

L Lecture (19) • Applications on Absorbing Markov chain

Pr 34.1 p.105

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 0.7 & 0.2 & 0.1 \\ 0 & 0 & 0.9 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

Non-abs.
 abs. → 3

trans. probs P_{ij}

Ans:

$$v_i = E [T | X_0 = i]$$

$i = 0, 1, 2$

Mean time to absorption

v_{03} ??

$$\Rightarrow v_0 = 1 + p_{00}v_0 + p_{01}v_1 + p_{02}v_2$$

$$v_1 = 1 + p_{10}v_0 + p_{11}v_1 + p_{12}v_2$$

$$v_2 = 1 + p_{20}v_0 + p_{21}v_1 + p_{22}v_2$$

$$\Rightarrow v_0 = 1 + 0.4v_0 + 0.3v_1 + 0.2v_2 \quad (1)$$

$$v_1 = 1 + 0.7v_1 + 0.2v_2 \quad (2)$$

$$v_2 = 1 + 0.9v_2 \quad (3)$$

$$(3) \Rightarrow v_2 = \frac{1}{0.1} = 10$$

$$(2) \Rightarrow v_1 = 1 + 0.7v_1 + 2$$

$$\therefore 0.3v_1 = 3$$

$$\therefore v_1 = \frac{3}{0.3} = 10$$

∴ By subst. values of v_1, v_2 in (1), we get

$$v_0 = 1 + 0.4v_0 + 3 + 2$$

$$0.6v_0 = 6$$

$$\therefore v_0 = 10 \Rightarrow v_0 = v_{03} = 10$$

المتوسط لزمن الانتظار (زمن الانتقال) من الحالة 0 الى الحالة 3 الخاصة (3)

Mean time to absorption

Sec 3.5 p 111 → p. 113

* Some special Markov chains

Consider the two-state Markov chain

where $P = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \end{matrix}$ $0 < a, b < 1$

is the transition M_X of two state Markov chain

⇒ the n -step transition M_X is given by

$$P^n = \frac{1}{a+b} \begin{bmatrix} b & a \\ b & a \end{bmatrix} + \frac{(1-a-b)^n}{a+b} \begin{bmatrix} a & -a \\ -b & b \end{bmatrix}$$

For long run ^{المرحلة}

∴ $(1-a-b)^n \rightarrow 0$ as $n \rightarrow \infty$, $|1-a-b| < 1$

$$\begin{aligned} \therefore \lim_{n \rightarrow \infty} P^n &= \frac{1}{a+b} \begin{bmatrix} b & a \\ b & a \end{bmatrix} \\ &= \begin{bmatrix} b/(a+b) & a/(a+b) \\ b/(a+b) & a/(a+b) \end{bmatrix} \end{aligned}$$

Note

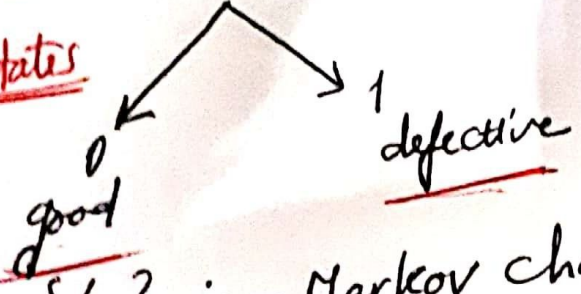
$\lim_{n \rightarrow \infty} \frac{1}{x^n} = 0$

$\lim_{x \rightarrow \infty} x^n = \infty$, $x > 0$

EX P.113

Given X_n denotes the quality of an item

2 states



$\{X_n\}$ is a Markov chain with transition M_X

$$P = \begin{matrix} & \begin{matrix} 0 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \end{matrix} & \begin{bmatrix} 0.99 & 0.01 \\ 0.12 & 0.88 \end{bmatrix} \end{matrix}$$

Red circles are drawn around the values 0.01, 0.12, and 0.88. Red arrows labeled 'a' and 'b' point to the 0.01 and 0.12 values respectively.

* for long run

$$\lim_{n \rightarrow \infty} P^n = \begin{bmatrix} \frac{b}{a+b} & \frac{a}{a+b} \\ \frac{b}{a+b} & \frac{a}{a+b} \end{bmatrix} = \begin{bmatrix} \frac{0}{13} & \frac{1}{13} \\ \frac{12}{13} & \frac{1}{13} \end{bmatrix}$$

The value 1/13 in the second row, second column is circled in red. A red arrow points downwards from this value.

Thus, for long run, the prob. of an item will be defective is $\frac{1}{13} \approx 0.08$ and the prob. that an item is good is 0.92

$$\frac{12}{13} \approx 0.92$$

