Solution of Final Examination

King Saud University: Mathematics Department Math-254
First Semester 1431-32 H Solution of Final Examination
Maximum Marks = 50 Time: 180 mins.

Name of the Student:—————————— **I.D. No.** ——————————

Name of the Teacher:—————————— **Section No.** ——————————

<u>The Answer Table for Q.1 to Q.15 :</u> Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	a	b	d	b	c	a	c	d	d	a	b	c	c	b	a

Quest. No.	Marks
Q. 1 to Q. 15	
Q. 16	
Q. 17	
Q. 18	
Q. 19	
Total	

Question 1: The error bound for the 5^{th} approximation to the solution of the nonlinear equation $f(x) = 0$ in $[1.5, 2]$ using bisection method is:

(a) $\frac{1}{64}$ (b) $\frac{1}{32}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

Question 2: If the root of the nonlinear equation $f(x) = 0$ in $[0.5, 2]$ is a fixed point of the equation $g(x) = \sqrt{2-x}$, then $f(x) = 0$ is:

(a) $x^2 + x - 2 = 0$ (b) $\frac{x}{\sqrt{2-x}} - x = 0$ (c) $\frac{\sqrt{2-x}}{x} - x = 0$ (d) $x^2 - x + 2 = 0$

Question 3: The rate of convergence of Newton's method to the root $\alpha = 0$ of the equation $\cos x - 1 - 0.5x^2 = 0$ is:

(a) order 1 (b) order 2 (c) order 3 (d) order 4

Note: The following information will be used in Questions 4 to 6:

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

Question 4: The solution of the linear system $A\mathbf{x} = \mathbf{b}$ using LU-decomposition ($l_{ii} = 1$) is:

(a) $[1.1, -0.7]^T$ (b) $[0.1, -0.7]^T$ (c) $[1.1, 0.7]^T$ (d) $[-1.1, -0.7]^T$

Question 5: The relative error with respect to the approximate solution $\hat{\mathbf{x}} = [0.4, -0.6]$ for l_∞-norm is bounded by:

(a) 2.6 (b) 2.7 (c) 2.8 (d) 2.9

Question 6: Using Jacobi iteration method with the initial approximation $[0, 0]^T$, the error bound $\|\mathbf{x} - \mathbf{x}^{(4)}\|$ is:

(a) $\frac{3}{32}$ (b) $\frac{3}{22}$ (c) $\frac{3}{26}$ (d) $\frac{3}{16}$

Question 7: If the best approximation of $f(1.5)$ using Newton's quadratic interpolating polynomial is 7 and $f[1, 2, 3, 4] = 8$, then the Newton's cubic polynomial $p_3(1.5)$ gives:

(a) 10.0 (b) 9.0 (c) 12.0 (d) 11.0

Question 8: Let $f(x) = \ln(x + 2)$ be given at the points $-1, 0, 4$, then the upper bound in approximating $\ln 3$ using a quadratic interpolating polynomial is:

(a) 2.0 (b) 1.0 (c) 3.0 (d) 4.0

Question 9: If a function $f(x)$ satisfies the conditions $f[-1,1]=1, f'(1)=5, f'(-1)=-1$, then $f[1,-1,1]$ equals:

(a) 2.0 **(b)** 3.0 **(c)** 4.0 **(d)** 5.0

Question 10: If $S(x)=\begin{cases} cx-2, & \text{if } 0\le x\le 1 \\ (4-c)x, & \text{if } 1\le x\le 2 \end{cases}$ is a linear spline of a function $f(x)$, then the value of c is:

(a) 3.0 **(b)** 2.0 **(c)** 4.0 **(d)** 1.0

Note: The following information will be used in Questions 11 to 13:

x	0.0	0.1	0.2	0.3	0.4	0.45	0.5
$f(x)$	−2.0	0.0	3.0	5.0	8.0	10.0	14.0

Question 11: The best approximate value of $f'(0.3)$ using 3-point difference formula is:

(a) 25.0 **(b)** 20.0 **(c)** 30.0 **(d)** 35.0

Question 12: The best approximate value of $f''(0.4)$ is:

(a) 300 **(b)** 250 **(c)** 350 **(d)** 400

Question 13: The best approximate value of $\int_0^{0.5} f(x)\,dx$ is:

(a) 2.2 **(b)** 1.8 **(c)** 2.0 **(d)** 1.6

Question 14: The error bound in approximating $\int_0^1 \frac{15}{x+1}\,dx$ using the composite Trapezoidal rule with $n=5$ is:

(a) 0.1 **(b)** 0.1×10^{-1} **(c)** 0.1×10^{-2} **(d)** 0.1×10^{-3}

Question 15: For IVP $y'+3y=4,\ y(0)=5$, the approximate value of $y(0.1)$ using Taylor's method of order two when $n=1$ is:

(a) 4.065 **(b)** 4.650 **(c)** 4.560 **(d)** 4.506

Question 16: Show that $\alpha = 1$ is the root of the nonlinear equation [5 points]

$$x^4 - x^3 - 3x^2 + 5x = 2.$$

Use quadratic convergent method to find its first approximation $x^{(1)}$ if $x^{(0)} = 0.5$.

Solution. Since

$$f(x) = x^4 - x^3 - 3x^2 + 5x - 2$$

and

$$f(1) = 1^4 - 1^3 - 3(1)^2 + 5(1) - 2 = 1 - 1 - 3 + 5 - 2 = 0$$

therefore, $\alpha = 1$ is the root of the given nonlinear equation. Now to check that the root is simple or multiple, we take the first derivative of the given function as follows:

$$f'(x) = 4x^3 - 3x^2 - 6x + 5$$

and its value at $\alpha = 1$ is

$$f'(1) = 4(1)^3 - 3(1)^2 - 6(1) + 5 = 4 - 3 - 6 + 5 = 0$$

which means that $\alpha = 1$ is the multiple root of the given nonlinear equation. To find its order of multiplicity, we take the higher derivatives as follows:

$$\begin{array}{rclrcl} f''(x) & = & 12x^2 - 6x - 6, & f''(1) & = & 0 \\ f'''(x) & = & 24x - 6, & f'''(1) & = & 24(1) - 6 = 18 \neq 0, \end{array}$$

so, the order of multiplicity of the given root is 3.
For the approximation of a multiple root of the nonlinear equation the quadratic convergent method is the modified Newton's method which is

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}, \qquad n = 0, 1, 2, \ldots$$

For the first approximation, we use

$$x_1 = x_0 - m\frac{f(x_0)}{f'(x_0)}, \qquad n = 0$$

which becomes

$$x_1 = x_0 - m\frac{[x_0^4 - x_0^3 - 3x_0^2 + 5x_0 - 2]}{[4x_0^3 - 3x_0^2 - 6x_0 + 5]}$$

Using $m = 3$ and $x_0 = 0.5$ in the above formula, we get

$$x_1 = 0.5 - 3\frac{[(0.5)^4 - (0.5)^3 - 3(0.5)^2 + 5(0.5) - 2]}{[4(0.5)^3 - 3(0.5)^2 - 6(0.5) + 5]}$$

and it gives

$$x_1 = 0.5 - 3\frac{[(0.5)^4 - (0.5)^3 - 3(0.5)^2 + 5(0.5) - 2]}{[4(0.5)^3 - 3(0.5)^2 - 6(0.5) + 5}$$

So

$$x_1 = 0.5 - 3\frac{(-0.3125)}{(1.7500)} = 0.5 + 0.5357 = 1.0357 \approx \alpha = 1.$$

Question 17: Solve the following system of linear equations using the Gaussian elimination with **partial pivoting** [5 points]

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & = & 1 \\ 2x_1 & + & 3x_2 & + & 4x_3 & = & 3 \\ 4x_1 & + & 9x_2 & + & 16x_3 & = & 11 \end{array}$$

Solution. For the first elimination step, since 4 is the largest absolute coefficient of first variable x_1, therefore, the first row and the third row are interchange, giving us

$$\begin{array}{rcrcrcr} 4x_1 & + & 9x_2 & + & 16x_3 & = & 11 \\ 2x_1 & + & 3x_2 & + & 4x_3 & = & 3 \\ x_1 & + & x_2 & + & x_3 & = & 1 \end{array}$$

Eliminate first variable x_1 from the second and third rows by subtracting the multiples $m_{21} = \frac{2}{4}$ and $m_{31} = \frac{1}{4}$ of row 1 from row 2 and row 3 respectively, gives

$$\begin{array}{rcrcrcr} 4x_1 & + & 9x_2 & + & 16x_3 & = & 11 \\ & - & 3/2x_2 & - & 4x_3 & = & -5/2 \\ & - & 5/4x_2 & - & x_3 & = & -7/5 \end{array}$$

For the second elimination step, $-3/2$ is the largest absolute coefficient of second variable x_2, so eliminate second variable x_2 from the third row by subtracting the multiple $m_{32} = \frac{5}{6}$ of row 2 from row 3, gives

$$\begin{array}{rcrcrcr} 4x_1 & + & 9x_2 & + & 16x_3 & = & 11 \\ & - & 3/2x_2 & - & 4x_3 & = & -5/2 \\ & & & & 1/3x_3 & = & 1/3 \end{array}$$

Obviously, the original set of equations has been transformed to an equivalent upper-triangular form. Now using backward substitution, gives

$$x_1 = 1, \quad x_2 = -1, \quad x_3 = 1,$$

which is the required solution of the given linear system. •

Question 18: Let $x_0 \in (a,b)$, where$f \in C^2[a,b]$ and that $x_1 = x_0 + h \in (a,b)$ for some $h \neq 0$, then show that [5 points]

$$f'(x_0) \approx \frac{f(x_0+h) - f(x_0)}{h}.$$

Use the above derived formula to find the approximate value of the derivative $f'(2.5)$ of the function $f(x) = (x+1)\ln(x+1)$, with $h = 0.05$.

Solution. Consider the linear Lagrange interpolating polynomial $p_1(x)$ which interpolate $f(x)$ at the given points is

$$f(x) \approx p_1(x) = \left(\frac{x - x_1}{x_0 - x_1}\right) f(x_0) + \left(\frac{x - x_0}{x_1 - x_0}\right) f(x_1) \tag{1}$$

By taking derivative of (1) with respect to x and at $x = x_0$, we obtain

$$f'(x)|_{x=x_0} \approx p_1'(x)|_{x=x_0} = -\frac{f(x_0)}{x_0 - x_1} + \frac{f(x_1)}{x_1 - x_0}$$

Simplifying the above expression, we have (taking $h = x_1 - x_0$)

$$f'(x_0) \approx -\frac{f(x_0)}{h} + \frac{f(x_0+h)}{h}$$

which can be written as

$$f'(x_0) \approx \frac{f(x_0+h) - f(x_0)}{h}. \tag{2}$$

It is called the **two-point formula**.

Using the above formula, with $x_0 = 2.5$, we have

$$f'(2.5) \approx \frac{f(2.5+h) - f(2.5)}{h}$$

Then for $h = 0.05$, we get

$$\begin{aligned} f'(2.5) &\approx \frac{f(2.55) - f(2.5)}{0.05} \\ &\approx \frac{(2.55+1)\ln(2.55+1) - (2.5+1)\ln(2.5+1)}{0.05} = 2.2599. \end{aligned}$$

Question 19: How many subintervals approximate the integral $\int_0^2 \frac{1}{x+4}\,dx$, to an accuracy 10^{-5} using the Simpson's rule ? Also, compute the approximation. [5 points]

Solution. To find the subintervals for the given accuracy, we use the following Simpson's formula

$$|E_{S_n}(f)| \leq \frac{(b-a)^5}{180n^4} M \leq 10^{-5}$$

where

$$|f^{(4)}(\eta(x))| \leq M = \max_{0 \leq x \leq 2} |f^{(4)}(x)|$$

and $\eta(x)$ is unknown point in $(0,2)$. Since the fourth derivative of the function is

$$f^{(4)}(x) = \frac{24}{(x+4)^5}$$

and therefore, we have $M = 0.0234$, at $x = 0$. Thus

$$\frac{(2-0)^5}{180n^4}(0.0234) \leq 10^{-5}$$

or

$$n^4 \geq \frac{2^5 \times 10^5 \times (0.0234)}{180}$$

It gives

$$\begin{aligned} n^4 &\geq 416 \\ n^2 &\geq 20.40 \\ n &\geq 4.52 \end{aligned}$$

Hence to get the required accuracy, we need $n = 6$ subintervals (because n should be even) that ensures the stipulated accuracy.
The composite Simpson's rule for $n = 6$, can be written as

$$S_6(f) = \frac{h}{3}\Big[f(x_0) + 4[f(x_1) + f(x_3) + f(x_5)] + 2[f(x_2) + f(x_4)] + f(x_6)\Big]$$

and since

$$x_0 = 0, x_1 = 1/3, x_2 = 2/3, x_3 = 3/3 = 1, x_4 = 4/3, x_5 = 5/3, x_6 = 6/3 = 2$$

because

$$h = \frac{2-0}{6} = \frac{1}{3}$$

Thus

$$S_6(f) = \frac{1/3}{3}\Big[f(0) + 4[f(1/3) + f(1) + f(5/3)] + 2[f(2/3) + f(4/3)] + f(2)\Big]$$

Since given, $f(x) = \frac{1}{x+4}$, therefore

$$S_6(f) = \frac{1}{9}\Big[(1/4) + 4[(3/13) + (3/15) + (3/17)] + 2[(3/14) + (3/16)] + (1/6)\Big]$$

or

$$S_6(f) = \frac{1}{9}\Big[3.6492\Big] = 0.4055$$

Hence

$$\int_0^2 \frac{1}{x+4}\,dx \approx S_6(f) = 0.4055$$

the required approximation. •

Solution of MCQ

Question 1. To compute the error bound for the nth approximation we use the formula

$$|\alpha - c_n| \leq \frac{b-a}{2^n}, \qquad n \geq 1$$

so to find the error bound for the 5^{th} approximation we have

$$|\alpha - c_5| \leq \frac{2-1.5}{2^5} = \frac{0.5}{2^5} = \frac{1}{64}.$$

Question 2. Since

$$f(x) = x - g(x) = 0$$

and given

$$x = g(x) = \sqrt{2-x} \quad \text{or} \quad x^2 = 2 - x$$

Thus

$$f(x) = x^2 + 2x - 2 = 0.$$

Question 3. Given

$$f(x) = \cos x - 1 - 0.5x^2$$

and at $\alpha = 0$, we have

$$f(0) = \cos(0) - 1 - 0.5(0)^2 = 0$$

which means that $\alpha = 0$ is the root of the given equation. Also,

$$f'(x) = -\sin x - 1.0x$$

and $f'(0) = 0$, shows that $\alpha = 0$ is the multiple root of the given equation. For multiple root the rate of convergence of Newton's method is **linear**.

Question 4. Using LU-decomposition ($l_{ii} = 1$), the factorization of the matrix is

$$A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.25 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 0 & 2.5 \end{bmatrix}.$$

Solving the lower system

$$L\mathbf{y} = \mathbf{b}$$

we get

$$y_1 = 3.0 \qquad \text{and} \qquad y_2 = -1.75$$

and the upper system

$$U\mathbf{x} = \mathbf{y}$$

gives

$$x_1 = 1.1 \qquad \text{and} \qquad x_2 = -0.7.$$

Question 5. Since we know the formula for the relative error with respect to the approximate solution is

$$\frac{\|\mathbf{x} - \mathbf{x}^*\|}{\|\mathbf{x}\|} \leq K(A)\frac{\|\mathbf{r}\|}{\|\mathbf{b}\|}, \quad \text{provided} \quad \mathbf{x} \neq 0, \ \mathbf{b} \neq 0$$

First we compute the condition number of the matrix as follows:

$$K(A) = \|A\|_\infty \||A^{-1}\|_\infty = (6)(0.5) = 3$$

The residual vector can be calculated as

$$\begin{aligned} \mathbf{r} &= \mathbf{b} - A\mathbf{x}^* \\ &= \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 0.4 \\ -0.6 \end{pmatrix} \end{aligned}$$

After simplifying, we get

$$\mathbf{r} = \begin{pmatrix} 2.6 \\ 0.4 \end{pmatrix}$$

and it gives

$$\|\mathbf{r}\|_\infty = 2.6$$

Thus

$$\frac{\|\mathbf{x} - \mathbf{x}^*\|}{\|\mathbf{x}\|} \leq (3)\frac{(2.6)}{3} = 2.6.$$

Question 6. The error bound formula using Jacobi method is

$$\|\mathbf{x} - \mathbf{x}^{(\mathbf{k})}\| \leq \frac{\|T_J\|^k}{1 - \|T_J\|} \|\mathbf{x}^{(1)} - \mathbf{x}^{(\mathbf{0})}\|$$

Since the first approximation by Jacobi method is $\mathbf{x}^{(1)} = [3/4, -1/3]^T$ and the Jacobi iteration matrix is

$$T_J = -D^{-1}(L + U) = \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{3} & 0 \end{pmatrix}$$

Then the l_∞ norm of the matrix T_J is

$$\|T_J\|_\infty = \max\left\{-\frac{1}{2}, -\frac{1}{3}\right\} = \frac{1}{2}$$

Now using $k = 4$, we have

$$\|\mathbf{x} - \mathbf{x}^{(\mathbf{4})}\| \leq \frac{(1/2)^4}{1 - 1/2} \left\| \begin{pmatrix} 3/4 \\ -1/3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\|$$

or

$$\|\mathbf{x} - \mathbf{x}^{(\mathbf{4})}\| \leq \frac{1}{8}(3/4) = \frac{3}{32}.$$

Question 7. The Newton cubic interpolatory polynomial $p_3(x)$ that fits at all four points $x_0 = 1, x_1 = 2, x_2 = 3, x_4 = 4$, is

$$p_3(x) = p_2(x) + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2)$$

Given $p_2(1.5) = 7$ and $f[1, 2, 3, 4] = 8$, we have

$$p_3(1.5) = p_2(1.5) + f[1, 2, 3, 4](1.5 - 1)(1.5 - 2)(1.5 - 3)$$

Thus

$$f(1.5) \approx p_3(1.5) = 7 + 8(0.5)(-0.5)(-1.5) = 7 + 3 = 10.$$

Question 8. To compute the error bound for the approximation of $\ln 3$ using the quadratic interpolating polynomial $p_2(x)$, we have

$$|f(x) - p_2(x)| = \frac{|f'''(\eta(x))|}{3!}|(x - x_0)(x - x_1)(x - x_2)|$$

Since the third derivative of the given function is

$$f'''(x) = \frac{2}{(x+2)^3}$$

and

$$|f'''(\eta(x))| = \left|\frac{2}{(\eta(x)+2)^3}\right|, \qquad \text{for} \quad \eta(x) \in (-1, 4)$$

Then

$$M = \max_{-1 \le x \le 4} \left|\frac{2}{(x+2)^3}\right| = 2.0$$

and

$$|f(1) - p_2(1)| \le \frac{(2)(6)}{6} = 2.0.$$

Question 9. Since we know the second divided difference of the function is

$$f[x_1, x_0, x_1] = f[x_0, x_1, x_1] = \frac{f[x_1, x_1] - f[x_0, x_1]}{x_1 - x_0}$$

Then

$$f[x_0, x_1, x_1] = \frac{f'(x_1) - f[x_0, x_1]}{x_1 - x_0}$$

Using the given information, we have

$$f[-1, 1, 1] = \frac{f'(1) - f[-1, 1]}{1 + 1}$$

or

$$f[-1, 1, 1] = \frac{5 - 1}{2} = 2.0.$$

Question 10. For linear spline, we know that

$$s(1) = f(1)$$

which gives

$$c(1) - 2 = (4 - c)(1) \qquad \text{gives} \qquad c = 3.0.$$

Or, the function must be continuous at $x = 1$, which means that the limit of the function must exists at $x = 1$. This implies that left hand limit and the right hand limit at $x = 1$ are equal, that is:

$$\begin{aligned}
\lim_{x \to 1^-} f(x) &= \lim_{x \to 1^+} f(x) \\
\lim_{x \to 1^-} cx - 2 &= \lim_{x \to 1^+} (4 - c)x \\
c - 2 &= 4 - c \\
2c &= 6 \\
c &= 3.0.
\end{aligned}$$

Question 11. The best approximate value of $f'(x_1)$ using 3-point difference formula will be **central difference formula** which is

$$f'(x_1) \approx \frac{f(x_1+h) - f(x_1-h)}{2h}$$

Now using $x_1 = 0.3$ and $h = 0.1$ in the above formula, we have

$$f'(0.3) \approx \frac{f(0.3+0.1) - f(0.3-0.1)}{2(0.1)} = \frac{f(0.4) - f(0.2)}{0.2}$$

which gives

$$f'(0.3) \approx \frac{f(0.4) - f(0.2)}{0.2} = \frac{8.0 - 3.0}{0.2} = 25.0.$$

Question 12. Since the **three-point central-difference formula** for the approximation of the second derivative of a function $f(x)$ at the given point $x = x_1$ is

$$f''(x_1) \approx \frac{f(x_1-h) - 2f(x_1) + f(x_1+h)}{h^2}$$

Now using $x_1 = 0.4$ and $h = 0.1$, we get

$$f''(0.1) \approx \frac{f(0.4-0.1) - 2f(0.4) + f(0.4+0.1)}{(0.1)^2}$$

which gives

$$f''(0.1) \approx \frac{f(0.3) - 2f(0.4) + f(0.5)}{(0.01)} = \frac{5.0 - 2(8.0) + 14.0}{(0.01)}$$

or

$$f''(0.1) \approx \frac{3.0}{(0.01)} = 300.$$

Question 13. The given points at equally spaced are

$$x_0 = 0.0, x_1 = 0.1, x_2 = 0.2, x_3 = 0.3, x_4 = 0.4, x_5 = 0.5$$

therefore, the best approximate value of the given integral can be obtained only by Composite Trapezoidal's rule because $n = 5$. Composite Trapezoidal's rule for six points or $n = 5$ is

$$T_5(f) = \frac{h}{2}\Big[f(x_0) + 2[f(x_1) + f(x_2) + f(x_3) + f(x_4)] + f(x_5)\Big]$$

Taking $h = 0.1$, we get

$$T_5(f) = \frac{0.1}{2}\Big[f(0.0) + 2[f(0.1) + f(0.2) + f(0.3) + f(0.4)] + f(0.5)\Big]$$

or

$$T_5(f) = \frac{0.1}{2}\Big[-2.0 + 2[0.0 + 3.0 + 5.0 + 8.0] + 14.0\Big]$$

Thus

$$\int_0^{0.5} f(x)dx \approx T_5(f) = \frac{0.1}{2}\Big[-2.0 + 32 + 14.0\Big] = \frac{0.1}{2}(44) = 2.2.$$

Question 14. The global error in the composite Trapezoidal rule is

$$E_{T_n}(f) = -\frac{h^2}{12}(b-a)f''(\eta(x)), \quad \eta(x) \in (a,b)$$

The second derivative of the function can be obtain as

$$f'(x) = -\frac{15}{(x+1)^2} \quad \text{and} \quad f''(x) = \frac{30}{(x+1)^3}$$

Since $\eta(x)$ is unknown point in $(0,1)$, therefore, the bound $|f''|$ on $[0,1]$ is

$$M = \max_{0 \leq x \leq 1} |f''(x)| = \left|\frac{30}{(x+1)^3}\right| = 30.0$$

Thus the error bound in approximating the given integral using the composite Trapezoidal rule is

$$|E_{T_5}(f)| \leq \frac{(1/5)^2}{12}(30) = 0.1.$$

Question 15. The Taylor's method of order two is

$$y_{i+1} = y_i + hf(x_i, y_i) + \frac{h^2}{2} f'(x_i, y_i), \quad \text{for} \quad i = 0, 1, 2, 3, 4$$

Since $f(x,y) = 4 - 3y$, $f'(x,y) = -12 + 9y$, and $x_0 = 0$, $y_0 = 5$, then for $i = 0$, we have

$$\begin{aligned} y(x_1) \approx y_1 &= y_0 + h(4 - 3y_0) + \frac{h^2}{2}(-12 + 9y_0) \\ y(0.1) \approx y_1 &= 5 + (0.1)(4 - 15) + (0.005)(-12 + 45) = 4.065. \end{aligned}$$

Solutions of Three-Types of MCQ

The Answer Table for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box. **(FInal)- First Type**

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	a	b	d	b	c	a	c	d	d	a	b	c	c	b	a

Ps. : Mark {a, b, c or d} for the correct answer in the box. **(FINal)- Second Type**

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	c	d	b	a	b	d	a	c	b	c	d	a	a	d	c

Ps. : Mark {a, b, c or d} for the correct answer in the box. **(FINAl)- Third Type**

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a,b,c,d	b	a	c	d	a	c	b	a	a	b	c	d	b	a	d