## King Saud University, Department of Mathematics
## Math 204 (3H), 40/40, Final Exam17/3/36

**Question 1[4,4] a)** Determine and sketch the largest region of the $xy$-plane for which the following initial value problem has a unique solution

$$\begin{cases} (x-2)(x+3)y' = 4\ln y \\ y(-5) = 2. \end{cases}$$

**b)** Test if the following equation is exact, if it is not, find the appropriate integrating factor and solve it.

$$(3x^2 + y)dx + (2x^2y - x)dy = 0.$$

**Question 2[4,4,5] a)** Solve the differential equation

$$\frac{dy}{dx} = \sqrt{3 + x + y},$$

**b)** Solve the initial value problem

$$\begin{cases} xy\prime - 2(1 + x + \sqrt{y})y = 0, \quad x >, y > 0 \\ y(1) = 1. \end{cases}$$

**c)** A building loses heat in accordance with Newton's law of cooling. Assume the inside temperature is $70^0F$ when the heating system fails. After 2 hours the inside temperature drops to $40^0F$. If the external temperature is $20^0F$, compute the interior temperature after 4 hours.

**Question 3[4,5] a)** Use the variation of parameters method to solve the differential equation

$$y^{''} - 2y' + y = \frac{e^x}{x^2+1}.$$

**b)** Use power series method to solve the nonhomogeneous equation

$$y^{''} + xy' - 2y = x,$$

about the ordinary point $x = 0$.

**Question 4[5,5] a)** Let

$$f(x) = \begin{cases} 1 + x, & -1 \le x \le 0 \\ -1 + x, & 0 < x \le 1 \end{cases}$$

where $f(x+2) = f(x)\ \forall x \in \mathbb{R}$. Sketch the graph of $f(x)$ on $(-1, 1)$ and find its Fourier series.

**b)** Find the Fourier integral of the function$f(x) = \begin{cases} -2, & -1 \le x \le 0 \\ 1, & 0 < x \le 1 \\ 0, & |x| > 1 \end{cases}$

and deduce that

$$\int_0^\infty \frac{(3 - 4\cos\lambda)\sin\lambda}{\lambda} d\lambda = \frac{\pi}{2}.$$