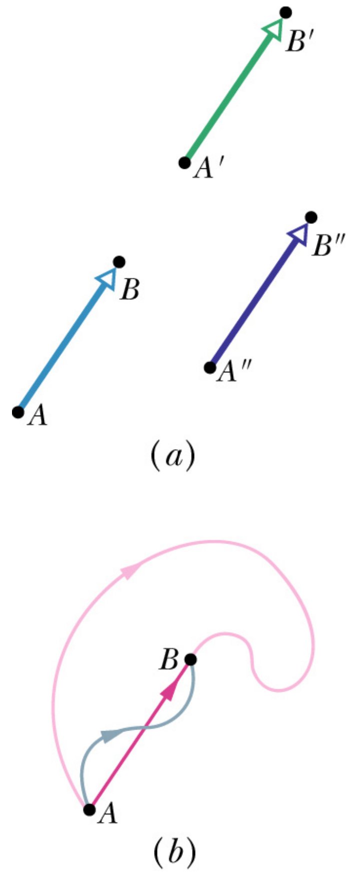
VECTORS

- Physical quantities are classified in two big classes: **vectors & scalars**.
- A **vector** is a physical quantity which is completely defined once we know precisely its **direction** and **magnitude** (for example: force, velocity, displacement)
- A **scalar** is a physical quantity which is completely defined once we know precisely **only** its **magnitude** (for example: speed, mass, density)

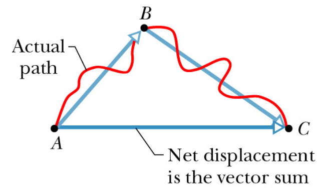
DISPLACEMENT

- Vectors are displayed by **arrows** having a "tip" and a "tail". In textbooks a vector quantity is normally denoted by bold letters (ex. **AB** or **A)**
- The simplest vector in nature is displacement. In figure (b) we see vector **AB** representing displacement from position A to B.
- A vector can be shifted without changing its value if its magnitude and direction are not changed.

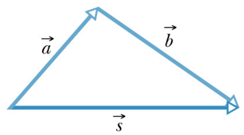


ADDING VECTORS GEOMETRICALLY

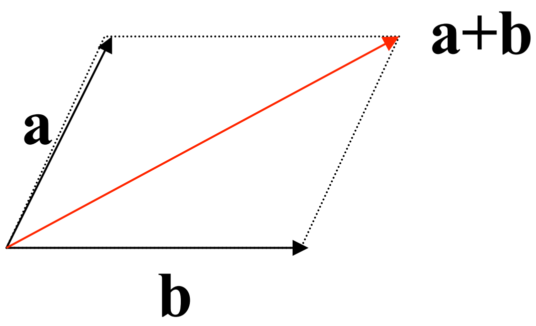
- The concept of displacement helps us very much to understand vector addition. There are **two** ways of vector addition:
- A) By putting the two vectors successively as in fig. (a), (b). In this case the resultant is the vector that runs from the tail of **a** to the tip of **b**
- B) By putting them with common tail, as in the lower figure.
- Both ways are equivalent.



(a)

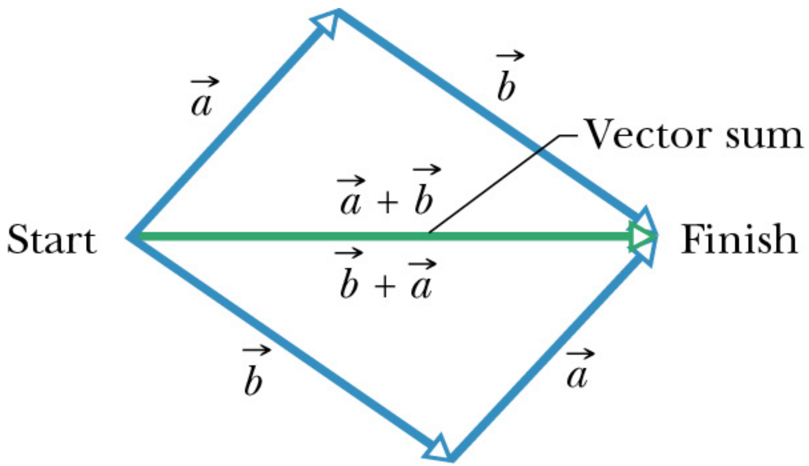


(b)



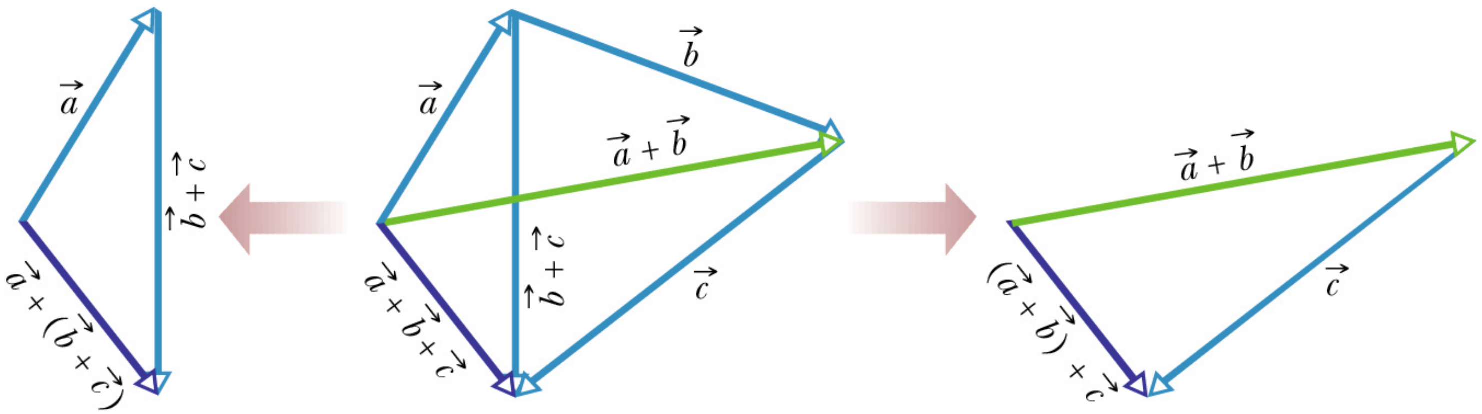
PROPERTIES OF ADDITION OF VECTORS-I

- The order of addition does not matter. Addition of vectors is **commutative**
- **a+b=b+a**



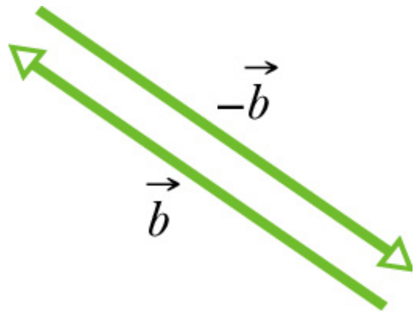
PROPERTIES OF ADDITION OF VECTORS-II

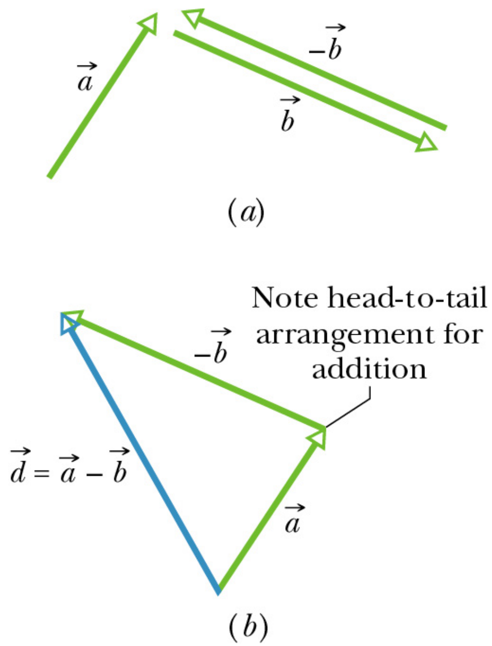
- When there are more than two vectors, we can group them in any order as we add them. The addition of vectors is **associative**:
    - **(a+b)+c = a+(b+c)**



SUBTRACTION OF VECTORS

- The vector –**b** is the vector with the same magnitude as **b** but opposite direction. Adding these two vectors would yield:
  - **b+(-b)=0**
- Vector **0** is a vector of zero magnitude called **null vector**.
- Thus subtraction is actually the addition of –**b**:
  - **a-b=a+(-b)**

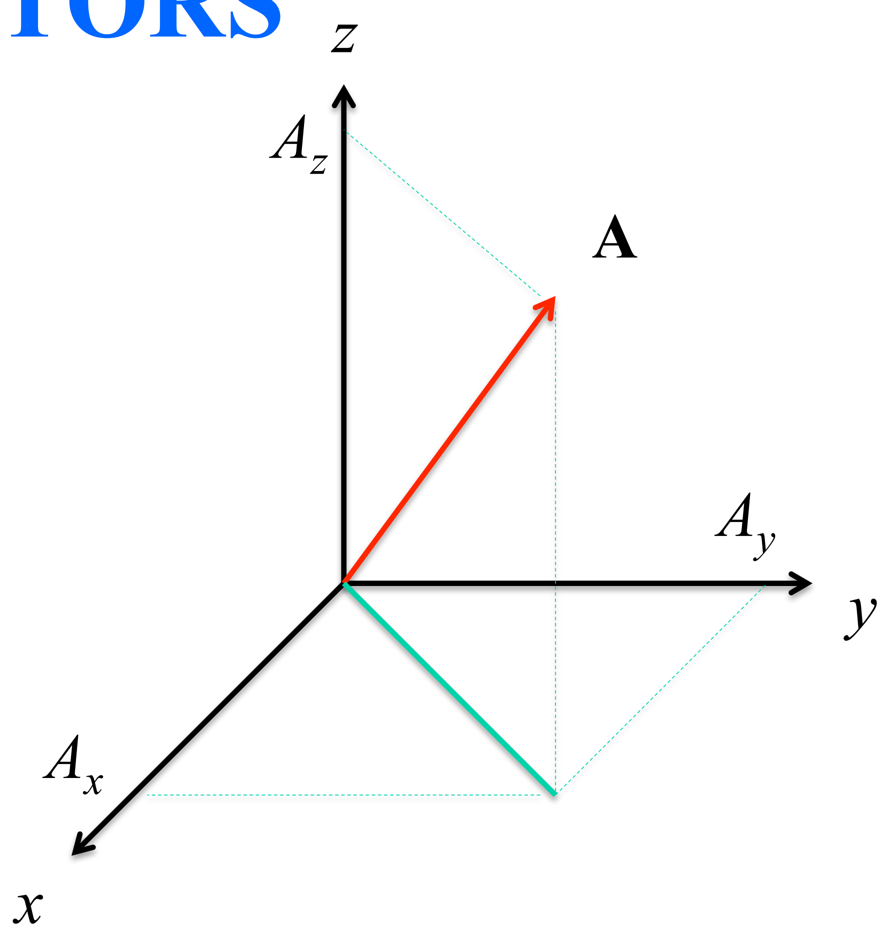




ALGEBRAIC DESCRIPTION OF VECTORS

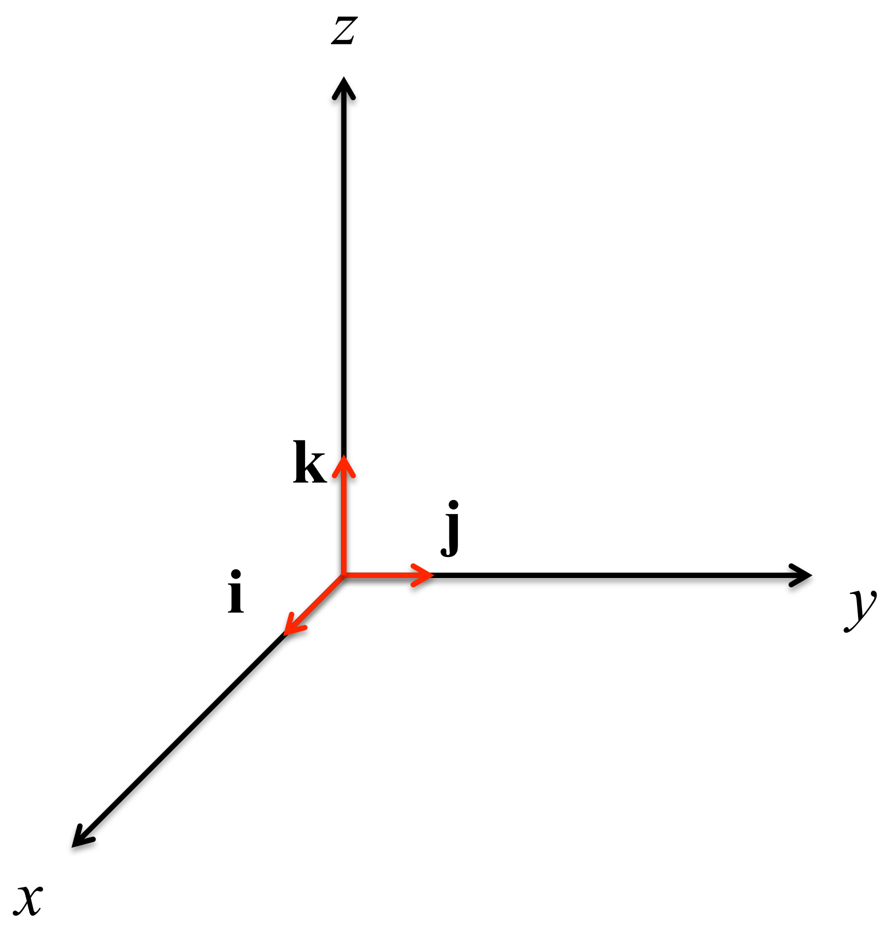
- A **component** of a vector is the projection of the vector on an axis. To find the projection we draw perpendicular lines from the two ends of the vector to the axis. The process of finding the components of a vector is called **resolving the vector**. Using geometry it is easy to see that:

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



UNIT VECTORS

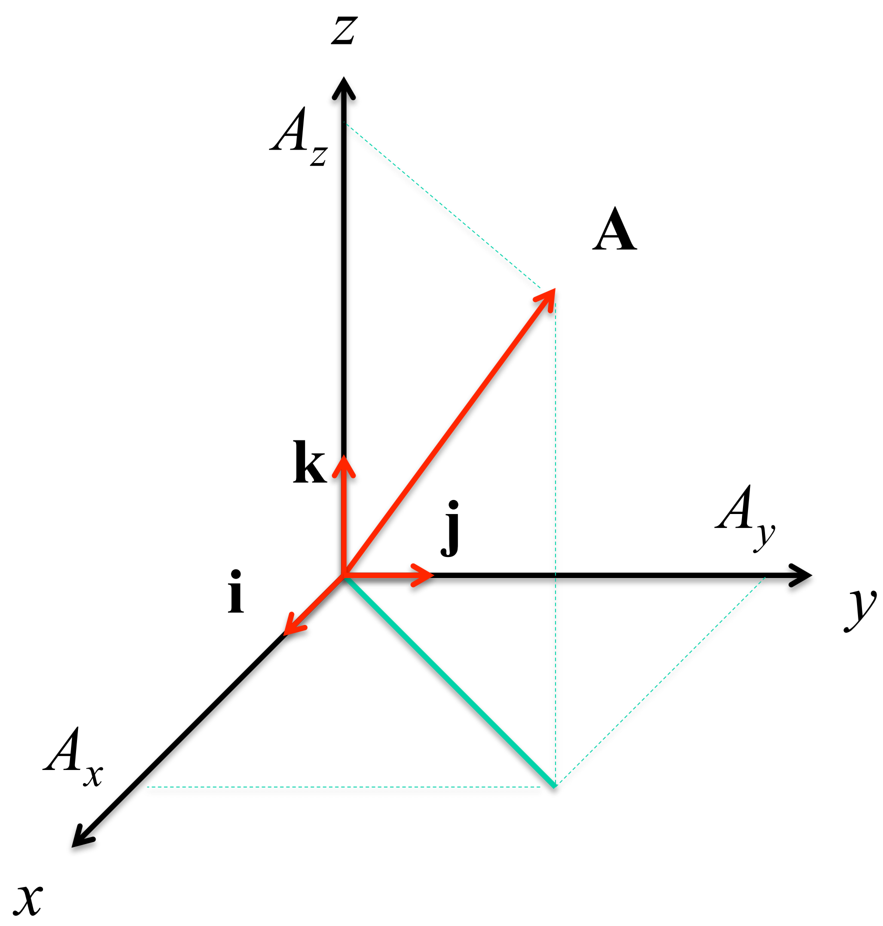
- A **unit** vector is a vector that has magnitude 1 and points in a particular direction. It lacks both dimension and unit.
- The arrangement shown in figure is called **right handed coordinate system**.



UNIT VECTORS

- With the help of unit vectors the vector **A** can be written as:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

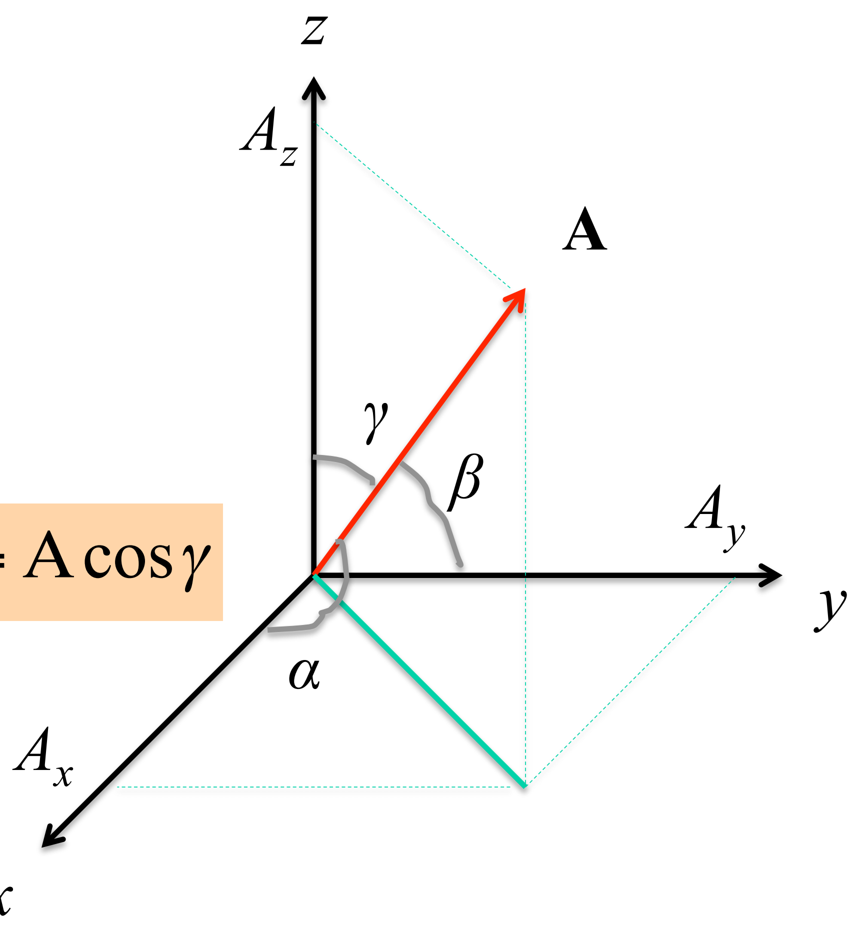


DIRECTION COSINES

- Sometimes a vector is specified by its magnitude and by the angles it makes with the axes. With the help of unit vectors the vector **A** can be written as:

$$A_x = \mathrm{A}\cos\alpha,\ \ A_y = \mathrm{A}\cos\beta,\ \ A_x = \mathrm{A}\cos\gamma$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$



THE SCALAR PRODUCT OF TWO VECTORS

- The **scalar** (or **dot**) **product** of two vectors is defined by the following relation:

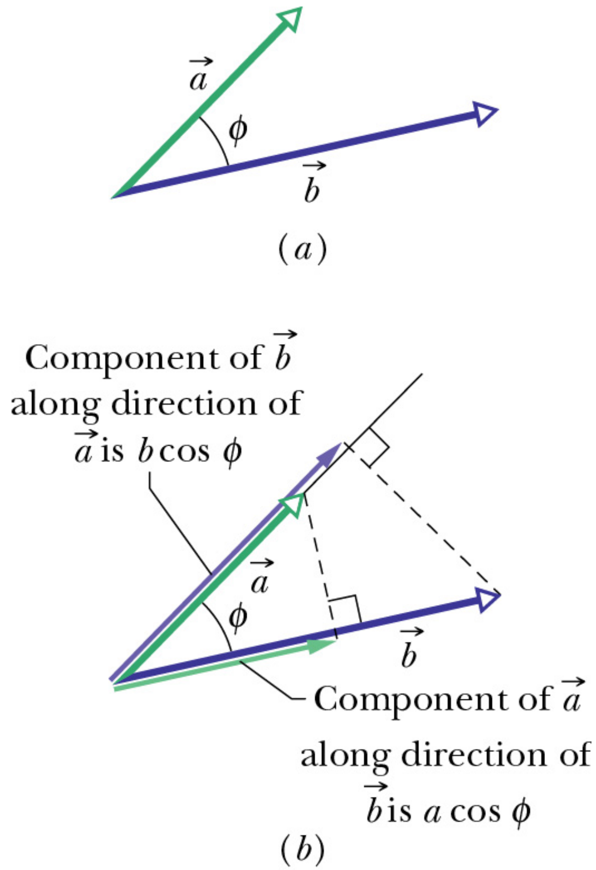
$$\vec{a} \cdot \vec{b} = ab\cos\varphi$$

- In unit-vector rotation it is defined as follows:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

- The scalar product of two vectors is commutative:

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$



MATRIX REPRESENTATION OF VECTORS-a

- A vector **A** can be represented by a single column matrix **a** whose elements are the components of **A**.

$$\mathbf{A} \Rightarrow \mathbf{a} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

- The rows of **a** are the coefficients of the individual members of the **basis** used to represent **A**, so the element A_i is associated with the corresponding basis unit vector.

$$\mathbf{G} = \mathbf{A} - 2\mathbf{B} \Rightarrow \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} - 2\begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} A_1 - 2B_1 \\ A_2 - 2B_2 \\ A_3 - 2B_3 \end{pmatrix}$$

- The vector operations of addition and multiplication by a scalar correspond exactly to the operations of the same names applied to the single-column matrices representing vectors.

MATRIX REPRESENTATION OF VECTORS-b

- The transpose of the matrix representing a vector **A** is a single-row matrix, called a **row vector**.

$$\mathbf{a}^T = \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix}$$

- The scalar product of two vectors **A** and **B** can be evaluated with the help of the transpose matrix.

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{a}^T \mathbf{b} =$$

$$\begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} = A_1 B_1 + A_2 B_2 + A_3 B_3$$

THE VECTOR PRODUCT OF TWO VECTORS

- The **vector** (or **cross**) **product** of two vectors is written as:

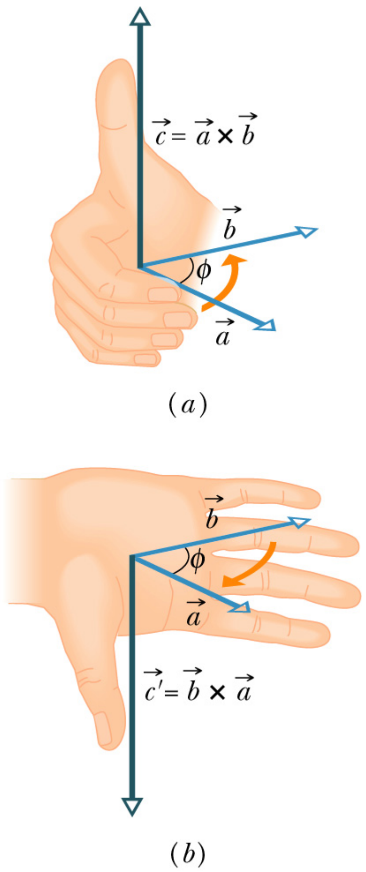
$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

- The magnitude of this vector is given by:

$$C = AB\sin\phi$$

- In unit-vector notation the vector product is given by:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$



THE VECTOR PRODUCT OF TWO VECTORS

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} A_2 & A_3 \\ B_2 & B_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} A_1 & A_3 \\ B_1 & B_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} A_1 & A_2 \\ B_1 & B_2 \end{vmatrix}$$

$$\mathbf{i}\left(A_2B_3 - A_3B_2\right) - \mathbf{j}\left(A_1B_3 - A_3B_1\right) + \mathbf{k}\left(A_1B_2 - A_2B_1\right)$$

$$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$$

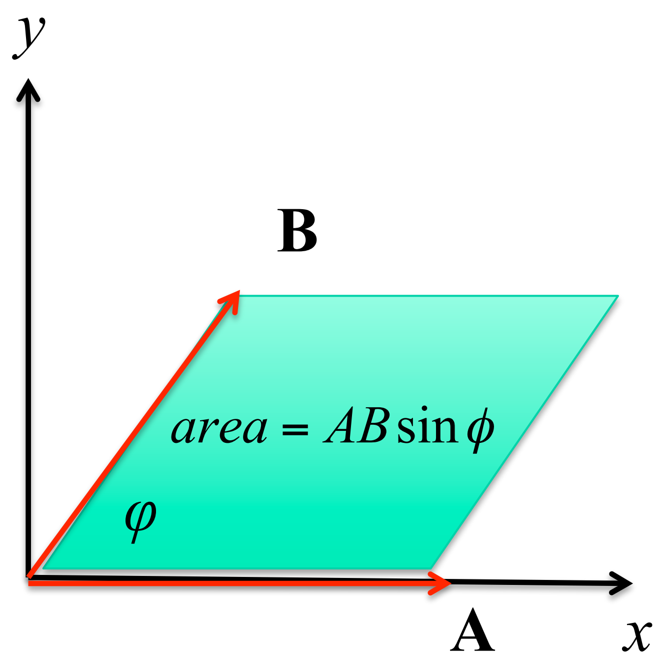
$$\mathbf{A}\times\left(\mathbf{B}+\mathbf{C}\right) = \mathbf{A}\times\mathbf{B} + \mathbf{A}\times\mathbf{C}$$

$$k\left(\mathbf{A}\times\mathbf{B}\right) = \left(k\mathbf{A}\right)\times\mathbf{B}$$

THE VECTOR PRODUCT OF TWO VECTORS

- The magnitude of the cross product of two vectors is equal to the area of the parallelogram formed by the two vectors.

$$C = AB\sin\phi$$

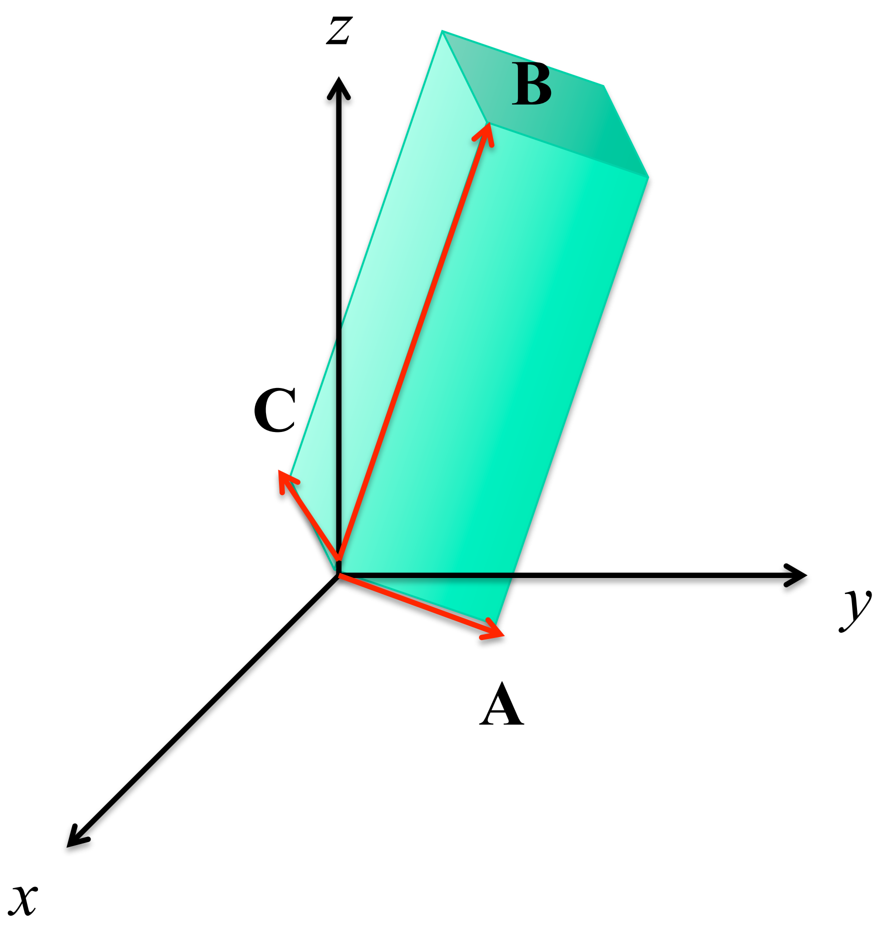


THE SCALAR TRIPLE PRODUCT

- The scalar triple product of three vectors is the quantity:

$$\mathbf{A}\cdot\left(\mathbf{B}\times\mathbf{C}\right)=\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

- The scalar triple product represents the volume of the parallelepiped defined by the three vectors.



THE VECTOR TRIPLE PRODUCT

- The vector triple product of three vectors is the quantity:

$$\mathbf{A}\times\left(\mathbf{B}\times\mathbf{C}\right)=\mathbf{B}(\mathbf{A}\cdot\mathbf{C})-\mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

- This is known as BAC-CAB rule.