

Department of Mathematics

King Saud University

# M-106

Second Semester(1427/14280)

## Final Exam

Name:	Number:
Name of Teacher:	Group No:

Max Marks: 50

Time: Three hours

Marks:

Multiple Choice(1-20)	
Question # 21	
Question # 22	
Question # 24	
Question # 25	
Question # 26	
Total	

## Multiple Choice

Q.NO:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
{a,b,c,d}	c	a	c	a	b	c	c	b	a	a	c	b	c	c	c	b	c	c	c	b

**Q.No:1** To evaluate the integral  $\int \frac{\sin 2x}{\sqrt{1 - \cos 2x}} dx$ , we use the substitution

- (a)  $u = \tan\left(\frac{x}{2}\right)$ ,      (b)  $u = \sqrt{1 - \cos 2x}$ ,      (c)  $u = 1 - \cos 2x$ ,      (d) None of these.

**Q.No:2**  $\frac{d}{dx} \left( \int_{x^2}^{2x} \frac{1}{t^2 + 1} dt \right)$  is equal to

- (a)  $\frac{2}{4x^2 + 1} - \frac{2x}{x^2 + 1}$       (b)  $\frac{2}{4x^2 + 1} - \frac{2x}{x^4 + 1}$ ,      (c)  $\frac{x}{x^2 + 1}$ ,      (d) None of these.

**Q.No:3** If  $2 \log|x| = \log|5x - 4|$  then the value of x equals to

- (a) 4 & -1      (b) -4 & 1      (c) 4 & 1      (d) None of these.

**Q.No:4** If  $\sum_{k=1}^4 (ak + 1) = 24$  then the value of a is equal to

- (a) 2      (b) 1      (c) -2      (d) None of these.

**Q.No:5**  $\int_{-1}^0 5^x dx$  is equal to

- (a)  $\frac{4 \ln 5}{5}$       (b)  $\frac{4}{5 \ln 5}$       (c)  $\frac{5}{4 \ln 5}$       (d) None of these.

**Q.No:6**  $\int \frac{e^{2x}}{\sqrt{1 + e^{4x}}} dx$  is equal to

- (a)  $\frac{1}{2} \sin^{-1}(e^{2x}) + c$       (b)  $\frac{1}{2} \cosh^{-1}(e^{2x}) + c$       (c)  $\frac{1}{2} \sinh^{-1}(e^{2x}) + c$       (d) None of these.

**Q.No:7** The domain of the function  $f(x) = \ln\left(\frac{1}{2-x}\right)$  is

- (a)  $(-\infty, 2)$       (b)  $[-2, 2]$       (c)  $(-\infty, 2]$       (d) None of these.

**Q.No:8** If  $f(x) = \cot^{-1}(\coth x)$ , then  $f'(x)$  is equal to

- (a)  $\frac{-\operatorname{csc}^2 x}{1 + \coth^2 x}$  (b)  $\frac{\operatorname{csc}^2 x}{1 + \coth^2 x}$  (c)  $\frac{1}{1 + \coth^2 x}$  (d) None of these.

**Q.No:9**  $\lim_{x \rightarrow \infty} (e^{-x} \sqrt{x})$  is equal to

- (a) 0 (b) -1 (c) 1 (d) None of these.

**Q.No:10**  $\int \frac{2}{x} e^{\ln x} \sin 2x dx$  is equal to

- (a)  $-\cos 2x + c$  (b)  $\cos 2x + c$  (c)  $-\frac{\cos 2x}{2} + c$  (d) None of these.

**Q.No:11** The partial fractions decomposition of  $\frac{x+2}{1+x^3}$  is

- (a)  $\frac{A}{x-1} + \frac{Bx+c}{x^2+x+1}$  (b)  $\frac{A}{x+1} + \frac{Bx+c}{x^2+x-1}$  (c)  $\frac{A}{x+1} + \frac{Bx+c}{x^2-x+1}$  (d) None of these

**Q.No:12** The integral  $\int \csc^4 x dx$  is equal to

- (a)  $\frac{\csc^5 x}{5} + c$  (b)  $-\cot x - \frac{\csc^3 x}{3} + c$  (c)  $\cot x + \frac{\csc^3 x}{3} + c$  (d) None of these

**Q.No:13** The substitution  $u = 1 - \sqrt{x}$  transforms  $\int \frac{2 + \sqrt{x}}{1 - \sqrt{x}} dx$

- (a)  $2 \int \frac{(3-u)(u-1)}{u} du$  (b)  $\int 2(u-1) du$  (c)  $2 \int \left( \frac{3-u}{u} - 2(1-u) \right) du$  (d) None of these.

**Q.No:14** The area of the region bounded by the graphs of  $y = \sin x$ ,  $x = \frac{\pi}{2}$  and  $y = 0$  is

- (a) 2 (b) 0 (c) 1 (d) None of these.

**Q.No:15** The length of the curve  $y = x$  from  $x = -2$  to  $x = 2$  equals to

- (a) 4 (b)  $2\sqrt{2}$  (c)  $4\sqrt{2}$  (d) None of these.

**Q.No:16** The arc length of the polar curve  $r = 2 \cos \theta$ ,  $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$  equals to

- (a)  $\pi$                       (b)  $2\pi$                       (c)  $\frac{\pi}{2}$                       (d) None of these.

**Q.No:17** The slope of the tangent line to the curve  $C : x = \cos t, y = \sin t$  at  $t = \frac{\pi}{2}$  equals

- (a) 0                      (b) 2                      (c) -1                      (d) None of these

**Q.No:18** If  $(r, \theta)$ -coordinate of points are  $\left(2, \frac{\pi}{4}\right)$  then  $(x, y)$ -coordinates are

- (a)  $(\sqrt{2}, 1)$                       (b)  $(1, 1)$                       (c)  $(\sqrt{2}, \sqrt{2})$                       (d) None of these

**Q.No:19** If  $(x, y)$ -coordinate of a point are  $(0, -2)$  then the  $(r, \theta)$ -coordinates are

- (a)  $\left(2, \frac{\pi}{2}\right)$                       (b)  $(-2, \pi)$                       (c)  $\left(2, \frac{3\pi}{2}\right)$                       (d) None of these

**Q.No:20** The polar curve  $r = 3 \sec \theta$  is

- (a) a circle                      (b) a line parallel to the line  $\theta = \frac{\pi}{2}$   
 (c) a line parallel to polar axis                      (d) None of these.

### Full Questions

**Question No: 21 Use the Simpson's rule to approximate  $\int_0^{\pi} \sqrt{\sin x} \, dx$  with  $n=4$ .**

**Solution:**  $\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$                        $x_0 = 0, x_1 = \frac{\pi}{4}, x_2 = \frac{\pi}{2}, x_3 = \frac{3\pi}{4}, x_4 = \pi$

$$\int_0^{\pi} \sqrt{\sin x} \, dx \approx \frac{\pi - 0}{3(4)} \left\{ \sin(0) + 4 \sin\left(\frac{\pi}{4}\right) + 2 \sin\left(\frac{\pi}{2}\right) + 4 \sin\left(\frac{3\pi}{4}\right) + \sin(\pi) \right\}$$

$$= \frac{\pi}{12} \{0 + 4(\sqrt{2}) + 2(1) + 4(\sqrt{2}) + 0\} = \frac{\pi}{12} (8\sqrt{2} + 2) = \frac{\pi}{6} (4\sqrt{2} + 1).$$

**Question No: 22** Evaluate the integral  $\int \frac{1}{x\sqrt{1+(\ln x)^2}} dx$ .

**Solution:** Put  $u = \ln x \Rightarrow du = \frac{1}{x} dx \Rightarrow$

$$\int \frac{1}{x\sqrt{1+(\ln x)^2}} dx = \int \frac{1}{\sqrt{1+u^2}} du = \sinh^{-1}(u) + c = \sinh^{-1}(\ln x) + c$$

**Question No: 23** Evaluate the integral  $\int \frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} dx$ .

**Solution:** Using the partial fraction decomposition

$$\frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

We can show easily that  $A=4$ ,  $B=1$ , and  $C=4$ .

$$\begin{aligned} \int \frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} dx &= \int \frac{4}{x+3} dx + \int \frac{1}{x-1} dx + \int \frac{4}{(x-1)^2} dx \\ &= 4 \ln|x+3| + \ln|x-1| - \frac{4}{x-1} + c. \end{aligned}$$

**Question No: 24** Let R be the region bounded by the graphs of

$$y = x^2 - 4 \text{ and } y = 4 - x^2.$$

**Set up an integral that can be used to find the volume generated by Revolving R about the line  $x=2$ . (Using Cylindrical shell method).**

**Solution:**

**Volume of one cylindrical shell =  $2\pi$ ( average radius)(height)(thickness**

$$= 2\pi(2-x)[(4-x^2)-(x^2-4)](\Delta x) \quad -2 \leq x \leq 2$$

$$\text{Full Volume} = 2\pi \int_{-2}^2 (2-x)[-2x^2 + 8]dx$$

$$= 2\pi \int_{-2}^2 [16 - 8x - 4x^2 + 2x^3]dx = \frac{256\pi}{3}$$

**Question N0: 25** Find the surface area generated by revolving the parametric curve  $C: x = \cos t \quad y = \sin t \quad 0 \leq t \leq \pi$  about the x-axis.

**Answer(  $4\pi$  ).**

**Question No: 26** Find the area of the region inside  $r = 4 \sin \theta$  an out side

$$r = 2.$$

**Answer: (  $2\pi$  ).**