thus S = 57.

The payoff for the 135-strike put is max(0, 135 – S) = max(0, 135 – 57) = 78.

49. Solution: D

The customer pays 500 to purchase the options. If the option is not in the money, the investor loses the 500. If the option is in the money the investor will have a payoff and thus a loss of less than 500. Hence the maximum possible loss is 500.

50. Solution: B

The investor received 7 (bought the 70 put for 1 and sold the 80 put for 8). To break even, the investor must lose 7 on the payoff. The purchased put cannot have a negative payoff. However, if the index is at 73 upon expiration, the investor will lose 7 on the 80 put (and have no positive payoff on the 70 put).

51. Solution: B

First, find the prepaid forward price as

$$F_{0,1}^{P} = 35 - PV(divs) = 35 - 0.32(e^{-0.04*2/12} + e^{-0.04*8/12}) = 34.37 .$$

Next, the forward price is

$$F_{0,1} = F_{0,1}^{P} e^{rt} = 34.37055 * e^{0.04} = 35.77 .$$

52. Solution: C

Consider the first three answers, which are identical except for the forward price. The short sale proceeds of 100 can be lent at 3%. At time one the investor has 103. If the forward price is less than 103, the investor can buy a share for less than 103 and use that shore to close out the short position, leaving an arbitrage profit. Hence (A) and (B) represent arbitrage opportunities while (C) does.

It is not necessary to evaluate (D) and (E). However, as a check, the stock is purchased for 100.5 and with interest at time one the investor will possess one share of stock and owe 103.565. If the short forward contract requires selling the share for more than 103.565 there will be an arbitrage opportunity. Both (D) and (E) have the forwards priced higher and therefore provide arbitrage opportunities.

53. Solution: D

If F is the forward price, K is the strike price, c is the call option price, p is the put option price, v is the annual discounting factor due to risk-free interest, and t is the number of years, then we have the put-call parity formula

$$c - p = v^{t}\left(F - K\right).$$

Using the data for the rice commodity,

$$110 - p = \left(e^{-0.065}\right)^{4}(300 - 400) \Rightarrow p = 110 + 100e^{-0.26} = 187.11. \; .$$

54. DELETED

55. Solution: A

This type of box spread is a long position in a synthetic forward (long call and short put) and a short position in a synthetic forward at a higher strike price (short call and long put). The payoff is the guaranteed positive difference between the strike prices. With $L > K$, the box spread is equivalent to $c(K) - p(K) - c(L) + p(L)$.

A bull spread using calls is $c(K) - c(L)$ and a bear spread using puts is $p(L) - p(K)$. To reproduce the box spread both spreads must be purchased (long position).

56. Solution: E

Cash flows like those of a short stock position are created by shorting both a forward and a zero-coupon bond.

57. DELETED

58. DELETED

59. Solution: E

The payoff table for the long stock, the long put and the short call is, where S is the stock price at expiration:

	$S <= 40$	$40 < S < 50$	$S >= 50$
Stock	S	S	S
Put	$40 – S$	0	0
Call	0	0	$– (S – 50)$
Total	40	S	50

Only Graph E is consistent with this table.

60. Solution: E

Buying a put at 8.60 and selling a call at 8.80 limits the sale price to the 8.60 -- 8.80 range. Brown also receives a premium from selling the call.

61. Solution: A

A put option is in-the-money if the current stock price is less than the strike price, at-the-money if these two prices are equal, and out-of-the-money if the current stock price is greater than the strike price.

Note that if the current stock price is less than the strike price 70, then the current stock price must be less than the strike price 80. Since option A is a 70-strike put and option B is an 80-strike put, we conclude that if option A is in-the-money, then option B must be in-the-money.

62. Solution: E

The future value of the put premium is 7(1.03) = 7.21. If the asset value falls the profit is max(0, 130 – 60) – 7.21 = 62.79. If the asset value rises, the profit is max(0, 130 – 125) – 7.21 = –2.21. The expected profit is 0.5(62.79) + 0.5(–2.21) = 30.29.

63. DELETED

64. DELETED

65. Solution: A

66. Solution: D

The current stock price, 80, is higher than the strike price, 65.

Since a call option provides the right (but not the obligation) to buy a share of the stock for only 65, a call option would have positive payoff if exercised immediately. So the option is in-the-money if it is a call option.

Since a put option provides the right (but not the obligation) to sell a share of the stock for only 65, a put option would have negative payoff if exercised immediately. So the option is out-of-the-money if it is a put option.

Therefore, the option is in-the-money if it is call option, but out-of-the-money if it is a put option.

67. Solution: C

When the stock price is 27, Payoff = -2 (30-27) + (35-27) =2.

When the stock price is 37, Payoff =0 (none of the puts would be exercised).

68. Solution: B

Long gains 1210 - 1150 = 60 (a short position will lose, not gain).

69. Solution: C

The reverse is true. Futures contracts are more useful for minimizing credit risk. This is due to the daily settlement of futures contracts.

70. Solution: C

Since the investors demand to be compensated for risk, on the average (or expected value) the stock should outperform the forward. So the expected value of the future stock price is greater than the forward price.

From the given data, the 2-year forward price of the stock is $S_0 e^{(r-\delta)T} = 100e^{(0.05-0.02)(2)} = 106.18$.
Since 1) X, the expected value of the stock price 2 years from now, exceeds the 2-year forward price, 2) X is assumed to be a whole number, and 3) there are no other restrictions on X, we conclude that the smallest possible value of X is 107.

71. Solution: D

$40 - 6/1.05 - 6/(1.05)^2 - 6/(1.05)^3 - 6/(1.05)^4 = 18.72$.

72. Solution; E

At time 0, CornGrower pays P to buy the put and receives C to sell the call, so the cash flow is C – P which is the premium 0.10 that CornGrower pays to set up the synthetic forward. Therefore, C – P = –0.10. By put-call parity, C – P = 3.59 – exp(–0.04)K. So, 3.59 – exp(–0.04)K = –0.10. Solving for K, we have K = 3.69exp(0.04) = 3.84059..

73. Solution: C

The term *arbitrage* refers to an opportunity for an investor to gain a riskless profit.

74. Solution: A

Buying calls allows a firm to insure against loss of profit as the price of their input increases

75. Solution: E

I is true. Entering into a short forward position means that the oil producer agrees to sell its goods for a predetermined price at a delivery time in the future, which protects the producer from drops in the goods' price.

II is true. Buying a put option allows the producer to sells its goods for a minimum price, the strike price, which protects the producer from drops in the goods' price below the strike price.

III is false. Buying a call option protects the buyer of oil, not the seller.

Thus, I and II only will hedge the producer's financial risk from the goods it sells.