1

Lecture (19) • Applications on Absorbing Markov chain

Ex. 3.4.1 p.105

$$P = \begin{array}{c} \\ \text{Non abs} \begin{cases} 0 \\ 1 \\ 2 \end{cases} \\ \text{abs} \to 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

trans. probs Mx

Ans:

$$v_i = E\left[T \mid X_0 = i\right]$$

$i = 0, 1, 2$

Mean time to absorption

v_{03} ??

$$\Rightarrow v_0 = 1 + P_{00} v_0 + P_{01} v_1 + P_{02} v_2$$

$$v_1 = 1 + \cancel{P_{10} v_0} + P_{11} v_1 + P_{12} v_2$$

$$v_2 = 1 + \cancel{P_{20} v_0} + \cancel{P_{21} v_1} + P_{22} v_2$$

$$\Rightarrow v_0 = 1 + 0.4 v_0 + 0.3 v_1 + 0.2 v_2 \quad (1)$$

$$v_1 = 1 + 0.7 v_1 + 0.2 v_2 \quad (2)$$

$$v_2 = 1 + 0.9 v_2 \quad (3)$$

$$(3) \Rightarrow \boxed{v_2} = \frac{1}{0.1} = 10$$

$$(2) \Rightarrow v_1 = 1 + 0.7 v_1 + 2$$

$$\therefore 0.3 v_1 = 3$$

$$\therefore \boxed{v_1} = \frac{3}{0.3} = 10$$

$\therefore$ By subs. values of v_1, v_2 in (1), we get

$$v_0 = 1 + 0.4 v_0 + 3 + 2$$

$$0.6 v_0 = 6$$

$$\therefore \boxed{v_0 = 10} \Rightarrow \boxed{v_0 = v_{03} = 10}$$

المتوسط الزمني للامتصاص (زمن الانتقال من حالة 0
إلى الحالة الماصة 3)

Mean time to absorption

2

Sec 3.5 p 111 → p.113

* Some Special Markov chains

Consider the two-state Markov chain

where
$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \end{array} \quad 0<a,b<1$$

is the transition $M\underline{X}$ of two stat Markov chain

$\Rightarrow$ the n-step transition $M\underline{x}$ is given by

$$P^n = \frac{1}{a+b}\begin{bmatrix} b & a \\ b & a \end{bmatrix} + \frac{(1-a-b)^n}{a+b}\begin{bmatrix} a & -a \\ -b & b \end{bmatrix}$$

For long run الأمد الطويل

$\because (1-a-b)^n \to 0$ as $n \to \infty$, $|1-a-b| < 1$

$$\therefore \lim_{n\to\infty} P^n = \frac{1}{a+b}\begin{bmatrix} b & a \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} b/(a+b) & a/(a+b) \\ b/(a+b) & a/(a+b) \end{bmatrix}$$

Note

$\lim_{n\to\infty} \frac{1}{x^n} = 0$

$\lim_{n\to\infty} x^n = \infty$, $x>0$

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EX (p. 113)

Given X_n denotes the quality of an item

2 states: $X_n \to$ 0 good, 1 defective

$\{X_n\}$ is a Markov chain with transition Mx

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix} \end{array}$$

a = 0.01, b = 0.12

* for long run

$$\lim_{n \to \infty} P_n = \begin{bmatrix} \frac{b}{a+b} & \frac{a}{a+b} \\ \frac{b}{a+b} & \frac{a}{a+b} \end{bmatrix}$$

$$= \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ & \begin{bmatrix} \frac{12}{13} & \frac{1}{13} \\ \frac{12}{13} & \frac{1}{13} \end{bmatrix} \end{array}$$

Thus, for long run, the prob. of an item will be defective is $1/13 \simeq 0.08$ and the prob. that an items is good is 0.92

$\frac{12}{13} \simeq 0.92$