Differential Equations of Order One

Dr. Bander Almutairi

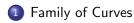
King Saud University

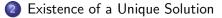
5 Feb 014

Dr. Bander Almutairi (King Saud University)

Differential Equations of Order One

5 Feb 014 1/5







Dr. Bander Almutairi (King Saud University) Differ

< A

Example:

$$(x-c)^{2} + (y-c)^{2} = 2c^{2}.$$
 (1)

represents a family of circles with their centers on y = x. If we assume that c in the equation (1) is arbitrary constant, then by using the elimination of arbitrary constant, then result equation is called *differential* equation of the family of curve (1).

Example:

$$(x-c)^2 + (y-c)^2 = 2c^2.$$
 (1)

represents a family of circles with their centers on y = x. If we assume that c in the equation (1) is arbitrary constant, then by using the elimination of arbitrary constant, then result equation is called *differential* equation of the family of curve (1).

We can rearrange equation (1) to be the following

 $x^2 + y^2 - 2c(x + y) = 0.$

Example:

$$(x-c)^2 + (y-c)^2 = 2c^2.$$
 (1)

represents a family of circles with their centers on y = x. If we assume that c in the equation (1) is arbitrary constant, then by using the elimination of arbitrary constant, then result equation is called *differential* equation of the family of curve (1).

We can rearrange equation (1) to be the following

 $x^2 + y^2 - 2c(x + y) = 0.$

So we get

$$\frac{x^2+y^2}{x+y}=2c, y\neq -x.$$

Q(1): Find a differential equation satisfied by the family of parabolas having their virtices at the origin and their foci (focus) on the *y*-axis. Q(2): Find the differential equation of the family of circles having their centers on the *y*-axis.

イロト 不得下 イヨト イヨト 三日

For a first order differential equation (IVP)

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_0) = y_0,$$

there exists a unique solution if

For a first order differential equation (IVP)

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_0) = y_0,$$

there exists a unique solution if

• f(x, y) and $\frac{\partial f(x, y)}{\partial y}$ are continous with in the region \mathbb{R}^2 of xy-plane.

4 / 5

For a first order differential equation (IVP)

$$\frac{dy}{dx} = f(x, y)$$
$$y(x_0) = y_0,$$

there exists a unique solution if

- f(x, y) and $\frac{\partial f(x, y)}{\partial y}$ are continous with in the region \mathbb{R}^2 of xy-plane.
- (x_0, y_0) be a point in the region \mathbb{R}^2 .

For a first order differential equation (IVP)

$$\frac{dy}{dx} = f(x, y)$$

$$y(x_0) = y_0,$$

there exists a unique solution if

- f(x, y) and $\frac{\partial f(x, y)}{\partial y}$ are continous with in the region \mathbb{R}^2 of xy-plane.
- (x_0, y_0) be a point in the region \mathbb{R}^2 .

Q: Find the largest region of the *xy*-plane for which the following initial value problems have unique solutions:

(a)
$$\sqrt{x^2 - 4}y' = 1 + \sin(x)\ln(y)$$
, with initial condition $y(3) = 4$.

(b)
$$\ln(x-2) \cdot \frac{dy}{dx} = \sqrt{y-2}$$
, with initial condition $y(\frac{5}{2}) = 4$.

(c)
$$\sqrt{\frac{x}{y}}y' = \cos(x+y); y \neq 0$$
, with initial condition $y(1) = 1$.

Consider a first-order differential equation of the form:

M(x,y)dx + N(x,y)dy = 0,

where M and N are two function of x, y.

A B A B A
 A
 B
 A
 A
 B
 A
 A
 B
 A
 A
 B
 A
 A
 B
 A
 A
 B
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A
 A

(2)

Consider a first-order differential equation of the form:

$$M(x, y)dx + N(x, y)dy = 0,$$
(2)

where M and N are two function of x, y. Sometimes we can write the equation (2) as follows:

$$F(x)dx + G(y)dy = 0, \qquad (3)$$

the variables separated here and we can find a solution immediately. Q(i): Find a solution for each of the following:

(a)
$$2x(y^2 + y)dx + (x^2 - 1)ydy = 0; y \neq 0$$

(b)
$$(xy+1)dx = (x^2y^2 + x^2 + y^2 + 1)dy$$
.

Q(ii): Solve the following IVP

$$e^{y} \frac{dy}{dx} = \cos(2x) + 2e^{y} \sin^{2}(x) - 1; y(\frac{\pi}{2}) = \ln(2).$$