Lecture (14) Markov chains

sec 3.2 p. 83

Defn The n-step transition probs of a Markov chain can be defined as

$$P_{ij}^{n} = pr\{\underbrace{X_{m+n}=j}_{\text{المستقبل}} \mid \underbrace{X_m = i}_{\text{الحاضر}}\}$$

Theorem

$$P_{ij}^{n} = \sum_{k=0}^{\infty} \underbrace{P_{ik}}_{\text{أعمدة}} \; \underbrace{P_{kj}^{n-1}}_{\text{صفوف}}$$

where
$$P_{ij}^{0} = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

clearly, the n-step transition probabilities P_{ij}^{n} are the entries (عناصر) in the matrix P^n (n th power of P)

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$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} \end{array}$$

a) Compute the two-step transition Mx P^2

2

$$P^2 = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}$$

$$P^2 = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0.47 & 0.13 & 0.40 \\ 0.42 & 0.14 & 0.44 \\ 0.26 & 0.17 & 0.57 \end{bmatrix} \end{array}$$

b) what's the prob. $\{X_3 = 1 \mid X_1 = 0\}$

$$pr\{X_3 = 1 \mid X_1 = 0\} = P^2_{01} = 0.13$$

c) what's the prob. $\{X_3 = 1 \mid X_0 = 0\}$

$$pr\{X_3 = 1 \mid X_0 = 0\} = P^3_{01}$$

$$\boxed{P^3 = P \cdot P^2}$$

$$P^3 = \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.47 & 0.13 & 0.40 \\ 0.42 & 0.14 & 0.44 \\ 0.26 & 0.17 & 0.57 \end{bmatrix} \end{array}$$

$$P^3 = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0.313 & 0.16 & 0.527 \\ 0.334 & 0.156 & 0.51 \\ 0.402 & 0.143 & 0.455 \end{bmatrix} \end{array}$$

$$\therefore P^3_{01} = 0.16$$

OR

$$P^3_{01} = \begin{bmatrix} 0.1 & 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} 0.13 \\ 0.14 \\ 0.17 \end{bmatrix} = 0.16$$