MATH 499 : Research Project

Introduction to Networks and Applications

Thamer bin Sharid bin Hajji Hanushi al - Mutairi

433103988

Under Supervision of Dr. Mohamad Alghamdi

King Saud University in 1437 - 1438 H

2016 - 2017

College of Science

Department of Mathematics

Contents

1 Introduction

Networks can be found in social and economic phenomena. The use of methods from graph theory has allowed network theory in many fields to improve our understanding of the outcomes. There has been substantial progress in network research reported in both theoretical and empirical literature. A network is formed by a set of vertices (nodes) and a set of edges (links) connecting these vertices [11, 6]. In the real world networks of business, there are massive examples, including diffusion of the knowledge among firms, job-contacts, sellers and buyers, and R&D cooperation among firms or between firms and institutions.

In a network, there are players represented by nodes and relationships represented by links. For example, in R&D cooperation network, the players (firms) are represented by nodes and the R&D partnerships (cooperation) are represented by links [4, 3, 15, 9, 1]. For the case of R&D cooperation network, we use the theoretical R&D model by Goyal and Moraga-Gonzalez. They described research and development (R&D) cooperation between firms as a network game. The total investment of each firm equals the own effort and efforts of other firms that are determined by a free R&D spillover. If firms are linked, the spillover between them is set one; otherwise it is set free less than one.

This research is organized as follows. In the first section, we introduce the network concepts by presenting sets of definitions. In the second section, we present samples of networks applications. In the third section, we provide examples of networks in economics that is R&D cooperation networks. In the fourth section, we conclude the research.

2 Definitions & Terminologies

Definition 1 *A network is an ordered pair $G(N, E)$ where N is a set of vertices (or nodes) and E is a set of edges (links) connecting these vertices.*

Let N be a set of all vertices labeled by letters $i, j, k, ...$ or numbers $1, 2, 3, etc.$ and let E be a set of all edges in the network. Then $G(N, E)$ denotes a network with nodes $N = \{i, j, k, ...\}$ and links $E = \{ij, jk, ...\}$ [11, 6]. We usually denote to the number of nodes by $|N| = n$ and to the number of links by $|E| = m$. For simplicity the network $G(N, E)$ is denoted by G.

Definition 2 *A network G is called undirected if each link between any two vertices runs in both directions.*

This means each two links ij and ji in the network G are the same.

Definition 3 *A simple network is a graph that has no parallel edges (edges that have the same end vertices) neither loops (edges where their start and end vertices are the same).*

Example 1 *Figure 1 illustrates the previous definitions.*

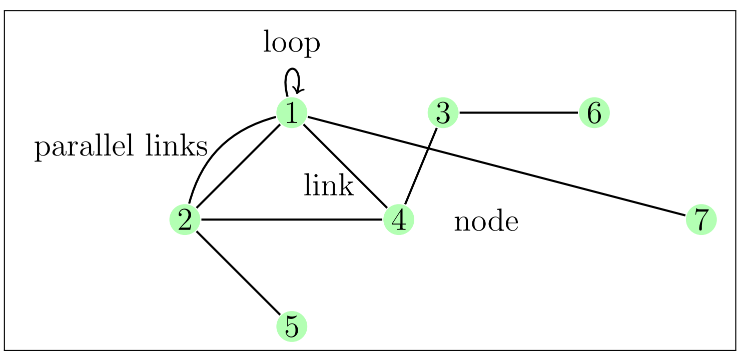

Figure 1: An example network with seven vertices and nine edges.

Any simple network G can be represented by an $n \times n$ adjacency matrix A with elements 0 or 1, depending on whether or not nodes are linked. More formally, each element a_{ij} of the adjacency matrix A can be written as

$$a_{ij} = \begin{cases} 1 & : \ ij \in E; \\ 0 & : otherwise. \end{cases}$$

For an undirected network the adjacency matrix, A is symmetric.

Definition 4 *Let G be an undirected network, a set of neighbors of node $i \in N$ is a set of all nodes that link to node i i.e.,*

$$N_i = \{j \in N : ij \in E\}$$

The length of the neighbors' set of node i is a degree of that node ($deg(i)$). Thus, the degree of each node $i \in N$ is denoted by $deg(i) = |N_i|$ where $0 \leq deg(i) \leq n - 1$. The degree of node i can also be calculated from the adjacency matrix A where $deg(i) = \sum_i^n a_{ij} = \sum_i^n a_{ji}$ since A is a symmetric matrix.

Example 2 *Let $N = \{1, 2, 3, 4, 5, 6\}$. The network given in Figure 2 can be represented by adjacency matrix A:*

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

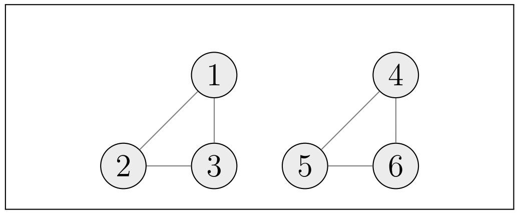

Figure 2: A network consists of six nodes.

From this example, the neighbor set of node 1 is $N_1 = \{2, 3\}$ *and the neighbor set of node 4 is* $N_4 = \{5, 6\}$.

There are different types of the network. In the following definition, we focus on some of them.

Definition 5

1. **Complete network**

   A complete network with n *nodes* (K_n) *is a graph such that each two different nodes in that network are linked.*

2. **Star network**

   A star network (S_n) *is characterized by a node in the center of the network (hub) linked to all* $n-1$ *other nodes (peripheral nodes) while none of the peripheral nodes has a link with any other.*

3. **Empty network**

   An empty network (E_n) *is a network containing* n *nodes without links between them.*

4. **Cycle network**

A simple cycle consists of a sequence of nodes starting and ending at the same node, with each two consecutive nodes in the sequence adjacent to each other in the graph. A cycle network (C_n) is a graph of n nodes containing a single cycle through all nodes.

5. **k-regular network**

   A k-regular network is a graph such that each node has the same degree, k.

6. *Bipartite network*

   A bipartite network is a graph whose vertices is divided into two disjoint sets where nodes that do not belong to the same set can be linked.

Remark 1

1. *The complete network is also defined as a $(n-1)$-regular network because each node has $n-1$ links.*
2. *The cycle network is a 2-regular network since each node has two links.*

Example 3 *Figure 3 shows the previous types of networks with different numbers of nodes.*

3 Samples of Networks Applications

There are massive applications of the networks in both the theoretical and empirical literature. This indicates that importance of the network concept in many scientific fields. In the following, we briefly consider some applications of the networks. For simplicity, we mention the applications after classifying them into four groups.

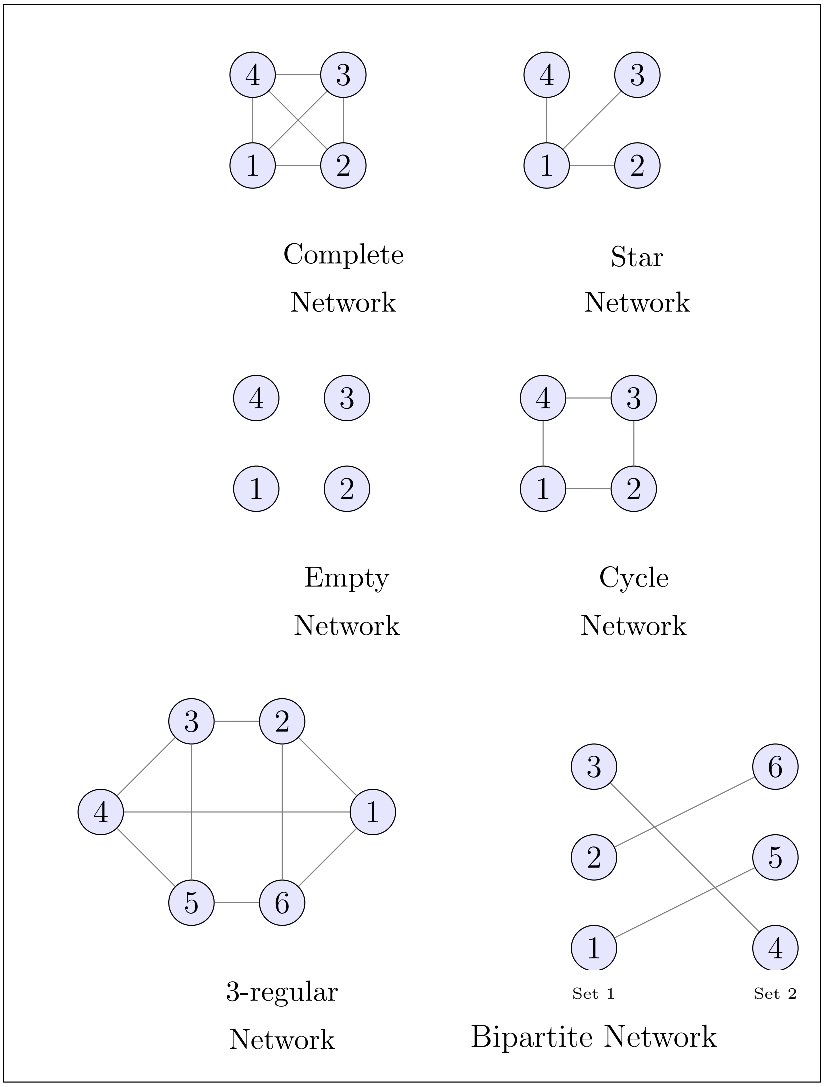

Figure 3: An example for different types networks.

1. **Social Networks**

   A social network is a set of people or groups of people with some pattern of contacts or interactions between them.

   The networks have been used for representing:

   (a) The intermarriages between individuals.

   (b) The friendships between individuals.

   (c) The business relationships between companies.

2. **Information networks**.

   (a) The citations between academic papers.

   The classic example of an information network is the network of citations between academic papers. Most learned articles cite previous work by others on related topics. These citations form a network in which the vertices are articles and edges are the citation.

   (b) The World Wide Web.

   The Web pages contain information and by linking them together by links from one page to another forms a network.

   (c) Sharing files between computers.

   (d) The relationship between classes.

3. **Technological networks**.

   They are man-made networks designed typically for distribution of some commodity or resource.

   (a) The electric power grid.

   (b) The telephone network (the network of who calls whom).

   (c) The delivery network.

   (d) The Internet (the network of physical connections between computers).

4. **Biological networks**.

   (a) The network of metabolic pathways.

   It is a representation of metabolic substrates and products with edges joining them if a known metabolic reaction exists that acts on a given substrate and produces a given product.

   (b) The genetic regulatory network.

   (c) The food web.

   In such networks, the vertices represent species in an ecosystem and an edge from species A to species B indicates that A preys on B.

4 Networks in Economics

4.1 Applications of Networks in Economics

In economics, the organizational networks have been applied for:

1. The relations between companies.

2. The diffusion of the knowledge among firms.

3. The job-contacts.

4. The relationship between sellers and buyers.

5. The research and development (R&D) cooperation among firms or between firms and institutions.

4.2 R&D Cooperation under Network Concept

The R&D is an essential source of innovation, which is in turn an important factor in the growth of output in the economy as a whole. For firms participating in R&D, this field is a fundamental element to continue the level of production which leads firms to invest in R&D in order to be able to maintain their positions in a market. Hence, firms may spend substantial amounts on R&D. They do this not only to reduce the cost of the production (cost-reducing alliances), but also to enable other objectives, such as enhancing or developing existing products, finding new processes or producing new technologies that could open up new markets [5, 8, 10, 12, 4].

Definition 6 *Research and development (R&D) is defined as activities that are achieved by one agent or a group of agents to create a new discovery or knowledge that may be used for new applications.*

The cooperation among firms in R&D is described as a network (called an R&D cooperation network) where firms are represented by nodes and the R&D partnerships (cooperation) are represented by links [4, 2, 15, 14].

4.2.1 R&D Network Model

In the theory of R&D cooperation, numerous papers have developed and discussed theoretical frameworks describing R&D expenditure and cooperation [13, 7, 3]. In this section, we focus on the model by [4]. They introduced the network concept to the R&D partnerships between firms.

Definition 7 *The R&D network is a graph represents the cooperation of firms in R&D.*

From the definition, in the R&D network, the cooperators are represented by nodes and the R&D agreements are represented by links.

Example 4 *Figure 4 shows an example of the R&D cooperation among firms.*

4.2.2 A Sample of the R&D Network Models

We apply the R&D network model by [4] as follows:

1. Firms have a choice to form a link between them or to delete it.

2. The formation of a link requires the cooperation of both firms where they have.

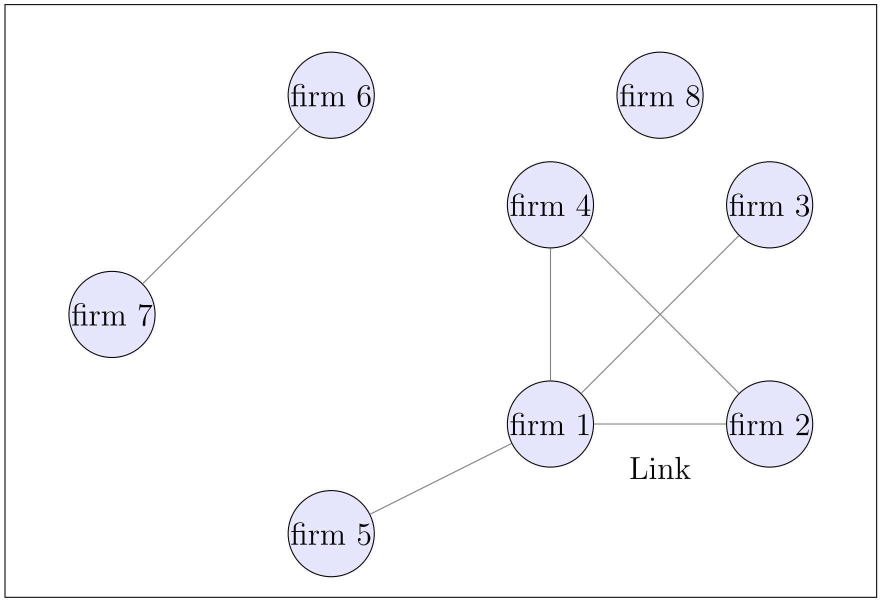

Figure 4: An example of the R&D cooperation.

3. If a link is established between any two cooperating firms, the cost of the link formation is zero.

4. If firms do not cooperate, they are not linked and there is an R&D spillover $\beta \in [0, 1)$ between them. The spillover is set to ensure partial benefits between non-cooperated firms.

The effective R&D investment for each firm i is defined by the following equation:

$$X_i = x_i + \sum_{j \in N_i} x_j + \beta \sum_{k \notin N_i} x_k, \quad i = 1, \ldots, n \ . \tag{1}$$

x_i: R&D investment of firm i where $x_i \geq 0$

N_i: set of firms participating in R&D with firm i

$\beta \in [0, 1)$: an exogenous parameter that captures knowledge spillovers acquired from firms not engaged in R&D with firm i.

The Objective of the R&D Cooperation:

We focus on the R&D cooperation to reduce the marginal cost. Usually, the marginal cost is denoted by $\overline{c}$.

Definition 8 *The marginal cost of a product is the cost to produce one unit of that product.*

If firm i produces a product i, the total R&D investment reduces the marginal cost of the product i. Therefore, the marginal cost of the product i becomes

$$\begin{aligned} c_i &= \overline{c} - X_i \qquad (2) \\ &= \overline{c} - x_i - \sum_{j \in N_i} x_j - \beta \sum_{k \notin N_i} x_k, \quad i = 1, \ldots, n \; . \qquad (3) \end{aligned}$$

4.2.3 An Application of the R&D Network Model

In this section, we apply the R&D network model by [4]. Assume that n firms work in a market. The demand function and the inverse demand function for each good i are given by the following equations:

Inverse demand function:

$$p_i = a - q_i - \lambda \sum_{j=1}^{n} q_j \; . \qquad (4)$$

Demand function:

$$q_i = \frac{(1-\lambda)a - (1+(n-2)\lambda)p_i + \lambda \sum_{j \neq i} p_j}{(1-\lambda)(1+(n-1)\lambda)} \ . \tag{5}$$

Definition 9 *The profit (payoff) of one firm is the difference between revenue and production cost for that firm.*

The definition indicates that the profit of firm i is the price minus the cost of its product.

The profit of firm $i \in N$ is usually denoted by π_i and defined by the following expression:

$$\pi_i = (p_i - c_i)q_i = \left(a - q_i - \lambda \sum_{j \neq i}^{n} q_j - c_i \right) q_i \ , \tag{6}$$

where p_i is the price of good i produced by firm i and c_i is the production cost.

Example 5 *Assume three firms produce homogeneous goods and compete in a market by their quantity of the production. For $n = 3$, the functions of the R&D investment and the profit of firms are given in Appendix. Let $a = 120$ and $\overline{c} = 100$ and $\gamma = 1$, then*

1. *Figure 5 shows the possible R&D networks generated by participating three firms in a market.*

2. *Figure 6 shows the R&D investment and the profit of firms.*

Note:

1. *With n firms, there are $2^{\binom{n}{2}}$ possible networks. The figure shows the distinct networks for $n = 3$.*

2. *Grouping the firms in the network in terms of the outcomes depends on their*

cooperative links. In network G_1, all firms have the same number of links, so their outcomes are equal. Also, firms 2 and 3 in network $G_3 have the same outcomes$.

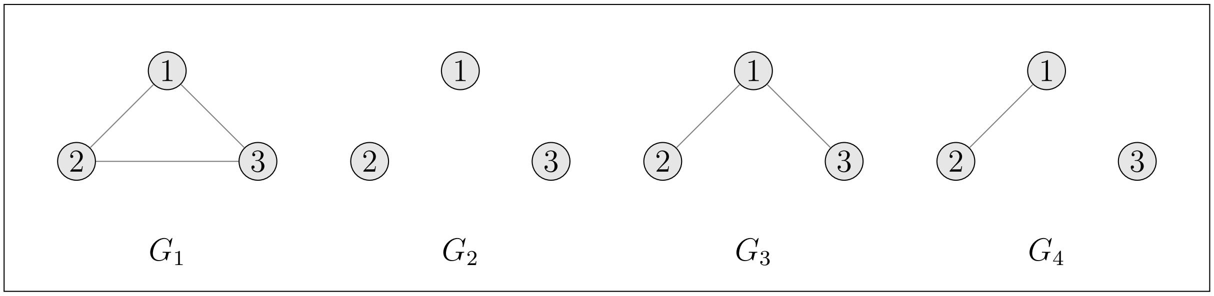

Figure 5: The distinct networks with three firms. In each of G_1 and G_2, there is one group of firms, but the other networks, there are two groups. In the network G_3, there is a hub (firm 1) and peripheries (firms 2 and 3) and in the network G_4, there are linked firms (firms 1 and 2) and an isolated firm (firm 3).

From Figure 6, we note the following:

1. The R&D investment decreases with growing the cooperative links.[1] For example, the investment of firm 1 in the network G_1 is less than the investment of firm 2 or 3 in the network G_3.

2. The profit of firms increases with the cooperative links. We can see this by comparing the profit before and after adding a link. For example, the profit of firm 1 in the network G_2 is less than the profit of firm 1 in the network G_1.

5 Conclusion

In this research, we have given an introduction to the network concept and its applications. Among the applications of the networks, we considered R&D cooperation networks between firms. We noted that the R&D model should be connected the characteristics

[1]Homogeneous products are goods that are produced by different producers where these goods are identical either physically or at least viewed as identical by buyers.

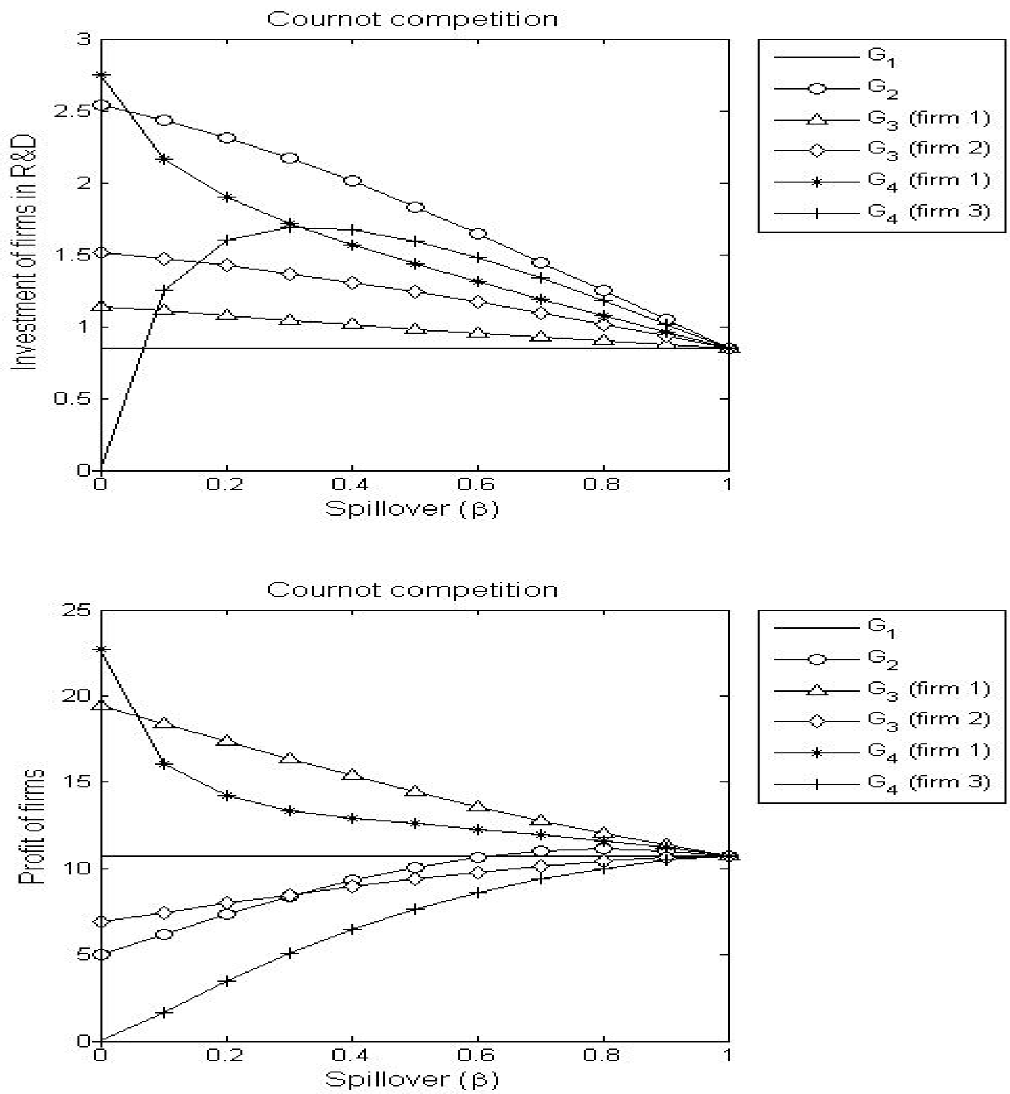

Figure 6: The outcomes of firms in the networks given in 5.

of networks. This allows us to study and compare the changes in the outcomes that accompany with the network structure. As the case of the R&D cooperation, we found that the network contributes in understand the impact of the R&D agreements on the investment and the profit of firms.

References

[1] Alghamdi, M., 2016. Expenditure of firms on R&D in different structural markets. Journal of Business Economics and Finance. 5(2): 191-205

[2] Cantner, U., and Graf, H., 2006. The network of innovators in Jena: An application of social network analysis. Research Policy., 35, 463-480.

[3] Goyal, S. and Joshi, S., 2003. Networks of collaboration in oligopoly. Games and Economic Behavior., 43 (1): 57-85.

[4] Goyal, S., and Moraga-Gonzalez, J. L., 2001. R&D Networks. Rand Journal of Economics., 32: 686-707.

[5] Hagedoorn, J., 1993. Understanding the rationale of strategic technology partnering: inter-organizational modes of cooperation and sectoral differences. Strategic Management Journal., 14, 371-385.

[6] Jackson, M., 2008. Social and Economic Networks. Princeton, NJ: Princeton University Press.

[7] Katz, M., 1986. An Analysis of Cooperative Research and Development. Rand Journal of Economics., 17, 527-543.

[8] Kamien, M., Muller, E., and Zang, I., 1992. Research joint ventures and R&D cartels. American Economic Review., 82, 1293-1306.

[9] Konig, M. D., Battiston, S., Napoletano, M. and Schweitzer, F., 2012. The efficiency and stability of R&D networks. Games and Economic Behavior., 75: 694-713.

[10] Mowery, D. C., Oxley, J. E., and Silverman, B. S., 1998. Technology overlap and interfirm cooperation: implications for the resource-based view of the firm. Research Policy., 27, 507-523.

[11] Newman, M. E. J., 2003. The Structure and Function of Complex Networks. SIAM Review., 45: 167-256.

[12] Powell, W. W., 1998. Learning from collaboration: Knowledge and networks in the biotechnology and pharmaceutical industries. California Management Review., 40 (3), 228-240.

[13] Spence, M., 1984. Cost Reduction, Competition, and Industry Performance. Econometrica., 52, 101-121.

[14] Tomasello, M. V., Napoletano, M., Garasz, A., and Schweitzer, F., 2013. The Rise and Fall of R&D Networks. arXiv:1304.3623v1.

[15] Westbrock, B., 2010. Natural concentration in industrial research collaboration. RAND Journal of Economics., 41 (2): 351-371.

6 Appendix

The R&D investment and the profit of firms in the four networks given in Figure 5 are provided in [1].

$$x_{G_1} = \frac{(a-\bar{c})}{((4\lambda^2+8\lambda+4)\gamma-3)}, \tag{7a}$$

$$q_{G_1} = \frac{(2\gamma(\lambda+1)(a-\bar{c}))}{((4\lambda^2+8\lambda+4)\gamma-3)}\ , \tag{7b}$$

$$x_{G_2} = \frac{(a-\bar{c})(\lambda(2\beta-1)-2)}{2+4\beta-8\gamma+(1-12\gamma-4\beta^2)\lambda+4\gamma\lambda^3}, \tag{8a}$$

$$q_{G_2} = \frac{2\gamma(a-\bar{c})(\lambda^2-\lambda-2)}{2+4\beta-8\gamma+(1-12\gamma-4\beta^2)\lambda+4\gamma\lambda^3}\ , \tag{8b}$$

$$x_{G_3}(firm1) = \frac{(a-\bar{c})(\beta^2\lambda-\beta\lambda-2\beta+2\gamma\lambda^3-6\gamma\lambda^2+8\gamma+2)}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{9a}$$

$$q_{G_3}(firm1) = \frac{(2\gamma(a-\bar{c})(\lambda+1)(\beta^2\lambda-\beta\lambda-2\beta+2\gamma\lambda^3-6\gamma\lambda^2+8\gamma+2))}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{9b}$$

$$x_{G_3}(firm2) = \frac{(2\gamma(\beta\lambda-2)(\lambda+1)(\lambda-2))(a-\bar{c})}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}, \tag{9c}$$

$$q_{G_3}(firm2) = \frac{2\gamma^2(a-\bar{c})(\lambda-\lambda^2+2)^2}{8\gamma^2\lambda^5-8\gamma^2\lambda^4-S_1\lambda^3+S_2\lambda^2+S_3\lambda+2(16\gamma^2-4(\beta+2)\gamma+\beta-1)}\ , \tag{9d}$$

$$x_{G_4}(firm1) = \frac{(\beta\lambda - 2)(a-\overline{c})(2\beta^2\lambda - 3\beta\lambda - 2\beta - 2\gamma\lambda^3 + 6\gamma\lambda^2 + \lambda - 8\gamma + 2)}{2(-4\gamma^2\lambda^6 + 12\gamma^2\lambda^5 + S_4\lambda^4 + S_5\lambda^3 + S_6\lambda^2 + S_7\lambda + 4(8\gamma^2 - 6\gamma - \beta^2 + 1))}, \tag{10a}$$

$$q_{G_4}(firm1) = \frac{(\gamma(a-\overline{c})(\lambda - \lambda^2 + 2)(3\beta\lambda - 2\beta^2\lambda + 2\beta + 2\gamma\lambda^3 - 6\gamma\lambda^2 - \lambda + 8\gamma - 2))}{-4\gamma^2\lambda^6 + 12\gamma^2\lambda^5 + S_4\lambda^4 + S_5\lambda^3 + S_6\lambda^2 + S_7\lambda + 4(8\gamma^2 - 6\gamma - \beta^2 + 1)}, \tag{10b}$$

$$x_{G_4}(firm3) = \frac{(a-\overline{c})(\lambda - 2\beta\lambda + 2)(\beta\lambda - \beta^2\lambda + 2\beta + \gamma\lambda^3 - 3\gamma\lambda^2 + 4\gamma - 2)}{2\big(-4\gamma^2\lambda^6 + 12\gamma^2\lambda^5 + S_4\lambda^4 + S_5\lambda^3 + S_6\lambda^2 + S_7\lambda + 4(8\gamma^2 - 6\gamma - \beta^2 + 1)\big)}, \tag{10c}$$

$$x_{G_4}(firm3) = \frac{(2\gamma(a-\overline{c})(\lambda - \lambda^2 + 2)(\beta\lambda - \beta^2\lambda + 2\beta + \gamma\lambda^3 - 3\gamma\lambda^2 + 4\gamma - 2))}{2\big(-4\gamma^2\lambda^6 + 12\gamma^2\lambda^5 + S_4\lambda^4 + S_5\lambda^3 + S_6\lambda^2 + S_7\lambda + 4(8\gamma^2 - 6\gamma - \beta^2 + 1)\big)}\ , \tag{10d}$$

where $S_1 = 2(20\gamma^2 + (2\beta+1)\gamma)$, $S_2 = 2(4\gamma^2 + (2\beta^2+7)\gamma)$, $S_3 = 64\gamma^2 + 4\beta(\beta-1)(4\gamma-1)$, $S_4 = 12\gamma^2 + (6\beta^2 - 4\beta + 1)\gamma$, $S_5 = -44\gamma^2 - (6\beta^2 + 12\beta - 3)\gamma$, $S_6 = (6 + 24\beta - 12\beta^2)\gamma - 24\gamma^2 - \beta(\beta^2 - 1)(2\beta - 1)$, $S_7 = 2(\beta(2\beta^2 - \beta - 3) + 24\gamma^2 - (10 - 16\beta)\gamma + 1)$.