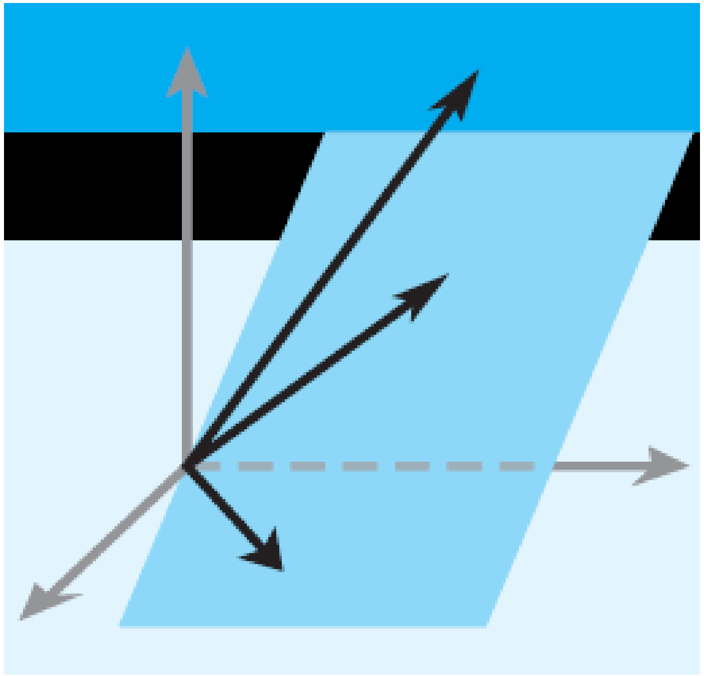
Faculty of Engineering
Mechanical Engineering Department

Linear Algebra and Vector Analysis
MATH 1120
Lecture 12

Elementary Linear Algebra



Chapter 3

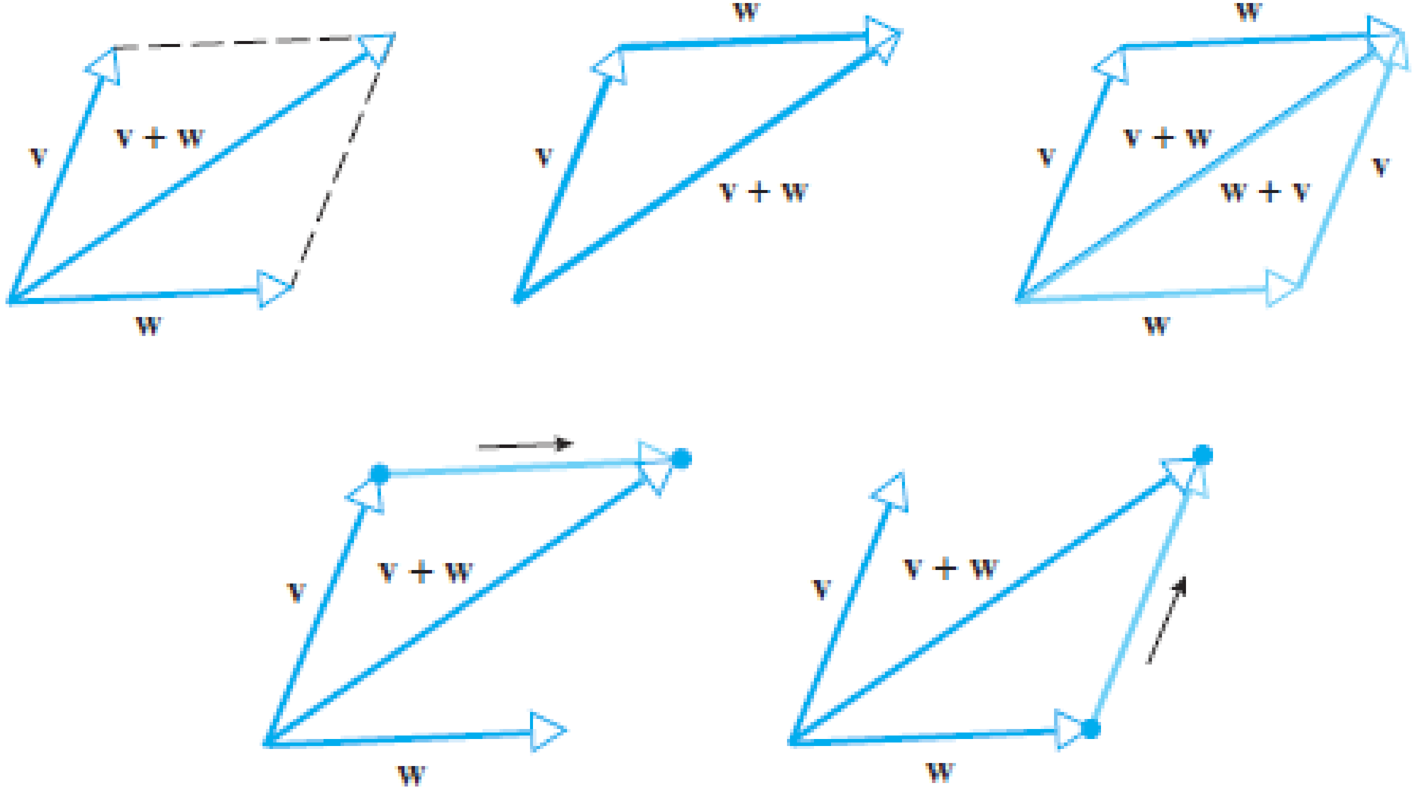
Howard Anton

Chapter 3
Euclidean Vector Spaces

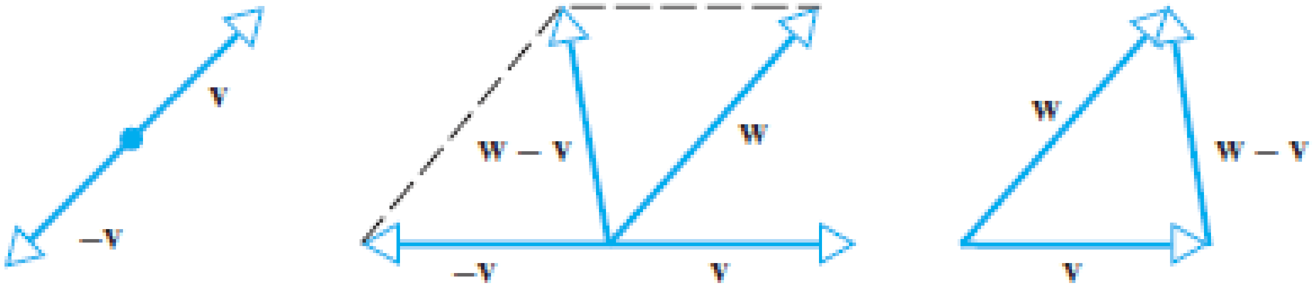
- 3.1 Vectors in 2-Space, 3-Space, and n-Space
- 3.2 Norm, Dot Product, and Distance in R^n
- 3.3 Orthogonality
- 3.4 The Geometry of Linear Systems
- 3.5 Cross Product

Section 3.1 Vectors

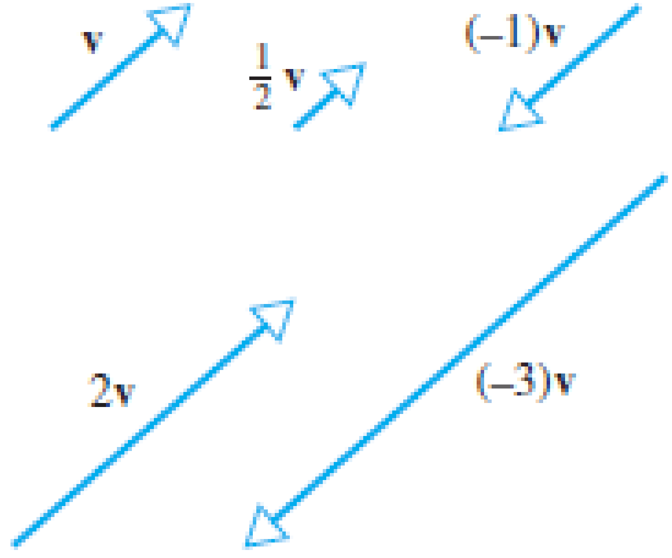
Addition of vectors by the parallelogram or triangle rules



Subtraction:



Scalar Multiplication:



Properties of Vectors

THEOREM 3.1.1 *If* $\mathbf{u}$, $\mathbf{v}$, *and* $\mathbf{w}$ *are vectors in* R^n, *and if* k *and* m *are scalars, then:*

(*a*) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(*b*) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(*c*) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

(*d*) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(*e*) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$

(*f*) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$

(*g*) $k(m\mathbf{u}) = (km)\mathbf{u}$

(*h*) $1\mathbf{u} = \mathbf{u}$

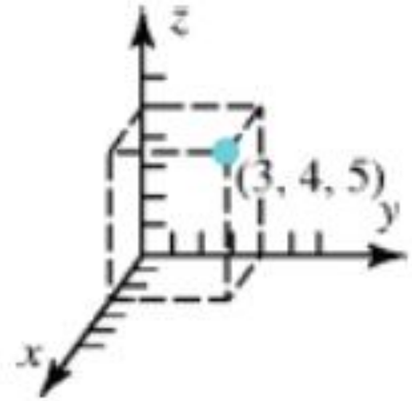
Example 1: Draw a coordinate system of the following:

1. (a) $(3, 4, 5)$

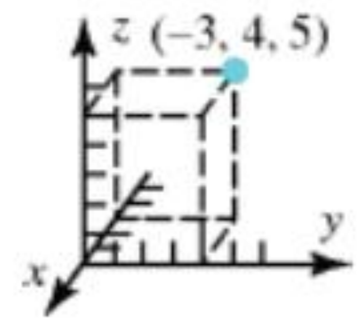
(b) $(-3, 4, 5)$

(c) $(3, -4, 5)$

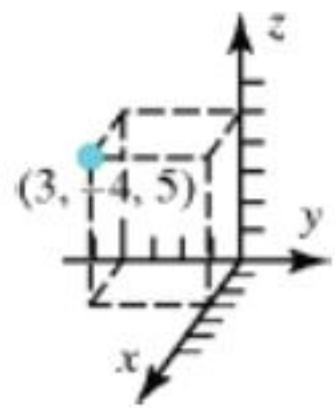
(a)



(b)



(c)



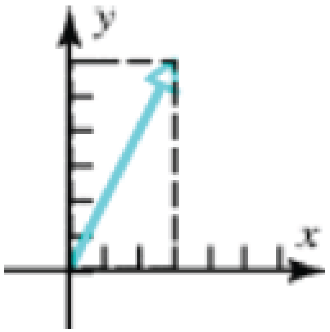
sketch the following vectors with the initial points located at the origin.

(a) $\mathbf{v}_1 = (3, 6)$

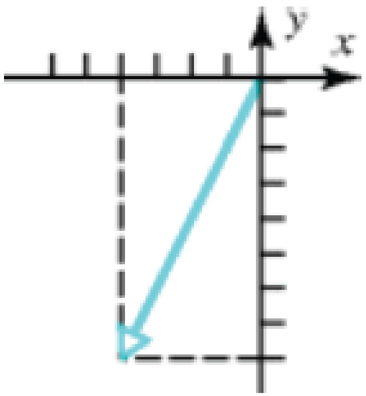
(b) $\mathbf{v}_2 = (-4, -8)$

(c) $\mathbf{v}_3 = (-4, -3)$

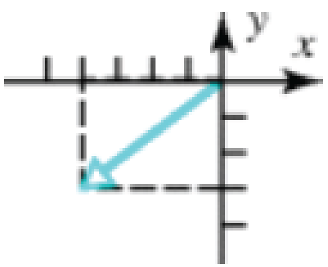
(a)



(b)



(c)



find the components of the vector $\overrightarrow{P_1P_2}$.

(a) $P_1(3, 5), \quad P_2(2, 8)$

(b) $P_1(5, -2, 1), \quad P_2(2, 4, 2)$

Answer:

(a) $\overrightarrow{P_1P_2} = (-1, 3)$

(b) $\overrightarrow{P_1P_2} = (-3, 6, 1)$

Examples

Example 1 Give the vector for each of the following.

(a) The vector from $(2, -7, 0)$ to $(1, -3, -5)$.

(b) The vector from $(1, -3, -5)$ to $(2, -7, 0)$.

(c) The position vector for $(-90, 4)$

Solution

(a) Remember that to construct this vector we subtract coordinates of the starting point from the ending point.

$$\left\langle 1-2, -3-(-7), -5-0 \right\rangle = \left\langle -1, 4, -5 \right\rangle$$

(b) Same thing here.

$$\left\langle 2-1, -7-(-3), 0-(-5) \right\rangle = \left\langle 1, -4, 5 \right\rangle$$

(c) Not much to this one other than acknowledging that the position vector of a point is nothing more than a vector with the point's coordinates as its components.

$$\left\langle -90, 4 \right\rangle$$

Example:

If $\mathbf{v} = (1, -3, 2)$ and $\mathbf{w} = (4, 2, 1)$, then

$$\mathbf{v} + \mathbf{w} = (5, -1, 3), \qquad 2\mathbf{v} = (2, -6, 4)$$

$$-\mathbf{w} = (-4, -2, -1), \qquad \mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w}) = (-3, -5, 1)$$

Section 3.2 Norm, Dot Product, and Distance in R^n

Norm:

DEFINITION 1 If $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is a vector in R^n, then the *norm* of $\mathbf{v}$ (also called the *length* of $\mathbf{v}$ or the *magnitude* of $\mathbf{v}$) is denoted by $\|\mathbf{v}\|$, and is defined by the formula

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2 + \cdots + v_n^2} \tag{3}$$

Unit Vectors:

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

Example 2 Determine the magnitude of each of the following vectors.

(a) $\vec{a} = \langle 3, -5, 10 \rangle$

(b) $\vec{u} = \left\langle \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}} \right\rangle$

(c) $\vec{w} = \langle 0, 0 \rangle$

(d) $\vec{i} = \langle 1, 0, 0 \rangle$

Solution

There isn't too much to these other than plug into the formula.

(a) $\left\|\vec{a}\right\| = \sqrt{9 + 25 + 100} = \sqrt{134}$

(b) $\left\|\vec{u}\right\| = \sqrt{\frac{1}{5} + \frac{4}{5}} = \sqrt{1} = 1$

(c) $\left\|\vec{w}\right\| = \sqrt{0 + 0} = 0$

(d) $\left\|\vec{i}\right\| = \sqrt{1 + 0 + 0} = 1$