Solution of Final Examination	Math 106	Semester II	1438-1439

Exercise 1 : (3+3+2)

a) $\frac{1}{4}\int_0^4 \frac{x}{\sqrt{x^2+9}}dx = \frac{1}{4}\left[\sqrt{x^2+9}\right]_0^4 = \frac{1}{2}.$ $1\frac{1}{2}$

$\frac{c}{\sqrt{c^2+9}} = \frac{1}{2} \iff c = \sqrt{3}.$ $1\frac{1}{2}$ $(-\sqrt{3} \notin [0,4])$

b) Use the trapezoidal rule with $n = 4$ to approximate the integral $\int_0^4 \frac{dx}{\sqrt{1+x^3}}$.

Solution

k	x_k	$f(x_k)$	m	$mf(x_k)$
0	0	1	1	1
1	1	0.707	2	1.414
2	2	0.333	2	0.666
3	3	0.189	2	0.378
4	4	0.124	1	0.124
				3.582

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$\int_0^4 \frac{dx}{\sqrt{1+x^3}} \approx \frac{1}{2}(3.582) = 1.791.$ 1

c) $\int \frac{x^2}{\sqrt{1+x^6}}dx \overset{t=x^3}{=} \frac{1}{3}\int \frac{dt}{\sqrt{1+t^2}}dx = \frac{1}{3}\sinh^{-1}(x^3) + c..$ $1+1$

Exercise 2 : (3+3+3)

a) $\int \frac{dx}{x\sqrt{x-9}} \overset{t=\sqrt{x-9}}{=} \int \frac{2dt}{9+t^2}dt = \frac{2}{3}\tan^{-1}(\frac{1}{3}\sqrt{x-9}) + c.$ $2+1$

Or $\int \frac{dx}{x\sqrt{x-9}} \overset{t=\sqrt{x}}{=} = \frac{2}{3}\sec^{-1}(\frac{\sqrt{x}}{3}) + c.$

b) $\lim_{x\to+\infty}(e^x+1)^{e^{-x}} \overset{t=e^x}{=} \lim_{t\to+\infty} e^{\frac{\ln(1+t)}{t}} = 1.$ $1+1+1$

c) By parts: $\int \cosh^{-1}(x)dx = x\cosh^{-1}(x) - \int \frac{x}{\sqrt{x^2-1}}dx = x\cosh^{-1}(x) - \sqrt{x^2-1} + c.$ $2+1$

Exercise 3 : (3+3+3)

a) $\int \sin^5(x)\cos^4(x)dx \overset{t=\cos(x)}{=} -\int t^4(1-t^2)^2dt = -\frac{1}{9}\cos^9(x) - \frac{1}{5}\cos^5(x) + \frac{2}{7}\cos^7(x) + c.$ $2+1$

b) $\int \frac{x^3}{\sqrt{x^2+1}}dx \overset{t^2=x^2+1}{=} \int (t^2-1)dt = \frac{1}{3}t^3 - t + c = \frac{1}{3}(x^2+1)^{\frac{3}{2}} - \sqrt{x^2+1} + c.$ $2+1$

We can do the change of variable $t = \tan\theta$.

c)

$$\begin{aligned}\int \frac{3x+2}{x^2+2x+10}dx &= \int \frac{3(x+1)}{(x+1)^2+9}dx - \int \frac{dx}{(x+1)^2+9} \\ &= \frac{3}{2}\ln(x^2+2x+10) - \frac{1}{3}\tan^{-1}(\frac{x+1}{3}) + c.\end{aligned}$$

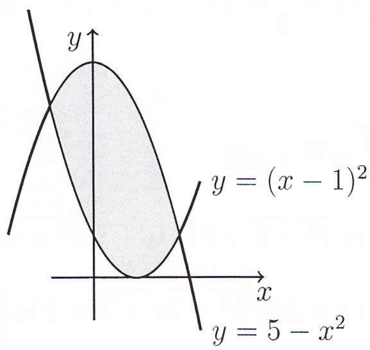
$1\frac{1}{2}$

$1\frac{1}{2}$

Exercise 4 : (2+3+3)

a) $\int \frac{dx}{\sqrt{x^2-8x}} = \int \frac{dx}{\sqrt{(x-4)^2-16}} = \cosh^{-1}(\frac{x-4}{4}) + c.$ 1+1

b) $\int_e^c \frac{dx}{x(\ln x)^{\frac{3}{2}}} \overset{t=\ln x}{=} \int_1^{\ln c} \frac{dt}{(t)^{\frac{3}{2}}} = \left[-\frac{1}{2\sqrt{t}}\right]_1^{\ln c} \underset{c\to+\infty}{\longrightarrow} \frac{1}{2}$. The integral converges and its value is $\frac{1}{2}$. 2+1



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c)

The area is equal to $\int_{-1}^{2}(5-x^2-(x-1)^2)dx = 9.$ 1+1

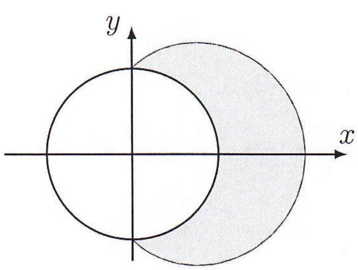
Exercise 5 : (3+3)

a) The volume is $\pi \int_0^2 25-(1+y^2)^2 dy = \pi(\frac{544}{15})$. 2+1
Or $V = 2\pi \int_0^4 (x+1)\sqrt{x}dx$.

b) The area of the region R is equal to

$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1+\cos\theta)^2 - 1d\theta = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(2\cos\theta+\cos^2\theta)d\theta = 2+\frac{\pi}{4}.$$

1+1



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