ch 4 تابع

The long run behavior of Markov chain

Sec 4.2 p.178

states (S,S), (S,C), (C,S), (C,C)

S → Sunny , C → cloudy

$(S,S) \xrightarrow{0.8} (S,S)$

$(C,S) \xrightarrow{0.6} (S,S)$

$(S,C) \xrightarrow{0.4} (C,S)$

$(C,C) \xrightarrow{0.1} (C,S)$

To state

From state (Y, Td)	Td Tm (S,S)	(S,C)	(C,S)	(C,C)
(S,S)	0.8	0.2	0	0
(S,C)	0	0	0.4	0.6
(C,S)	0.6	0.4	0	0
(C,C)	0	0	0.1	0.9
	↓ π_0	↓ π_1	↓ π_2	↓ π_3

* For long run the limitting dist^n

is $\pi = (\pi_0, \pi_1, \pi_2, \pi_3)$

$$\pi_j = \sum_{k=0}^{3} \pi_k P_{kj} \quad , j = 0,1,2,3$$

at $j=0 \Rightarrow \pi_0 = 0.8\pi_0 + 0.6\pi_2$

$0.2\pi_0 = 0.6\pi_2 \quad \therefore \boxed{\pi_2 = \frac{1}{3}\pi_0}$ ①

at $j=1 \Rightarrow \pi_1 = 0.2\pi_0 + 0.4\pi_2$

① $\Rightarrow \pi_1 = \frac{1}{5}\pi_0 + \frac{2}{5}\left(\frac{1}{3}\pi_0\right) = \frac{5}{15}\pi_0$

$\boxed{\pi_1 = \frac{1}{3}\pi_0}$ ②

at $j=2 \Rightarrow \pi_2 = 0.4\pi_1 + 0.1\pi_3 \quad (\times 10)$

$\boxed{10\pi_2 = 4\pi_1 + \pi_3}$

$\therefore \pi_3 = 10\pi_2 - 4\pi_1$ ③

2. Substitute (1) and (2) in (3)

$$\pi_3 = 10\left(\frac{1}{3}\pi_0\right) - 4\left(\frac{1}{3}\pi_0\right)$$

$$\boxed{\pi_3 = 2\pi_0} \quad (4)$$

$\because \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \quad (5)$

Subs. (1), (2), (3) and (4) in (5) we get,

$$\pi_0 + \frac{1}{3}\pi_0 + \frac{1}{3}\pi_0 + 2\pi_0 = 1$$

$$\frac{11}{3}\pi_0 = 1 \quad \therefore \boxed{\pi_0 = \frac{3}{11}}$$

(1), (2), (4)

$$\Rightarrow \pi_1 = \pi_2 = \frac{1}{3}\left(\frac{3}{11}\right) = \frac{1}{11}, \quad \pi_3 = \frac{6}{11}$$

$\therefore$ For long run, the limitting dist$\underline{n}$ is

$$\pi = \left(\underbrace{\frac{3}{11}}_{\pi(S,S)}, \underbrace{\frac{1}{11}}_{\pi(S,C)}, \underbrace{\frac{1}{11}}_{\pi(C,S)}, \underbrace{\frac{6}{11}}_{\pi(C,C)}\right)$$

- What is the long run fraction of days in which it sunny

$$\pi(S,S) + \pi(S,C) = \pi_0 + \pi_1 = \frac{3}{11} + \frac{1}{11} = \frac{4}{11}$$

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