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| Math 373: Sheet #1 |

1. Let $\{τ\_{α}:αϵΔ\}$ be any collection of topologies on a set $X$. Prove $\bigcap\_{α}^{}τ\_{α}$ is a topology on $X$.
2. Let $X=N$. Let $τ$ be the collection of subsets of $X$ consisting of $∅$ and all subsets of $X$ with the form $U\_{n}=\left\{n,n+1,n+2,…\right\}$, $n\in N$.
3. Prove that $τ$ is a topology on $X$.
4. List the closed subsets of $X$.
5. Let $A=\left\{5,7,50\right\}$, find $\overline{A}$, $Int(A)$, and $Bd(A)$. **JUSTEFY YOUR ANSWER**
6. Let $(X,τ)$ be topological space such that there exists a countable bases $B$ for $τ$. Prove there exists a countable dense subset of $X$. Show that $R$ with Co-finite topology does not have a countable base.
7. Prove the following” Let $f:(X,ζ)\rightarrow (Y,η)$ be open and onto function, and let $B$ be a base for $ζ$. Then $\left\{f\left(B\right):B\in B\right\}$ is a base for $η$.
8. If $A$ and $B$ are subsets of the space $X$.
9. Prove that $\left(A∪B\right)^{'}=A'∪B'$.
10. Give an example to show $\left(A∩B\right)^{'}\ne A^{'}∩B^{'}.$
11. Let $A$ and $B$ be subsets of the space $X$.
12. Prove that $Bd(A∪B)⊆Bd(A)∪Bd(B)$.
13. Give an example to show the reverse inclusion in part (i) need not hold.
14. Let $A$ and $B$ be subsets of the space $X$, and suppose $Bd\left(A\right)∩Bd\left(B\right)=∅$. Prove $Int\left(A∪B\right)=Int(A)∪Int(B)$.
15. In usual topology, do $Q$An open set? Closed set? Neither? Both?
16. Prove that if $A$ is a dense subset of a space $X$, and $B$ is a non-empty open set of $X$, then $A∩B\ne ∅$.
17. Show that every infinite subset of infinite co-finite space $X$ is dense in $X$.