

EXERCISE SHEET-2

1) Determine whether the following sets are linearly independent or dependent.

- (i) $f_1(x) = 1 - 2x$, $f_2(x) = x^2$, $f_3(x) = 1 + 2x$, $-\infty < x < \infty$.
- (ii) $f_1(x) = \sec^2 x$, $f_2(x) = \tan^2 x$, $f_3(x) = 2$, $-\infty < x < \infty$.
- (iii) $f_1(x) = \sin x$, $f_2(x) = \cos x$, $f_3(x) = \sin\left(\frac{\pi}{4} + x\right)$, $-\infty < x < \infty$.
- (iv) $f_1(x) = e^x$, $f_2(x) = \sinh x$, $f_3(x) = e^{-x}$, $-\infty < x < \infty$.
- (v) $f_1(x) = \ln x$, $f_2(x) = e^x$, $f_3(x) = x$, $x > 0$.
- (vi) $f_1(x) = \sin x$, $f_2(x) = \cos x$, $f_3(x) = e^x$, $-\infty < x < \infty$.

Ans: (i) LI (ii) LD (iii) LD (iv) LD (v) LI (vi) LI

2) In the following you are given one solution y_1 of the second order differential equation, find the second solution and hence the general solution.

- (i) $y_1 = x^{-\frac{1}{2}} \cos 2x$, $x^2 y'' + xy' + (4x^2 + \frac{1}{4})y = 0$, $x > 0$.
- (ii) $y_1 = \ln x$, $x^2 y'' + xy' = 0$, $x > 0$.
- (iii) $y_1 = e^{\sqrt{x}}$, $xy'' + \frac{1}{2}y' - \frac{1}{4}y = 0$, $x > 0$.
- (iv) $y_1 = x^{-\frac{3}{2}}$, $4x^2 y'' + 4xy' - 9y = 0$, $x > 0$.

Ans: (i) $x^{-\frac{1}{2}} \sin 2x$ (ii) 1 (iii) $e^{-\sqrt{x}}$ (iv) $x^{\frac{3}{2}}$

2) In the following you are given two linearly independent solutions y_1, y_2 of the homogeneous second order differential equation, find differential equation.

- (i) $y_1 = \cos(2 \ln x)$, $y_2 = \sin(2 \ln x)$, $x > 0$
- (ii) $y_1 = x$, $y_2 = x \ln x$, $x > 0$
- (iii) $y_1 = e^{-x} \cos(\ln x)$, $y_2 = e^{-x} \sin(\ln x)$, $x > 0$

Ans: (i) $x^2 y'' + xy' + 4y = 0$ (ii) $x^2 y'' - xy' + y = 0$ (iii) $x^2 y'' + 3xy' + 2y = 0$