

Lecture 6: "p. solving 1.4.7 p. 33"  
Bivariate Random Variable

\* pb 1.4.7 p. 33

Given independent exponentially distributed random variables  $S$  and  $T$  with common parameter  $\lambda$ , determine the probability density function of the sum  $R = S + T$  and identify its type by name.

Ans:  $\because S, T \sim \text{exp}(\lambda)$

$$\therefore R = S + T$$

$$\therefore R \sim \text{Gamma}(2, \lambda)$$

$$\therefore f_R(r) = \frac{\lambda^2}{\Gamma(2)} r^{2-1} e^{-\lambda r}, \quad r \geq 0$$

$$\therefore f_R(r) = \frac{\lambda^2}{1!} r e^{-\lambda r}$$

$$\therefore f_R(r) = \lambda^2 r e^{-\lambda r}, \quad r \geq 0$$

which is the Gamma prob. density  $f_n$ .

remember

$$f_X(x) = \frac{\lambda^n}{\Gamma(n)} x^{n-1} e^{-\lambda x}, \quad x \geq 0$$

$$\begin{matrix} X \rightarrow R \\ x \rightarrow r \end{matrix}$$

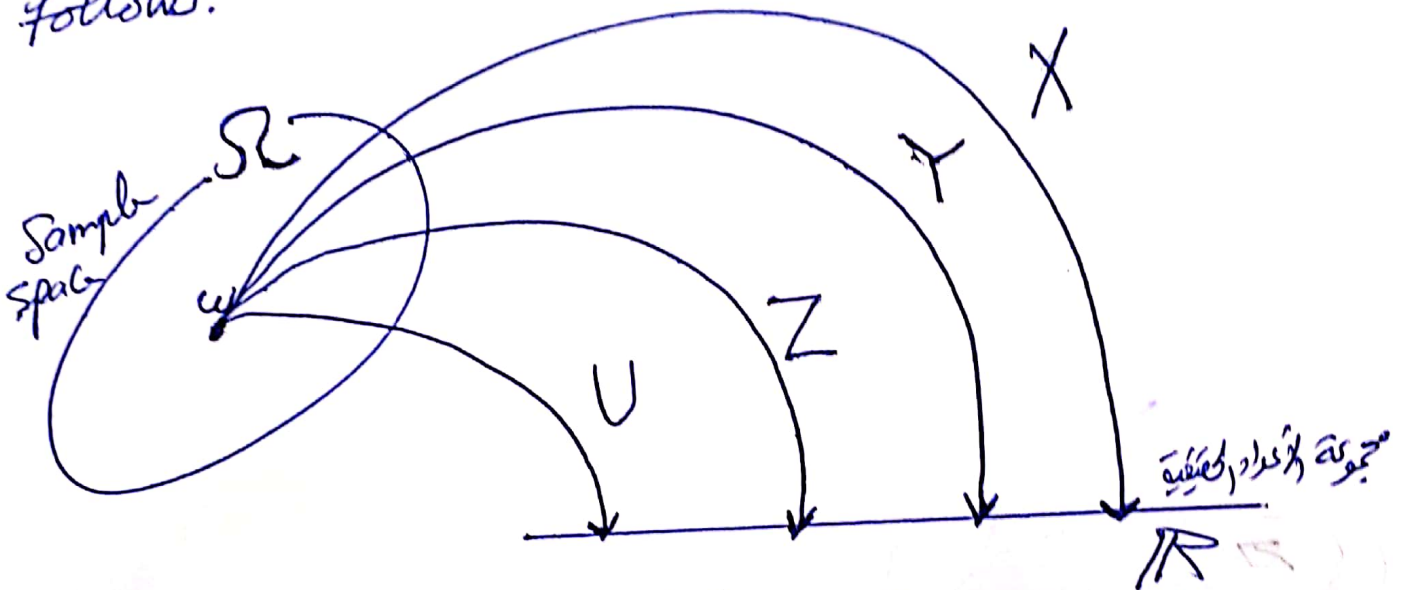
2

## Bivariate Random Variable

Random experiment.

\* Example

Consider a selection of a student  $\omega$  in KSU as follows.



where  $X, Y, Z$  and  $U$  are r.v.s.

represent the age, weight, height and family members for the student then we say that

$(X, Y, Z, U)$  is a multivariate r.v.

$(X, Y)$  is a bivariate r.v.

and  $X$  is a univariate r.v.

### 3/ Discrete Bivariate distn

The joint prob. mass fn for discrete Bivariate r.v  
(X, Y) is

$$P(x, y) = \text{pr}(X=x, Y=y)$$

X, Y

such that

$$(i) P(x, y) \geq 0 \quad (ii) \sum_X \sum_Y P(x, y) = 1$$

$$\text{and } \text{pr}(a \leq X \leq b, c \leq Y \leq d)$$

$$= \sum_{X=a}^b \sum_{Y=c}^d P(x, y)$$

• cdf for discrete Bivariate r.v (X, Y)

$$F(x, y) = \text{pr}(X \leq x, Y \leq y)$$

$$= \sum_{X \leq x} \sum_{Y \leq y} P(x, y)$$

• the marginal prob. mass fns for discrete Bivariate r.v (X, Y)

$$P_X(x) = \sum_y P(x, y) \quad \text{and} \quad P_Y(y) = \sum_x P(x, y)$$

• For Continuous Bivariate distribution

\* The Joint prob. density fn for <sup>Continuous</sup> Bivariate r.v.  $(X, Y)$  is  $f_{X,Y}(x, y)$

s.t (i)  $f_{X,Y}(x, y) \geq 0$

(ii)  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{X,Y}(x, y) dx dy = 1$

\* CdF for Continuous Bivariate r.v.  $(X, Y)$

is  $F(x, y) = \text{pr}(X \leq x, Y \leq y)$

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$$= \int_{-\infty}^y \int_{-\infty}^x f(x, y) dx dy$$

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$$\Rightarrow f(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$$

Joint prob. density  $f_{X,Y}$

• The marginal density  $f_X(x)$  for r.v.  $X$  and  $f_Y(y)$  are

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy \text{ and } f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

respectively.

Some Definitions for Bivariate r.v (X, Y)

Defn ①: Covariance  
 (القovariance)  

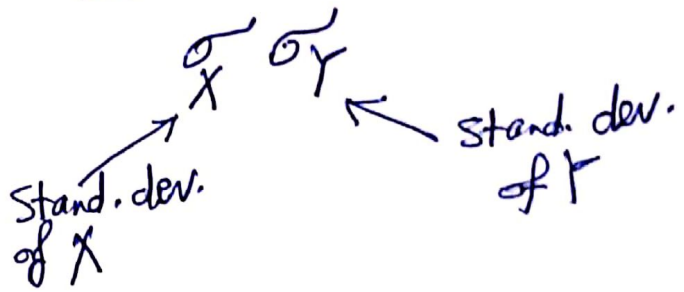
$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

$$= E(XY) - \mu_X \mu_Y$$

X, Y must be related and dependent (القovariance)

Defn ②: Correlation Coefficient  
 (القovariance)

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$



where  $-1 \leq \rho \leq 1$ .

Some props

① For independent r.v X and Y,  $\text{Cov}(X, Y) = 0$   
 and consequently  $\rho(X, Y) = 0$  i.e indep r.v  $\Rightarrow \rho(X, Y) = 0$

Note that For  $\rho(X, Y) = 0$ , it's not necessary for X and Y to be independent r.v

②  $\text{Cov}(X, X) = \text{Var}(X) = \sigma_X^2$

③  $\text{Cov}(\alpha X, \beta Y) = \alpha \beta \text{Cov}(X, Y)$  where  $\alpha, \beta \in \mathbb{R}$

④  $\text{Var}(X + Y) = \sigma_X^2 + \sigma_Y^2 + 2\text{Cov}(X, Y)$

Note that: For  $Z = aX + bY$ ,  $\text{Var}(aX + bY) = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\text{Cov}(X, Y)$   
 or  $\text{Var}(Z) = a^2\sigma_X^2 + 2ab\rho\sigma_X\sigma_Y + b^2\sigma_Y^2$  See p.32 Textbook