Phys 570

Lecture #4

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ELECTRICAL CONDUCTIVITY AND OHM'S LAW

The momentum of a free electron is related to the wavevector by $m\mathbf{v} = \hbar\mathbf{k}$. In an electric field E and magnetic field B the force F on an electron of charge –e *can be written as:*

$$F = m\frac{dv}{dt} = \hbar\frac{dk}{dt} = -e\left[E + \frac{1}{c}v \times B\right] \qquad (39)$$

This equation is the Newton's second law of motion for the electron of charge –e and mass m_e in both of E and B.
We want to find the Electrical Conductivity (From Ohm's Law).
Hence, we set B = 0 (no magnetic Field):

$$\hbar\frac{dk}{dt} = -e\left[E\right] \Rightarrow dk = -eEdt/\hbar$$

by intigrating both sides:

$$k(t) - k(0) = -eEt/\hbar \qquad (40)$$

If the force **F = -e*E*** *is applied at time t = 0 to an electron gas that fills* the Fermi sphere centered at the origin of k space, then at a later time *t the* sphere will be displaced to a new center at:

$$\delta k = -eEt / \hbar \qquad (41)$$

Notice that the Fermi sphere is displaced as a whole because every electron is displaced by the same δk.
Because of collisions of electrons, the displaced sphere may be maintained in a steady state in an electric field. If the collision time is τ, the displacement of the sphere is given by (41) with *t* = τ. The velocity is: **v** =**P**/m = *ħ**k*** /m = -eE τ /m.
If ***E*** = constant; there are n electrons of charge -e per unit volume, the electric current density is:

$$j = nqv = n(-e)v = n(-e)(-eE\tau / m) = ne^2\tau E / m \qquad (42)$$

ELECTRICAL CONDUCTIVITY AND OHM'S LAW

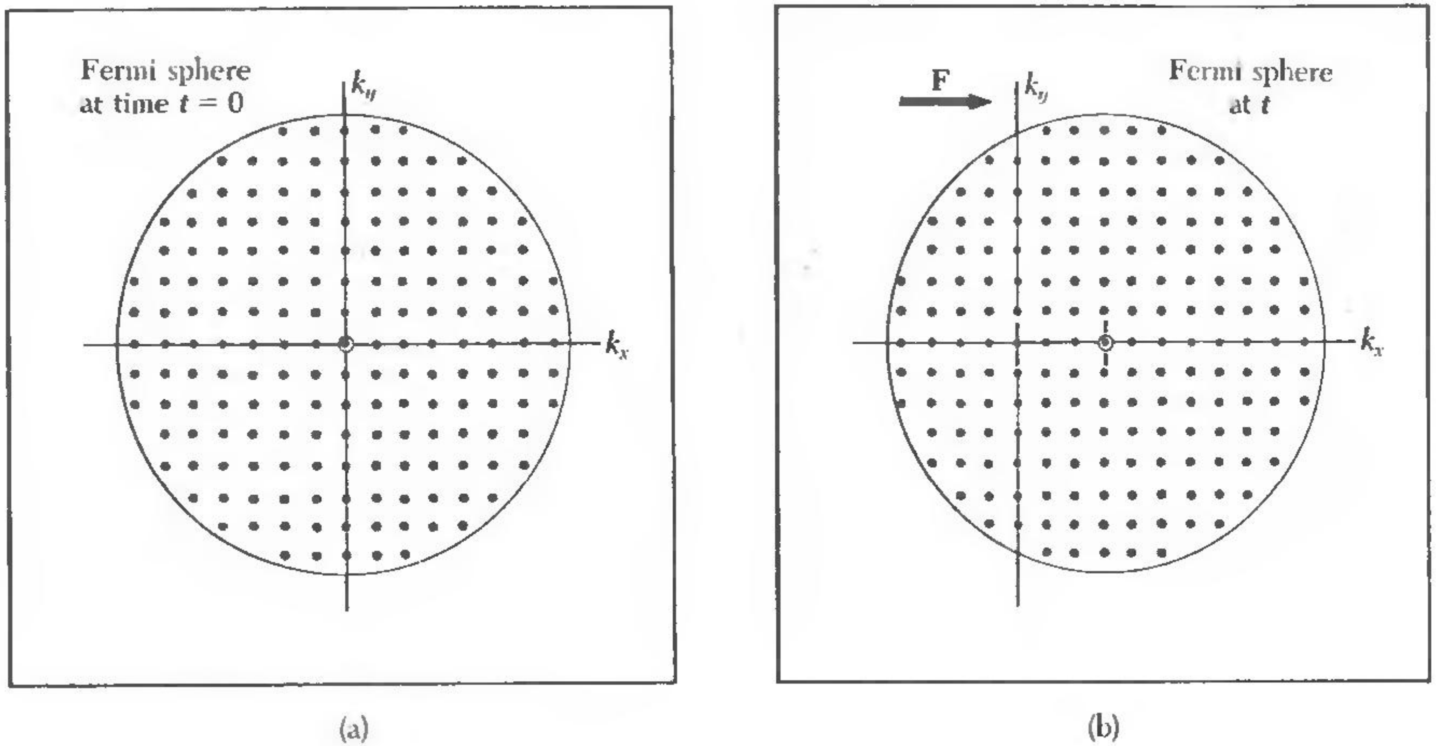


Figure 10 (a) The Fermi sphere encloses the occupied electron orbitals in **k** space in the ground state of the electron gas. The net momentum is zero, because for every orbital **k** there is an occupied orbital at $-\mathbf{k}$. (b) Under the influence of a constant force **F** acting for a time interval t every orbital has its **k** vector increased by $\delta\mathbf{k} = \mathbf{F}t/\hbar$. This is equivalent to a displacement of the whole Fermi sphere by $\delta\mathbf{k}$. The total momentum is $N\hbar\delta\mathbf{k}$, if there are N electrons present. The application of the force increases the energy of the system by $N(\hbar\delta\mathbf{k})^2/2m$.

Equation (42) is called Ohm's Law.
We can find the electrical conductivity σ defined by $\boldsymbol{j} = \sigma \boldsymbol{E}$, so by (42):

$$\sigma = \frac{ne^2\tau}{m} \qquad (43)$$

The electrical resistivity ρ is defined as the reciprocal of the conductivity, so that: (see table 3)

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad (44)$$

It is easy to understand the result (43). Charge transported is proportional to the density *ne;* e/m is because the acceleration is proportional to **e** and inversely proportional to the mass **m**.

Li 1.07 9.32	Be 3.08 3.25	
Na 2.11 4.75	Mg 2.33 4.30	
K 1.39 7.19	Ca 2.78 3.6	S 0 4
Rb 0.80 12.5	Sr 0.47 21.5	Y 0 5
Cs 0.50 20.0	Ba 0.26 39.	L 0 7
Fr	Ra	A

Conductivity in units of 10^5 (ohm-cm)$^{-1}$.
Resistivity in units of 10^{-6} ohm-cm.

 50 .9	Cr 0.78 12.9	Mn 0.072 139.	Fe 1.02 9.8	Co 1.72 5.8	Ni 1.43 7.0	Cu 5.88 1.70
b 69 .5	Mo 1.89 5.3	Tc ~0.7 ~14.	Ru 1.35 7.4	Rh 2.08 4.8	Pd 0.95 10.5	Ag 6.21 1.61
a 76 .1	W 1.89 5.3	Re 0.54 18.6	Os 1.10 9.1	Ir 1.96 5.1	Pt 0.96 10.4	Au 4.55 2.20

O	F	Ne
S	Cl	Ar
Se	Br	Kr
Te	I	Xe
Po 0.22 46.	At	Rn

Tm 0.16 62.	Yb 0.38 26.4	Lu 0.19 53.
Md	No	Lr

ELECTRICAL CONDUCTIVITY AND OHM'S LAW

- According the Table 3, conductivity of cupper could be:

$5.88x10^5$ /Ω.cm at room temp.

- This value could be as high as **10^5** times larger at low temperatures.
- Hence: cupper crystals become more pure when cooled and vise versa. This applies to all crystals.
- This leads to large increase in relaxation time t that can reach values: **$2x10^{-9}$ s** at very low temp.
- We have a quantity that depends on τ which is ℓ (mean free path) which represents the mean distance between every tow collisions.
- ℓ is expressed as: $\ell = v_F\tau$

if the electric field were switched off; the momentum distribution would relax back to its ground state with the net relaxation rate:

$$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i} \qquad (45)$$

where τ_L and τ_i are the collision times for scattering by phonons and by imperfections, respectively.

Total resistance from phonons and impurities is:

$$\rho = \rho_L + \rho_i \qquad (46)$$

First term is independent of impurities (when their concentration is small) .
and 2nd term is independent of temperature.

Chapter 6: Free Electron Fermi Gas

Resistance of Potassium

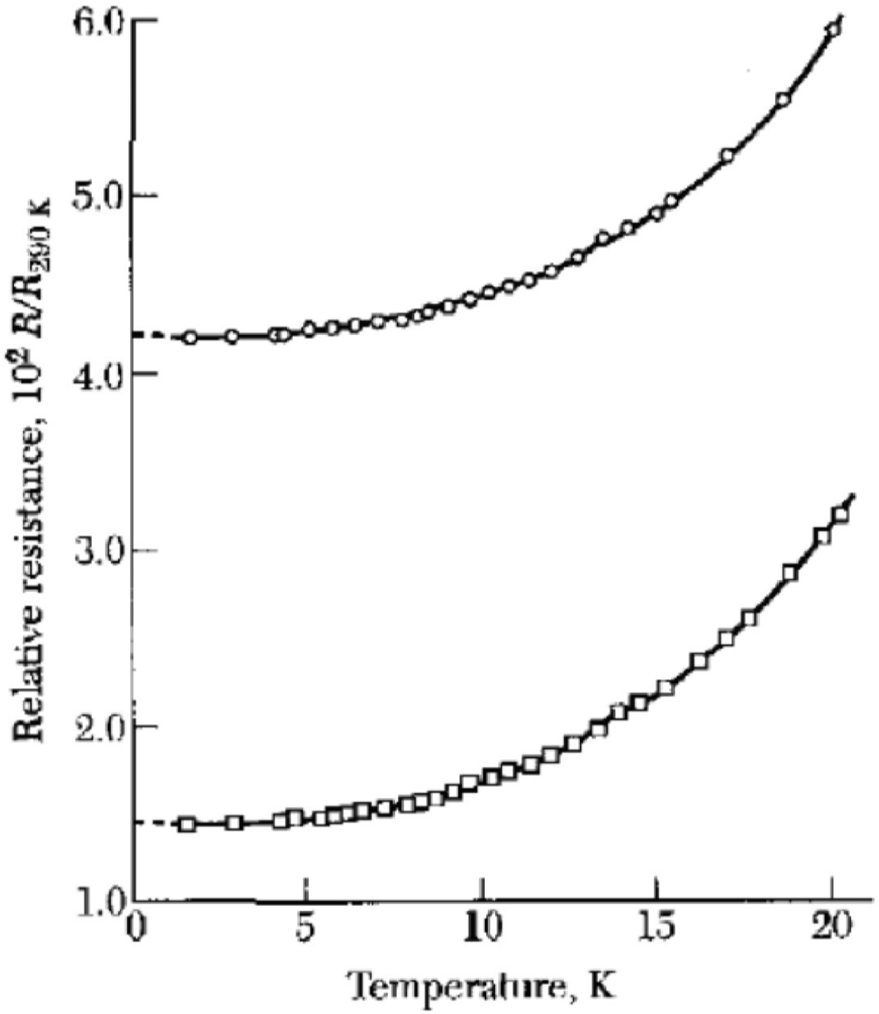


Figure 12 Resistance of potassium below 20 K, as measured on two specimens by D. MacDonald and K. Mendelssohn. The different intercepts at 0 K are attributed to different concentrations of impurities and static imperfections in the two specimens.

The relation $\ell = v_F\tau$ shows clearly that Fermi velocity v_F is the same as the velocity of electrons in the conductor because all collisions involve only electrons near the Fermi surface.

From Table 1 we have $v_F = 1.57 \times 10^8$ cm S^{-1} for Cu, thus the mean free path is $\ell(4\ K) = 0.3$ *cm. Mean free paths as long as* 10 cm have been observed in very pure metals in the liquid helium temperature range.

Since v_F is very high as we showed previously; and because ℓ is large (as large as 10 cm) then we expect that τ is very small. Usually this time is opposite to Fermi velocity.

When electrons move under both of B and E:

$$F = -e\left(\vec{E} + \frac{1}{c}\vec{\mathrm{v}} \times \vec{\mathrm{B}}\right) \qquad (49)$$

1st term: eE is coulumbic force, 2nd term is Lorentz formce.

if mv=$\hbar\delta$k then we have: $\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v = -e\left(\vec{E} + \frac{1}{c}\vec{\mathrm{v}} \times \vec{\mathrm{B}}\right)$ (50)

if B lie along the z axis. Then the component equations of motion are:

$$\left.\begin{aligned}
\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_x &= -e\left(E_x + \frac{\mathrm{B}}{c}\mathrm{v}_{\mathrm{y}}\right) \\
\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_y &= -e\left(E_y + \frac{\mathrm{B}}{c}\mathrm{v}_x\right) \\
\mathrm{m}\left(\frac{d}{dt} + \frac{1}{\tau}\right)v_z &= -eE_z
\end{aligned}\right\} \qquad (51)$$

MOTION of electrons IN MAGNETIC FIELDS

In the steady state in a static electric field the time derivatives are zero, so that the drift velocity is:

$$\left.\begin{aligned} v_x &= \frac{-e\tau E_x}{m} - \omega_c \tau v_y \\ v_y &= \frac{-e\tau E_y}{m} + \omega_c \tau v_x \\ v_z &= \frac{-e\tau E_z}{m} \end{aligned}\right\} \qquad (52)$$

where $\omega_c = \dfrac{eB}{mc}$ is the cyclotron frequency

This means that when electron moves in the existence of magnetic field B, it will rotate with this frequency. We notice the linear dependence on B, when B increases w will increase.

Since we are talking about motion of electrons under the effect of B, let us imagine that these electrons move inside a conductor. The Hall field is the electric field developed across two faces of a conductor, in the direction jxB, when a current j flows across a magnetic field B.

** Let us consider a conductor in a form of *rectangular parallelepiped* with current flowing in x-direction. Hence we have: E_x.
We also have a B perpendicular on this conductor.
Current cannot move in y direction ➔ $v_y = 0$.
Hence, 2nd equation in (52) = 0.
Accordingly; we have:

$$0 = -\frac{e\tau}{m}E_y + \omega_c \tau v_x = -\frac{e\tau}{m}E_y + \frac{eB\tau}{mc}v_x$$

$$\Rightarrow E_y = \frac{B}{c}v_x = \frac{B}{c}\left[-\frac{e\tau}{m}E_x\right] \Rightarrow E_y = -\frac{eB\tau}{mc}E_x \qquad (53)$$

Hall coefficient is defined as:

$$R_H = \frac{E_y}{j_x B} \qquad (54)$$

using: $j_x = \frac{ne^2\tau E_x}{m}$ we can get:

$$R_H = -\frac{eB\tau E_x / mc}{ne^2\tau E_x / m} = -\frac{1}{nec} \qquad (55)$$

Hence; R_H is negative for free electron.

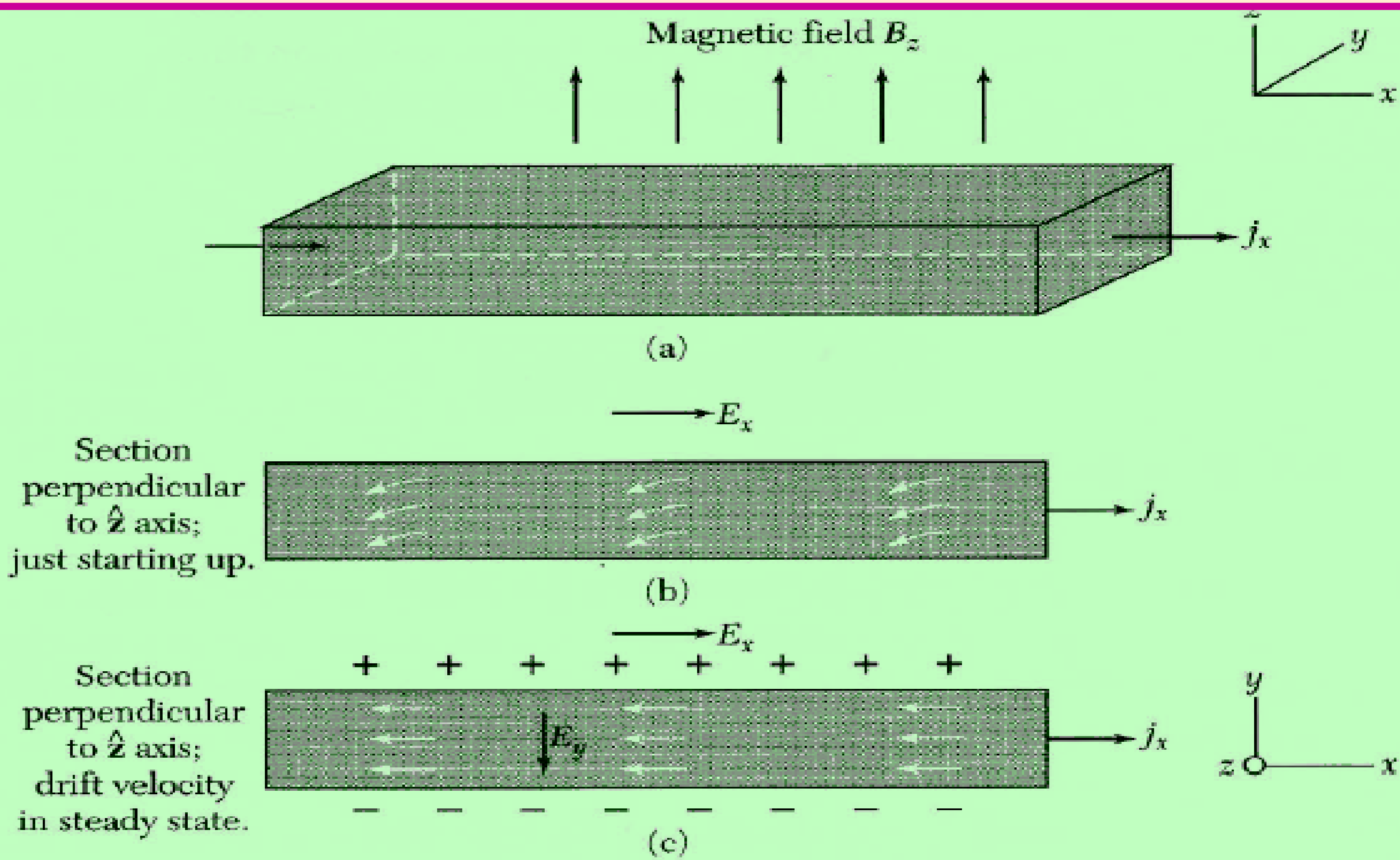


Figure 14 The standard geometry for the Hall effect: a rod-shaped specimen of rectangular cross-section is placed in a magnetic field B_z, as in (a). An electric field E_x applied across the end electrodes causes an electric current density j_x to flow down the rod. The drift velocity of the negatively-charged electrons immediately after the electric field is applied as shown in (b). The deflection in the $-y$ direction is caused by the magnetic field. Electrons accumulate on one face of the rod and a positive ion excess is established on the opposite face until, as in (c), the transverse electric field (Hall field) just cancels the Lorentz force due to the magnetic field.

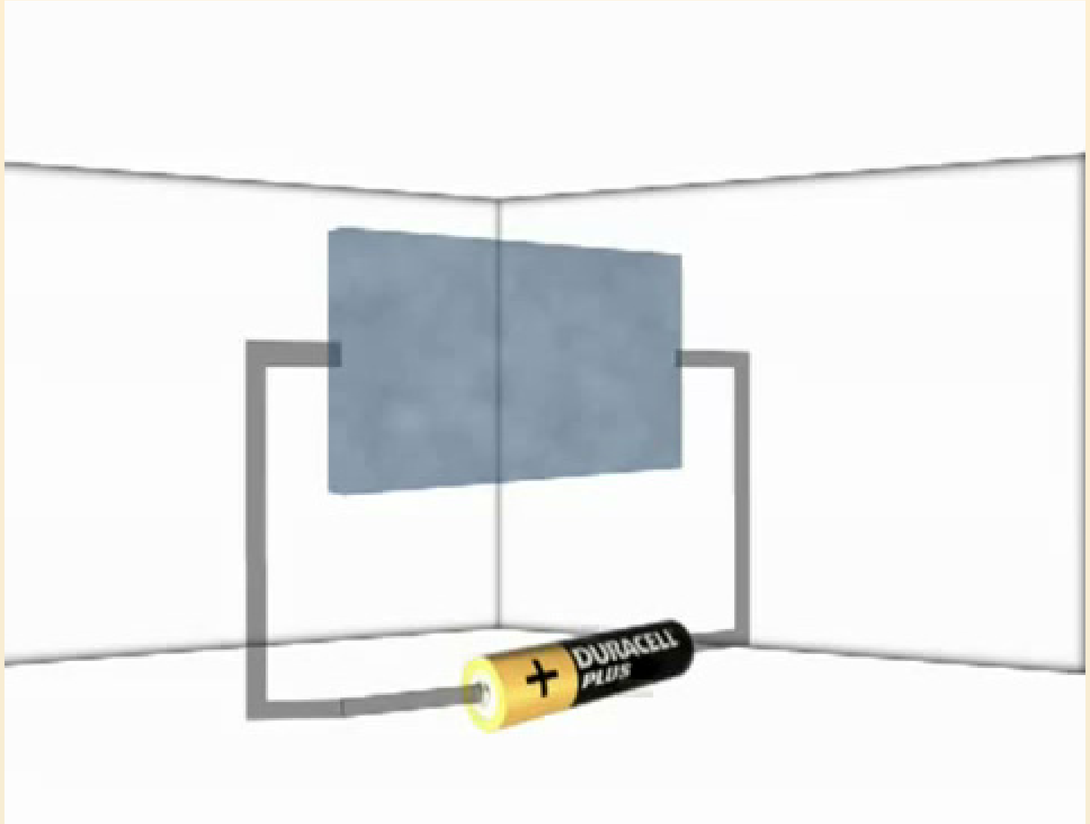
Table 4 Comparison of observed Hall coefficients with free electron theory

[The experimental values of R_H as obtained by conventional methods are summarized from data at room temperature presented in the Landolt-Bornstein tables. The values obtained by the helicon wave method at 4 K are by J. M. Goodman. The values of the carrier concentration n are from Table 1.4 except for Na, K, Al, In, where Goodman's values are used. To convert the value of R_H in CGS units to the value in volt-cm/amp-gauss, multiply by 9×10^{11}; to convert R_H in CGS to m^3/coulomb, multiply by 9×10^{13}.]

Metal	Method	Experimental R_H, in 10^{-24} CGS units	Assumed carriers per atom	Calculated $-1/nec$, in 10^{-24} CGS units
Li	conv.	−1.89	1 electron	−1.48
Na	helicon	−2.619	1 electron	−2.603
	conv.	−2.3		
K	helicon	−4.946	1 electron	−4.944
	conv.	−4.7		
Rb	conv.	−5.6	1 electron	−6.04
Cu	conv.	−0.6	1 electron	−0.82
Ag	conv.	−1.0	1 electron	−1.19
Au	conv.	−0.8	1 electron	−1.18
Be	conv.	+2.7	—	—
Mg	conv.	−0.92	—	—
Al	helicon	+1.136	1 hole	+1.135
In	helicon	+1.774	1 hole	+1.780
As	conv.	+50.	—	—
Sb	conv.	−22.	—	—
Bi	conv.	−6000.	—	—

Hall Effect Animation

- From table we notice that: the lower the concentration, the greater R_H.
- Measuring R_H is important for measuring the carrier concentration.
- Eq. (55) follows from the assumption that τ for all electrons are equal, independent of the velocity of the electron.



Thermal conductivity coefficient K is defined as:

$$j_u = -K \frac{dT}{dx}$$

J_u is the Thermal Energy Flux (Amount of thermal energy flown cross unit area in 1 sec. From previous lectures:

$$C_{el} = \frac{1}{2}\pi^2 N k_B \frac{T}{T_F} \quad with: T_F = \frac{\varepsilon_F}{k_B} \qquad \Rightarrow C_{el} = \frac{1}{2}\pi^2 N k_B \frac{T}{\varepsilon_F} k_B$$

$$\text{from Chapt. 5: } K = \frac{1}{3} C v l$$

$$\Rightarrow K_{el} = \frac{1}{3}.\frac{1}{2}\pi^2 N k_B \frac{T}{\varepsilon_F} k\, v l = \frac{1}{3}.\frac{1}{2}\pi^2 N k_B \frac{T}{m v_F^2}.2.k_B v_F l$$

$$\Rightarrow K_{el} = \frac{\pi^2 n k_B{}^2 T \tau}{3m} \quad (n \, for \, N \;\; and \; l = v_F \tau) \tag{56}$$

- Do the electrons or the phonons carry the greater part of the heat current? in a metal?
- In pure metals the electronic contribution is dominant at all temperatures.
- In impure metals or in disordered alloys, the electron mean free
- path is reduced by collisions with impurities, and the phonon contribution may be comparable with the electronic contribution.