King Saud University | Math. Department | College of Science

Q1: Let V ={0} and define the addition and scalar multiplications as follows:

0+0=0 and k0=0 for any scalar k.

Show (in details) that V is a vector space. (5 marks)

Q2: Let **V** be an inner product space. Let **W** be a subspace of **V** and define a set **S** as follows: **S**={**s**∈**V**| the vectors **s** and **w** are orthogonal for all **w**∈**W**}. Show that **S** is a subspace of **V**. (3 marks)

Q3: Let S={v_1=(1,1,1,1), v_2=(1,0,1,0), v_3=(1,0,0,1), v_4=(0,2,1,1)} be a subset of the Euclidean space $\mathbb{R}^4$. Find a subset of S which is a basis of span(S). (4 marks)

Q4- Let T: V →$\mathbb{R}$ be a map from an inner product space V to the set of real numbers $\mathbb{R}$ defined by $T(v)=<3v,3v_o>$ for all v∈ V, where v_o is a fixed vector in V. (6 marks)

(a) Show that T is a linear transformation.

(b) Find ker(T).

(c) Show that if v_o∈ ker(T) then ker(T)=V.

Q5- Let $\mathbb{R}^4$ be the Euclidean space with a weighted inner product which 3,2,1,1 are the weights respectively. Let v=(1,1,1,1) and u=(1,2, -1,1). (2 marks)

Find:

(i) $<u,v>$.

(ii) $\|v\|$.