

King Saud University,  
College of Sciences  
Mathematical Department.

Mid-Term1 /S1/2015  
Full Mark:25. Time 1H30mn  
02/01/1437

**Question 1[4,4].** a) Find and sketch the largest region of the  $xy$ -plane for which the initial value problem

$$\begin{cases} (x^2 + y^2) \frac{dy}{dx} = x\sqrt{y-1} \\ y(-2) = 4. \end{cases}$$

has a unique solution.

b) Find the solution of the differential equation:

$$\frac{dy}{dx}(y-1)\sqrt{x^2+1} + x^3 + x = y(x^3 + x); \quad y \neq 1.$$

**Question 2[4,4].** a) Solve the following differential equation

$$(3xy - x + y^2) + (x^2 + xy) \frac{dy}{dx} = 0; \quad x > 0, \quad x + y \neq 0.$$

b) Find the solution of the initial value problem

$$\begin{cases} [x \cos^2(\frac{y}{x}) - y] dx + x dy = 0 \\ y(1) = \frac{\pi}{4}. \end{cases}; \quad x > 0$$

**Question 3[4].** Find the general solution of the differential equation

$$y^3 \frac{dy}{dx} + xy^4 = xe^{-x^2}; \quad x > 0, \quad y \neq 0.$$

**Question 5[5].** Find the family of orthogonal trajectories for the family of curves

$$y = e^{C \sin x}; \quad 0 < x < \frac{\pi}{2},$$

where  $C$  is an arbitrary constant such that  $C \neq 0$ .