# Discrete Mathematics and Its Applications

## Rules of Inference

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# Outline

1. Valid Arguments in Propositional Logic
2. Argument Form.
3. Rules of Inference for Propositional Logic
4. Examples

# Outline

# Valid Arguments in Propositional Logic

**Definition**

1. An argument in propositional logic is a sequence of propositions. All but the final proposition in the argument are called premises and the final proposition is called the conclusion.
2. An argument is valid if the truth of all its premises implies that the conclusion is true.
3. An argument form in propositional logic is a sequence of compound propositions involving propositional variables.
4. An argument form is valid if no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.
5. The conclusion is true if the premises are all true

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# Argument Form

**Example**

$(p \to q) \wedge p \longrightarrow q$, or

$$\begin{cases} p \to q \\ p \\ \therefore q \end{cases}$$

1. The validity of an argument follows from the validity of the form of the argument.
2. When both $p \to q$ and $p$ are true, then $q$ must also be true.
3. We say this form of argument is true (i.e the conclusion must also be true) whenever all its premises (all statements in the argument other than the final one, the conclusion) are true,
4. If one of the premises is false, we cannot conclude that the conclusion is true.

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**Example**

Either team A or Team B will win the match
Team B lost
Therefore Team A won

The general form of this argument is:
Either P or Q
Not P
Therefore Q
$(P \vee Q) \wedge \neg P \longrightarrow Q$, or

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# Rules of Inference for Propositional Logic

We can always use a truth table to show that an argument form is valid. We do this by showing that whenever the premises are true, the conclusion must also be true.

**Example**

Determine whether the following argument is valid or invalid (Hint: use the truthe table)

$$\begin{cases} p \to q \\ p \\ \therefore q \end{cases}$$

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**Example**

Determine whether the following argument is valid or invalid (Hint: use the truthe table)

$$\begin{cases} p \to q \\ p \\ \therefore q \end{cases}$$

1. We make all premises true: $p \to q = T, P = T$
2. See in the table where is $p = T$ and $p \to q = T$. You see it in the raw 1 so this is the critical row.
3. Now see what about the conclusion? The conclusion is true, Therefore, the argument is valid.

**TABLE 5** The Truth Table for the Conditional Statement $p \to q$.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

4.jpg

**Example**

Determine whether the following argument is valid or invalid (Hint: use informal method)

$$\begin{cases} p \to q \\ \neg p \\ \therefore \neg q \end{cases}$$

**Solution**

put all premises true:

$p \rightarrow q = T$

$\neg p = T$, then $p = F$

Now, put $q = T$. So, we still have $p \rightarrow q = T$, but $q = T$, Then $\neg q = F$.

The conclusion is False whereas all premises are true, Therefor, the argument is invalid.

**Example**

Determine whether the following argument is valid or invalid (Hint: use informal method)

$$\begin{cases} p \to q \\ q \to (p \to r) \\ p \\ \therefore r \end{cases}$$

# Examples

**Solution**

put all premises true: $(p \to q) = T$
$q \to (p \to r) = T$
$p = T$
......................................
Since $p = T$ and $(p \to q) = T$, then $q = T$
Now, since $q = T$ and $q \to (p \to r) = T$ then $Q := (p \to r) = T$
We have $p = T$ and $(p \to r) = T$ then $r$ must be true. Therefore $r = T$ and the argument is valid.

# Examples

## Example

Determine whether the following argument is valid or invalid (Hint: use informal method)

$$\begin{cases} p \to \neg r \\ r \to (p \to q) \\ r \to p \\ \therefore \neg p \to \neg q \end{cases}$$