

**Question 1.[4,4].** a) Find and sketch the largest region of the  $xy$ -plane for which the initial value problem

$$\begin{cases} (1 + y^3) \frac{dy}{dx} = x^2 \\ y(1) = -2 \end{cases}$$

has a unique solution.

b) Solve the initial value problem

$$\begin{cases} x^2 \frac{dy}{dx} = y - xy, \\ y(-1) = -1. \end{cases}$$

**Question 2.[4,4].** a) Find the general solution of the differential equation

$$(x^2 + y^2)dx + (x^2 - xy)dy = 0.$$

b) Solve the differential equation

$$\sqrt{y} \cdot y' + \frac{2}{3} y \sqrt{y} = \frac{2}{3}.$$

**Question 3.[5].** If 200 grams of a radioactive substance is present initially and if its half life is 3000 years, then how much will be left of the substance after 4000 years.

**Question 4.[4].** Find the largest interval for which the following initial value problem has a unique solution

$$\begin{cases} (x - 2)(x + 1)y'' - \frac{x}{x-3}y' + y = 0, \\ y(0) = 0, \quad y'(0) = 2. \end{cases}$$

**Question 5.[5].** Use the undetermined coefficients method to solve the differential equation

$$y'' + y = \sin x.$$