M - 107

KING SAUD UNIVERSITY **FULL MARKS: 40**

DEPARTMENT OF MATHEMATICS **TIME: 90min**

(SEMESTER I, 1431-1432) FIRST MID-TERM

Question: 1 (a) Solve the system of linear equations by Gauss - Jordan method

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$ [8]

(b) Find condition on a, b, and c for which the following system is consistent,

$$\begin{aligned} x + y + 2z &= a \\ x \quad + z &= b \\ 2x + y + 3z &= c \end{aligned}$$ [6]

Question: 2 (a) Let

$$\begin{aligned} x + y + 2z &= 4 \\ x + 2y + 3z &= 5 \\ 2x + y - z &= 1 \end{aligned}$$ [8]

i. **Write the above system of linear equations in the form** $AX=B$ **,**
ii. **find** A^{-1} **using elementary row operations, and**
iii. **use** A^{-1} **to solve the above system of equations.**

(b) Given

$$A = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$$ **and det(A) = 5,** [3]

evaluate the determinant $\det\left[(3A^{-1})\right]^T$

Question: 3 (a) Find the angle between $\overrightarrow{PQ}$ **and** $\overrightarrow{RS}$ **where P(2, 3, -1), Q(2, 1, 3), R(1, 2, 1) and S(2, 1, 1).** [5]

(b) Find the values of α **such that** u **and** v **orthogonal to each other, where**

$$u = 5i - 4j + 2\alpha k, \quad v = 4i - 3j - 4\alpha k$$ [5]

(c) Evaluate the following determinant by reducing it to row echelon form

$$\begin{vmatrix} 1 & 3 & 5 \\ 4 & 14 & 12 \\ -2 & -3 & -20 \end{vmatrix}$$ [5]