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Lecture 20

one dimensional random walker

P. 116

The transition M_X of a random walk has the form

| | | | | | | | | | |
|----------|-------|---------|-------|-----|----------|----------|----------|-----|----------|
| | 0 | 1 | 2 | ... | $i-1$ | i | $i+1$ | ... | N |
| 0 | b | β | 0 | ... | 0 | ... | | | \vdots |
| 1 | q_1 | r_1 | p_1 | ... | 0 | ... | | | \vdots |
| 2 | 0 | q | r | ... | 0 | ... | | | \vdots |
| \vdots | | | | | \vdots | \vdots | \vdots | | \vdots |
| i | | | | | q_i | r_i | p_i | | \vdots |
| \vdots | | | | | \vdots | \vdots | \vdots | | \vdots |
| N | 0 | 0 | 0 | ... | 0 | 0 | 0 | ... | 1 |

where $p_i \geq 0, q_i \geq 0, r_i \geq 0$

and $q_i + r_i + p_i = 1, i = 1, 2, \dots, N$

$$p_i = \text{pr} \{ X_{n+1} = i+1 \mid X_n = i \}$$

forward transition
تحوّل للأمام

$$q_i = \text{pr} \{ X_{n+1} = i-1 \mid X_n = i \}$$

backward transition
تحوّل للخلف

$$r_i = \text{pr} \{ X_{n+1} = i \mid X_n = i \}$$

the same state transition
نفس التحوّل



Defn: Random walk

The process of a random walk describes the path of a person moving randomly on step forward or step backward. Also, it describes Gambler's ruin, and Fluctuation of insurance Company assets over time and Modelling some physical processes.

تدوير المقامر
تقلب أصول شركة التأمين مع الزمن
وغيره من الامثلة الفيزيائية

Gambler's ruin

player A

Starting

with fortune \$i

فيرة / مبلغ / اموال

* Tossing a coin

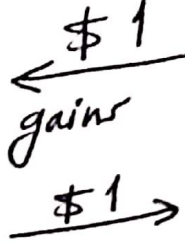
H head
It means, player A gains
T tail

player B

\$ N - i

, N is the total amount

bit
رهان



It means, player B gains

احتمال نجاح / فشل
السبب A في مباراة الواحدة
(المسابقة الواحدة)
one trial (Contest)



$q = 1 - p$

احتمال فشل / نجاح السبب B
في المباراة الواحدة

State 0 ستمر حظه، للملحة الى انه سيل والسبب A الى State 0
السؤال هو

what's the prob. that player A loses his money?

State N انه ستمر السبب A الى State N
what's the prob. that player A wins all money?

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Defn

$$u_i = \Pr \{ X_n \text{ reaches state 0 before state } N \mid X_0 = i \}$$

$$= \begin{cases} \frac{N-i}{N}, & p=q=1/2 \\ \frac{(q/p)^i - (q/p)^N}{1 - (q/p)^N}, & p \neq q \end{cases}$$

be player A's ruin probabilities where X_n is player A's fortune at stage n

حالات تدمير اللاعب A قبل أن يذهب إلى 0

Pb 3.5.1 p.121 $i = \$5$ fortune for player A

Total amount $\rightarrow N = \$5 + \$10 = \$15$

$$p = 0.4929 \Rightarrow q = 1 - p = 1 - 0.4929$$

$$q = 0.5071$$

$$u_i = \Pr \{ X_n \text{ reaches state 0 before state } N \mid X_0 = i \}$$

حالات التدمير قبل الوصول إلى 15

$$u_i = \frac{(q/p)^i - (q/p)^N}{1 - (q/p)^N}, \quad p \neq q$$

$$u_i = \frac{\left[\left(\frac{0.5071}{0.4929} \right)^5 - \left(\frac{0.5071}{0.4929} \right)^{15} \right]}{\left[1 - \left(\frac{0.5071}{0.4929} \right)^{15} \right]}$$

$$\therefore u_i = 0.71273 \quad \#$$