Department of Mathematics
King Saud University

M-106

Second Semester(1427/1428)

Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50 Time: Three hours

Marks:

Multiple Choice(1-20)	
Question # 21	
Question # 22	
Question # 24	
Question # 25	
Question # 26	
Total	

Multiple Choice

Q.N0:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
{a,b,c,d}	c	a	c	a	b	c	c	b	a	a	c	b	c	c	c	b	c	c	c	b

Q.No:1 To evaluate the integral $\int \frac{\sin 2x}{\sqrt{1-\cos 2x}}dx$, we use the substitution

(a) $u = \tan\left(\frac{x}{2}\right)$, (b) $u = \sqrt{1-\cos 2x}$, (c) $u = 1-\cos 2x$, (d) None of these.

Q.No:2 $\frac{d}{dx}\left(\int_{x^2}^{2x} \frac{1}{t^2+1}dt\right)$ is equal to

(a) $\frac{2}{4x^2+1} - \frac{2x}{x^2+1}$ (b) $\frac{2}{4x^2+1} - \frac{2x}{x^4+1}$, (c) $\frac{x}{x^2+1}$, (d)None of these.

Q.No:3 If $2\log|x| = \log|5x-4|$ then the value of x equals to

(a) 4 & -1 (b) -4 & 1 (c) 4 & 1 (d)None of these.

Q.No:4 If $\sum_{k=1}^{4}(ak+1) = 24$ then the value of a is equal to

(a) 2 (b) 1 (c) -2 (d)None of these.

Q.No:5 $\int_{-1}^{0} 5^x dx$ is equal to

(a) $\frac{4\ln 5}{5}$ (b) $\frac{4}{5\ln 5}$ (c) $\frac{5}{4\ln 5}$ (d)None of these.

Q.No:6 $\int \frac{e^{2x}}{\sqrt{1+e^{4x}}}dx$ is equal to

(a) $\frac{1}{2}\sin^{-1}\left(e^{2x}\right)+c$ (b) $\frac{1}{2}\cosh^{-1}\left(e^{2x}\right)+c$ (c) $\frac{1}{2}\sinh^{-1}\left(e^{2x}\right)+c$ (d)None of these.

Q.No:7 The domain of the function $f(x) = \ln\left(\frac{1}{2-x}\right)$ is

(a) $(-\infty,2)$ (b) $[-2,2]$ (c) $(-\infty,2]$ (d)None of these.

Q.No:8 If $f(x)=\cot^{-1}(\coth x)$, then $f'(x)$ is equal to

(a) $\dfrac{-\operatorname{csc}h^2 x}{1+\coth^2 x}$ (b) $\dfrac{\operatorname{csc}h^2 x}{1+\coth^2 x}$ (c) $\dfrac{1}{1+\coth^2 x}$ (d)None of these.

Q.No:9 $\lim\limits_{x\to\infty}\left(e^{-x}\sqrt{x}\right)$ is equal to

(a) 0 (b) -1 (c) 1 (d)None of these.

Q.No:10 $\int \dfrac{2}{x}e^{\ln x}\sin 2x\,dx$ is equal to

(a) $-\cos 2x+c$ (b) $\cos 2x+c$ (c) $-\dfrac{\cos 2x}{2}+c$ (d)None of these.

Q.No:11 The partial fractions decomposition of $\dfrac{x+2}{1+x^3}$ is

(a) $\dfrac{A}{x-1}+\dfrac{Bx+c}{x^2+x+1}$ (b) $\dfrac{A}{x+1}+\dfrac{Bx+c}{x^2+x-1}$ (c) $\dfrac{A}{x+1}+\dfrac{Bx+c}{x^2-x+1}$ (d)None of these

Q.No:12 The integral $\int \operatorname{csc}^4 x\,dx$ is equal to

(a) $\dfrac{\operatorname{csc}^5 x}{5}+c$ (b) $-\cot x-\dfrac{\operatorname{csc}^3 x}{3}+c$ (c) $\cot x+\dfrac{\operatorname{csc}^3 x}{3}+c$ (d)None of these

Q.No:13 The substitution $u=1-\sqrt{x}$ transforms $\int \dfrac{2+\sqrt{x}}{1-\sqrt{x}}dx$

(a) $2\int \dfrac{(3-u)(u-1)}{u}du$ (b) $\int 2(u-1)du$ (c) $2\int\left(\dfrac{3-u}{u}-2(1-u)\right)du$ (d)None of these.

Q.No:14 The area of the region bounded by the graphs of $y=\sin x$, $x=\dfrac{\pi}{2}$ and $y=0$ is

(a) 2 (b) 0 (c) 1 (d)None of these.

Q.No:15 The length of the curve $y=x$ from $x=-2$ to $x=2$ equals to

(a) 4 (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d)None of these.

Q.No:16 The arc length of the polar curve $r = 2\cos\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ equals to

(a) π (b) 2π (c) $\frac{\pi}{2}$ (d)None of these.

Q.No:17 The slope of the tangent line to the curve $C : x = \cos t, y = \sin t$ at $t = \frac{\pi}{2}$ equals

(a) 0 (b) 2 (c) -1 (d)None of these

Q.No:18 If $(r,\theta) - coordinate$ of points are $\left(2, \frac{\pi}{4}\right)$ then $(x, y) - coordinates$ are

(a) $(\sqrt{2},1)$ (b) $(1,1)$ (c) $(\sqrt{2}, \sqrt{2})$ (d)None of these

Q.No:19 If $(x, y) - coordinate$ of a point are $(0,-2)$ then the $(r,\theta) - coordinates$ are

(a) $\left(2, \frac{\pi}{2}\right)$ (b) $(-2, \pi)$ (c) $\left(2, \frac{3\pi}{2}\right)$ (d)None of these

Q.No:20 The polar curve $r = 3\sec\theta$ IS

(a) a circle (b) a line parallel to the line $\theta = \frac{\pi}{2}$

(c) a line parallel to polar axis (d) None of these.

Full Questions

Question No: 21 Use the Simpson's rule to approximate $\int_0^{\pi} \sqrt{\sin x}\ dx$ with n=4.

Solution: $\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$ $x_0 = 0, x_1 = \frac{\pi}{4}, x_2 = \frac{\pi}{2}, x_3 = \frac{3\pi}{4}, x_4 = \pi$

$$\int_0^{\pi} \sqrt{\sin x}dx \approx \frac{\pi - 0}{3(4)}\left\{\sin(0) + 4\sin(\frac{\pi}{4}) + 2\sin(\frac{\pi}{2}) + 4\sin(\frac{3\pi}{4}) + \sin(\pi)\right\}$$

$$= \frac{\pi}{12}\left\{0 + 4(\sqrt{2}) + 2(1) + 4(\sqrt{2}) + 0\right\} = \frac{\pi}{12}\left(8\sqrt{2} + 2\right) = \frac{\pi}{6}(4\sqrt{2} + 1).$$

Question No: 22 **Evaluate the integral** $\int \frac{1}{x\sqrt{1+(\ln x)^2}} dx$.

Solution: **Put** $u = \ln x \Rightarrow du = \frac{1}{x} dx \Rightarrow$

$$\int \frac{1}{x\sqrt{1+(\ln x)^2}} dx = \int \frac{1}{\sqrt{1+u^2}} dx = \sinh^{-1}(u) + c = \sinh^{-1}(\ln x) + c$$

Question No: 23 **Evaluate the integral** $\int \frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} dx$.

Solution: **Using the partial fraction decomposition**

$$\frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

We can show easily that A=4, B=1, and C=4.

$$\int \frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} dx = \int \frac{4}{x+3} dx + \int \frac{1}{x-1} dx + \int \frac{4}{(x-1)^2} dx$$

$$= 4\ln|x+3| + \ln|x-1| - \frac{4}{x-1} + c.$$

Question No: 24 Let R be the region bounded by the graphs of $y = x^2 - 4$ and $y = 4 - x^2$.

Set up an integral that can be used to find the volume generated by Revolving R about the line x=2. (Using Cylindrical shell method).

Solution:

Volume of one cylindrical shell= $2\pi($ **average radius)(height)(thickness**

$$= 2\pi(2-x)[(4-x^2)-(x^2-4)](\Delta x) \quad -2 \le x \le 2$$

Full Volume $= 2\pi \int_{-2}^{2} (2-x)[-2x^2+8)]dx$

$$= 2\pi \int_{-2}^{2} [16-8x-4x^2+2x^3]dx = \frac{256\pi}{3}$$

Question N0: 25 Find the surface area generated by revolving the parametric curve $C: x = \cos t \quad y = \sin t \quad 0 \le t \le \pi$ about the x-axis.

Answer(4π **).**

Question No: 26 Find the area of the region inside $r = 4\sin\theta$ an out side

$r = 2$.

Answer: (2π **).**