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Lecture 18

Absorbing Markov chain تاع

Case 1

Given states 0, 1 and 2

0 — absorbed, 1 — non-absorbed, 2 — absorbed

as before.

$$\Rightarrow P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ P_{10} & P_{11} & P_{12} \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

* starting Markov process in state 1

① Let $T = \min\{n \geq 0 : X_n = 0 \text{ or } X_n = 2\}$ be the time of absorption of the process

② The prob. that Markov chain ends in state 0 given that the Markov process starting in state 1 is

$T \to$ terminate

$$u = pr\{X_T = 0 \mid X_0 = 1\}$$

$$u = u_{10}$$

$$\boxed{u = P_{10} + P_{11}\, u}$$

③ the mean time to absorption المتوسط زمن الامتصاص is

$$v = E[T \mid X_0 = 1]$$

$$v = v_{10}$$

$$\boxed{v = 1 + P_{11}\, v}$$

2 pb 3.4.2 p.105

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} \text{abs} \to 0 \\ \text{non abs} \to 1 \\ \text{abs} \to 2 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

u_{10} ?

a) $u = pr\{X_T = 0 \mid X_0 = 1\}$

$u = P_{10} + P_{11} u$

$u = 0.1 + 0.6u$

$0.4u = 0.1 \Rightarrow \therefore \boxed{u = \frac{1}{4}}$

$\therefore u = u_{10} = \frac{1}{4}$

b) $v = E[T \mid X_0 = 1]$

$v = 1 + P_{11} v$

$v = 1 + 0.6v$

$0.4v = 1$ $(\times 10)$

$\therefore v = \frac{10}{4} = 2.5$ is the mean time of absorption

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Case 2:

* For more general case

$$u_i = pr\{X_T = 0 \mid X_0 = i\}, \quad i = 1, 2$$

$$\Rightarrow \boxed{\begin{aligned} u_1 &= P_{10} + P_{11} u_1 + P_{12} u_2 \\ u_2 &= P_{20} + P_{21} u_1 + P_{22} u_2 \end{aligned}}$$

where states 1 and 2 are non-absorbing

$u_1 = u_{10}$, $u_2 = u_{20}$

Also,

$$v_i = E[T \mid X_0 = i], \quad i = 1, 2$$

$$\Rightarrow \boxed{\begin{aligned} v_1 &= 1 + P_{11} v_1 + P_{12} v_2 \\ v_2 &= 1 + P_{21} v_1 + P_{22} v_2 \end{aligned}}$$

$v_1 = v_{10}$, $v_2 = v_{20}$

pb 3.4.6 [illegible]
p.106

pb 3.4.3 p 106

$$P \underset{\text{non-abs}}{=} \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 0 & 0 & 0 \\ 1 & 0.1 & 0.6 & 0.1 & 0.2 \\ 2 & 0.2 & 0.3 & 0.4 & 0.1 \\ 3 & 0 & 0 & 0 & 1 \end{array}$$

a) $u_1 = u_{10}$??

$$u_i = pr\{X_T = 0 \mid X_0 = i\}, \quad i = 1, 2$$

4:

$$\Longrightarrow u_1 = P_{10} + P_{11}u_1 + P_{12}u_2$$

احتمال الوصول الى الحالة 0
بشرط البداية كانت عند
$X_0=1$

$$u_2 = P_{20} + P_{21}u_1 + P_{22}u_2$$

احتمال الوصول الى الحالة 0
بشرط البداية كانت عند
$X_0=2$

$$\Longrightarrow \boxed{\begin{aligned} u_1 &= 0.1 + 0.6u_1 + 0.1u_2 \\ u_2 &= 0.2 + 0.3u_1 + 0.4u_2 \end{aligned}}$$

$$\Longrightarrow 0.4u_1 - 0.1u_2 = 0.1 \quad \times 10$$

$$0.6u_2 - 0.3u_1 = 0.2$$

$$0.3u_1 - 0.6u_2 = -0.2 \quad \times 10$$

$$\Longrightarrow 4u_1 - u_2 = 1 \quad (1)$$

$$3u_1 - 6u_2 = -2 \quad (2)$$

Solving (1) and (2)

$$(1)\times 6 - (2) \Longrightarrow 21u_1 = 8$$

$$\therefore u_1 = u_{10} = \frac{8}{21}$$

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b) $\nu_i = E[T \mid X_0 = i]$, $i = 1, 2$

is the mean time to absorption

$$\Rightarrow \begin{cases} \nu_1 = 1 + P_{11}\nu_1 + P_{12}\nu_2 \\ \nu_2 = 1 + P_{21}\nu_1 + P_{22}\nu_2 \end{cases}$$

$$\Rightarrow \begin{cases} \nu_1 = 1 + 0.6\nu_1 + 0.1\nu_2 \\ \nu_2 = 1 + 0.3\nu_1 + 0.4\nu_2 \end{cases}$$

ν_{10} ??

$$\Rightarrow \quad 0.4\nu_1 - 0.1\nu_2 = 1 \quad ①$$

$$0.3\nu_1 - 0.6\nu_2 = -1 \quad ②$$

$$① \times 6 - ② \Rightarrow 2.1\nu_1 = 7$$

$$\therefore \nu_1 = \frac{7}{2.1} = \frac{70}{21} = \frac{10}{3}$$

$$\therefore \nu_1 = \nu_0 = \frac{10}{3}$$

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