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| **Question** | **1** | **2** | **3** | **4** | **Total** |
| **Mark** |  |  |  |  |  |
| **Out of** | **5** | **7** | **16** | **12** | **40** |

**Question 1**

a) (2 marks) Let and let be defined by for . Prove that and that .

b) (2 marks) Using Reimann sum evaluate

c) (1 mark) Let be bounded on , Prove that

**Question 2:**

a)(2 marks) Let be defined by

i) Prove that

ii) Is this converge uniform? Prove your answer.

b) ( 3 point) Give examples of

* 1. Functions , with But (You can define the domain of the functions as you like).
  2. but .
  3. but . (Example 9.8 p 316)

b) (2 mark) Let and and Then .

**Question 3:**

1. (4 marks) Explain why (Give proof):

a) The set [0,1] is not countable. Corollary 10.6.2

b) is not a measure.

c) Every finite ring is a -ring. Remark 10.3

d) If is Lebesegue integrable on , then so is .

1. (1 mark) Let be a ring. Show that Remark 10.1
2. (2 points) Let be a nonempty class of subsets of Prove that:

is an algebra is closed under complementation and finite union . Exercise 10.1.1.

1. (2 marks) a) Give an example of a -ring which is not a -algebra.

b) Give an example of a function which is Lebesgue integrable but not Riemann integrable.

1. (2 mark) Recall that if is represented by a finite unoun of disjoint subsets of intervals , then the length if is defined by Show that is well defined.
2. ( 2 Marks) Prove that the outer measure is monotone. That is if then Theorem 10.6.ii.
3. (2 Marks) Let defined by

a) Is an outer measure?

b) Is a measure?

1. (1 marks) Let be a decreasing sequence of subset, then find and Example 10.5

**Question 4:**

1. (2 Marks) Let and Prove that :
   1. .
   2. If for all , then
2. (2 Marks) Given , Using the definition of Reman integrals and Lebesque integrals find: and .
3. ( 2 Marks) Let and (a.e) on , then and
4. ( 2 Mark) Let then
5. ( 2 Mark) Let and (a.e) on , and , then (a.e) on
6. (2 Marks) Let . Prove that the function defined by is continuous. Example 11.5.