Differential Equation of Order One

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1. Initial-Value Problem

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The equation (2) can be written as follows

$$M(x, y)dx + N(x, y)dy = 0, \tag{3}$$

where M and N are two functions of x and y.

We are interested in problems in which we seek a solution $y(x)$ of differential equation which satisfies some conditions imposed on the unknown $y(x)$ or its derivatives. On some interval I containing x_0, the problem

$$\begin{aligned} \text{Solve:} \quad & \frac{d^n y}{dx^n} = f(x, y, y', \ldots, y^{(n-1)}) \\ \text{Subject to:} \quad & y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-1)}(x_0) = y_{n-1}, \end{aligned}$$

where $y_0, y_1, \ldots, y_{n-1}$ are arbitrary specified real constants, is clled an *initial-values problem (IVP)* and its $n-1$ derivatives at a single point x_0: $y(x_0) = y_0, y'(x_0) = y_1, \ldots, y^{(n-1)}(x_0) = y_{n-1}$ are called *initial conditions*.

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Special case:

$$\begin{aligned} \text{Solve:} \quad & \frac{dy}{dx} = f(x, y) \\ \text{Subject to:} \quad & y(x_0) = y_0, y'(x_0) = y_1. \end{aligned}$$

(1) Solve: $y' = y$

Subject to: $y(0) = 4.$

(2) Solve: $y' + 2xy^2 = 0$

Subject to: $y(0) = -1.$

(3) Solve: $\frac{dy}{dx} = xy^{\frac{1}{2}}$

Subject to: $y(0) = 0.$