

**King Saud University**  
**Faculty of Sciences**  
**Department of Mathematics**

First Examination	Math 106	Semester I	1439-1440
Time: 1H30			

**Exercise 1 :** (3+3+2)

a) If  $F(x) = \ln(2x) \int_1^{4x^2} (1+t^2)^{10} dt$ . Find  $F'(\frac{1}{2})$ .

b) Use Riemann sums to find the value of  $\int_0^2 3x^2 dx$ .

c) Use Trapezoid rule with  $n = 4$  to approximate the integral  $\int_0^\pi \sin^4(x) dx$ .

**Exercise 2 :** (2+3+3)

a) If  $f(x) = \log_2(\sin^{-1}(x))$ ,  $x > 0$ . Find  $f'(x)$ .

b) Compute the integral  $\int \frac{4^{-\ln(x)}}{x} dx$ .

c) If  $y = x^{2x^2} (x-1)^{\frac{3}{2}}$ ,  $x > 1$ . Find  $y'$ .

**Exercise 3 :** (3+3+3)

a) Evaluate the integral  $\int \frac{2x+3}{\sqrt{4-x^2}} dx$ .

b) Find  $\int \frac{e^{\frac{x}{2}}}{7+e^x} dx$ .

c) Compute the integral  $\int \frac{\sin(x)}{\sqrt{e^{\cos(x)} - 1}} dx$ .

Exercise 1 :

a) If  $f = \ln(2x)$  and  $g(x) = \int_1^{4x^2} (1+t^2)^{10} dt$ . Since  $f(\frac{1}{2}) = 0$  and  $g(\frac{1}{2}) = 0$ , then  $F'(\frac{1}{2}) = 0$ .       $\uparrow$

$$(F'(x) = \frac{1}{x} \int_1^{4x^2} (1+t^2)^{10} dt + 8x \ln(2x)(1+16x^4)^{10}.) \quad \text{2}$$

b)

$$\begin{aligned} \int_0^2 3x^2 dx &= \lim_{n \rightarrow +\infty} 3 \frac{2}{n} \sum_{k=1}^n \frac{4k^2}{n^2} \quad \text{2} \\ &= \lim_{n \rightarrow +\infty} 24 \frac{1}{n^3} \left( \frac{n(n+1)(2n+1)}{6} \right) = 8. \quad \uparrow \end{aligned}$$

c)  $f(x) = \sin^4(x)$

$k$	$x_k$	$f(x_k)$	$m$	$mf(x_k)$
0	0	0	1	0
1	$\frac{\pi}{4}$	$\frac{1}{4}$	2	$\frac{1}{2}$
2	$\frac{\pi}{2}$	1	2	2
3	$\frac{3\pi}{4}$	$\frac{1}{4}$	2	$\frac{1}{2}$
4	$\pi$	0	1	0
				3

1,5

$$\int_0^\pi \sin^4(x) dx \approx \frac{3\pi}{8}. \quad \text{0,5}$$

Exercise 2 :



$$\text{a) } f'(x) = \frac{1}{(\ln 2)(\sin^{-1}(x))\sqrt{1-x^2}}.$$

2

$$\text{b) } \int \frac{4^{-\ln(x)}}{x} dx \stackrel{t=-\ln(x)}{=} - \int 4^t dt = -\frac{4^{-\ln(x)}}{\ln 4} + c.$$

2 + 1

$$\text{c) } y = e^{2x^2 \ln(x)} (x-1)^{\frac{3}{2}}, \ln(y) = 2x^2 \ln(x) + \frac{3}{2} \ln(x-1).$$

1

$$\frac{y'}{y} = 4x \ln(x) + 2x + \frac{3}{2(x-1)} \text{ and } y' = \left( 4x \ln(x) + 2x + \frac{3}{2(x-1)} \right) y.$$

1,5

0,5

Exercise 3 :

a)

$$\begin{aligned} \int \frac{2x+3}{\sqrt{4-x^2}} dx &= \int \frac{2x}{\sqrt{4-x^2}} dx + 3 \int \frac{dx}{\sqrt{4-x^2}} && 1 \\ &= -2\sqrt{4-x^2} + 3 \sin^{-1}\left(\frac{x}{2}\right) + c. && 2 \end{aligned}$$

$$\text{b) } \int \frac{e^{\frac{x}{2}}}{7+e^x} dx \stackrel{t=e^{\frac{x}{2}}}{=} 2 \int \frac{dt}{7+t^2} = \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{e^{\frac{x}{2}}}{\sqrt{7}}\right) + c. \quad 2 + 1$$

c)

$$\begin{aligned} \int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}} dx &\stackrel{t=\cos(x)}{=} - \int \frac{dt}{\sqrt{e^t-1}} && 1,5 \\ &\stackrel{u^2=e^t-1}{=} - \int \frac{2du}{1+u^2} = -2 \tan^{-1}(\sqrt{e^{\cos(x)}-1}). && 1,5 \end{aligned}$$

Or

$$\begin{aligned} \int \frac{\sin(x)}{\sqrt{e^{\cos(x)}-1}} dx &\stackrel{t=e^{\frac{1}{2}\cos(x)}}{=} -2 \int \frac{dt}{t\sqrt{t^2-1}} && 2 \\ &= -2 \sec^{-1}(e^{\frac{1}{2}\cos(x)}) + c. && 1 \end{aligned}$$