King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 470 Exam I
Semester I, 2009- (밍)
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| Serial NO: |  |
| :---: | :--- |
| ID: |  |
| Name: |  |


| Q |  | Points |
| :---: | :---: | :---: |
| 1 |  | 4 |
| 2 |  | $\mathbf{1 0}$ |
| 3 |  | 23 |
| 4 |  | $\mathbf{1 8}$ |
| 5 |  | $\mathbf{1 3}$ |
| 6 |  | $\mathbf{1 0 0}$ |
| 7 |  |  |
| Total |  |  |

(ْ) Say Bismillah \& Good luck $\because$
(1) Classify the partial differential equations:
(a) $u_{x x}-8 u_{x y}+2 u_{y y}+x u_{x}-y u_{y}=0$
(b) $3 u_{x x}+2 u_{x y}-u_{y y}+y u_{x}-u_{y}=0$
(c) $3 u_{x x}-8 u_{x y}+2 u_{y y}+(x+y) u_{y}=0$
(d) $2 u_{x x}-2 u_{x y}-3 u_{y y}+y^{2} u_{x}-u=0$
(2) Classify the equation as linear, quasi-linear and non-linear ; then find the order of the PDE

|  |  | linear | quasi <br> linear | non linear | order |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $(x-y) u_{x}^{2}+2 u_{y}=4 y$ | $(7(\mathrm{~d}) / \mathrm{pg} 4)$ |  |  |  |  |
| $u_{x}+u_{y}^{2}-u_{x x}=4$ |  |  |  |  |  |
| $\Delta^{2} u+u_{x} \Delta u_{y}-u_{y} \Delta u_{x}=0$, |  |  |  |  |  |
| where $\Delta=\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}$ |  |  |  |  |  |
| $\Delta u+u u_{x}+u u_{y}+P_{x}=f$, |  |  |  |  |  |
| $\Delta u+v_{x} u_{x}+v_{y} u_{y}=g$, |  |  |  |  |  |
| $v$ is a given function |  |  |  |  |  |

(3) Determine the canonical form of the elliptic equation

$$
2 u_{x x}-4 u_{x y}+2 u_{y y}-y^{2} u_{x}+u_{y}-x u=0 .
$$

(4) Determine the canonical form of the hyperbolic equation

$$
3 u_{x x}+6 u_{x y}+2 u_{y y}=0 .
$$

(5) Use the method of characteristics to find a solution of the

$$
x u_{x}+u_{y}=e^{u}
$$

that passes through the curve $\Gamma$ given by $y=x-1, u=0$.
(6) Let $a, b$, and $c$ be constants and suppose that

$$
u_{x x}-u_{y y}+a u_{x}+b u_{y}+c u=0
$$

Let $v(x, y)=e^{\alpha x+\beta y} u(x, y)$. Determine constants $\alpha, \beta$ and $h$ so that

$$
v_{x x}-v_{y y}=-h v .
$$

(7) Discuss the following statement: (is it true or false then justify your answer)
The problem:

$$
\begin{aligned}
& y^{2} u_{x}+x^{2} u y=y^{2} \\
& u(x, y)=-2 y \text { on } y^{3}=x^{3}-2
\end{aligned}
$$

has a unique solution.

