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Lecture 32

Pb 6.2.1 p. 293

A pure death process starting from $\boxed{X(0)=3}$ has death parameters $\mu_0=0$, $\mu_1=3$, $\mu_2=2$ and $\mu_3=5$.
Determine $P_n(t)$ for $n=0,1,2,3$.

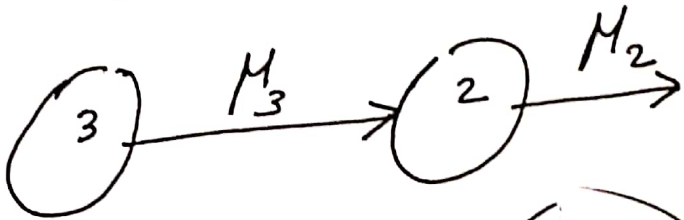
Ans

For $N=3 \overset{①}{\Longrightarrow} P_3(t)=e^{-\mu_3 t}$

$$P_3(t)=e^{-5t} \qquad (\mathrm{I})$$

For $n=2$

$$② \Longrightarrow P_2(t)=\mu_3\left[A_{2,2}\,e^{-\mu_2 t}+A_{3,2}\,e^{-\mu_3 t}\right]$$



$$A_{2,2}=\frac{1}{\mu_3-\mu_2}$$

$$A_{2,2}=\frac{1}{5-2}=\boxed{\frac{1}{3}}$$

$$A_{3,2}=\frac{1}{\mu_2-\mu_3}=\frac{1}{2-5}=\boxed{\frac{-1}{3}}$$

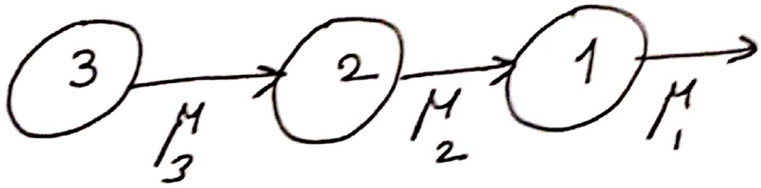
$\mu_0=0$
$\mu_1=3$
$\mu_2=2$
$\mu_3=5$

$$\therefore P_2(t)=5\left[\frac{1}{3}e^{-2t}-\frac{1}{3}e^{-5t}\right] \qquad (\mathrm{II})$$

For $n = 1$

② →

$$P_1(t) = \mu_2 \mu_3 \left[A_{1,1} e^{-\mu_1 t} + A_{2,1} e^{-\mu_2 t} + A_{3,1} e^{-\mu_3 t} \right]$$



$\mu_0 = 0$

$\mu_1 = 3$

$\mu_2 = 2$

$\mu_3 = 5$

$$A_{1,1} = \frac{1}{(\mu_3 - \mu_1)(\mu_2 - \mu_1)}$$

$$= \frac{1}{(5-3)(2-3)}$$

$$A_{1,1} = \boxed{\frac{-1}{2}}$$

$$, A_{2,1} = \frac{1}{(\mu_3 - \mu_2)(\mu_1 - \mu_2)}$$

$$A_{2,1} = \frac{1}{(5-2)(3-2)} = \boxed{\frac{1}{3}}$$

and

$$A_{3,1} = \frac{1}{(\mu_1 - \mu_3)(\mu_2 - \mu_3)}$$

$$= \frac{1}{(3-5)(2-5)} = \boxed{\frac{1}{6}}$$

$$\therefore P_1(t) = 2(5) \left[\frac{-1}{2} e^{-3t} + \frac{1}{3} e^{-2t} + \frac{1}{6} e^{-5t} \right]$$

$$\therefore P_1(t) = 10 \left[\frac{1}{6} e^{-5t} + \frac{1}{3} e^{-2t} - \frac{1}{2} e^{-3t} \right] \quad (\text{III})$$

$$\therefore \; P_0(t) = 1 - \left[P_1(t) + P_2(t) + P_3(t) \right] \qquad \text{V}$$

By Substitute (I), (II), (III) in V

$$P_0(t) = 1 - \left[\left(1 \cancel{- \tfrac{5}{3} + \tfrac{10}{6}}\right) e^{-5t} + \underbrace{\left(\tfrac{5}{3} + \tfrac{10}{3}\right)}_{5} e^{-2t} - \underbrace{\tfrac{10}{2}}_{5} e^{-3t} \right]$$

$$\therefore \; P_0(t) = 1 + 5e^{-3t} - 5e^{-2t} - e^{-5t}$$

$$3 \xrightarrow{\mu_3} 2 \xrightarrow{\mu_2} 1 \xrightarrow{\mu_1} 0$$

OR You can verify that:

(2) $\Rightarrow$

$$P_0(t) = \mu_1 \mu_2 \mu_3 \left[A_{0,0}\, e^{-\mu_0 t} + A_{1,0}\, e^{-\mu_1 t} + A_{2,0}\, e^{-\mu_2 t} + A_{3,0}\, e^{-\mu_3 t} \right] \qquad \text{VI}$$

where

$$A_{0,0} = \frac{1}{(\mu_3 - \mu_0)(\mu_2 - \mu_0)(\mu_1 - \mu_0)} = \boxed{\frac{1}{30}}$$

$$A_{1,0} = \frac{1}{(\mu_3 - \mu_1)(\mu_2 - \mu_1)(\mu_0 - \mu_1)} = \boxed{\frac{1}{6}}$$

$$A_{2,0} = \frac{1}{(\mu_3 - \mu_2)(\mu_1 - \mu_2)(\mu_0 - \mu_2)} = \boxed{-\frac{1}{6}}$$

$$A_{3,0} = \frac{1}{(\mu_2 - \mu_3)(\mu_1 - \mu_3)(\mu_0 - \mu_3)} = \boxed{-\frac{1}{30}}$$

$$\therefore \; P_0(t) = 1 + 5e^{-3t} - 5e^{-2t} - e^{-5t} \quad \#$$