MID TERM EXAMINATION, SEMESTER I, 1442
DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH: 107 FULL MARK: 30 TIME: 2 HOURS

[N. B.: Marks: Q1. [4]; Q2. [5+3]; Q3. [4+4]; Q4. [2+2]; Q5. [3+3]]

Q1. Solve the system of linear equations by Gauss-Jordan elimination:

$$\begin{aligned} 2x_1 + 2x_2 - 2x_3 &= 4 \\ 3x_1 + 5x_2 + x_3 &= -8 \\ -4x_1 - 7x_2 - 2x_3 &= 13 \end{aligned}$$

Q2. Consider the following system of linear equations

$$\begin{aligned} x + y + 2z &= 0 \\ -x + 3y - z &= 1 \\ -x + y &= 2 \end{aligned}$$

(a) Find the inverse of the matrix A of the coefficients of the above system by elementary matrix method.

(b) Use A^{-1} to solve the above system.

Q3. (a) Let

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 1 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

Find values of λ such that the matrix $A - \lambda I$ is *not* invertible, where I is the 3×3 identity matrix.

(b) Use Cramer's Rule to solve the linear system:

$$\begin{aligned} -x + 2y - 3z &= 1 \\ 2x \quad + z &= 0 \\ 3x - 4y + 4z &= 2 \end{aligned}$$

Q4. (a) Find the value of m so that the vector $\langle 2, 1, m\rangle$ is orthogonal to the sum of the vectors $\langle 1, -1, 2\rangle$ and $\langle 3, 2, 1\rangle$.

(b) The magnitude and direction of a constant force are given by $\vec{a} = 4\vec{i} + 7\vec{j} + 4\vec{k}$. Find the work done if the point of application of the force moves along the line of action from $P(1, 1, 1)$ to $Q(3, 5, 4)$.

Q5. (a) Find parametric equations of the line l through the point $P(1, 3, 0)$ and perpendicular to the vectors $\vec{u} = -\vec{i} - \vec{k}$ and $\vec{v} = 2\vec{i} + \vec{j} + 4\vec{k}$.

(b) Find the equation of the plane through the points $P(1, 0, -2)$ and $Q(0, -2, 0)$ and containing the vector $\vec{a} = 3\vec{i} - \vec{j} + 2\vec{k}$.